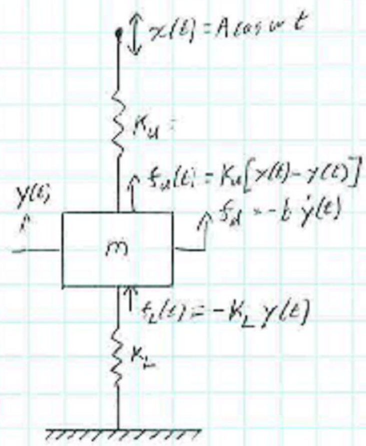
Appendix B: Linearized Analysis

FREQ DOMAIN RESPONSE OF MASS
BETWEEN TWO SPRINGS



$$x(t) = Re[X e^{i\omega t}] = X^R \cos\omega t + X^I \sin\omega t$$

$$\hookrightarrow X = X^R + iX^I \quad : \quad X^R = A,\ X^I = 0$$

$$y(t) = Re\{Y e^{i\omega t}\} = Y^R \cos\omega t + Y^I \sin\omega t$$

$$\dot{y}(t) = Re[i\omega Y e^{i\omega t}] \qquad \ddot{y}(t) = Re[-\omega^2 Y e^{i\omega t}]$$

$$f_u(t) = K_u[x(t) - y(t)] = Re[F_u e^{i\omega t}] \qquad F_u = K_u[X - Y]$$

$$f_L(t) = -K_L y(t) = Re[F_L e^{i\omega t}] \qquad F_L = -K_L Y$$

$$f_d(t) = -b\,\dot{y}(t) = Re[F_D e^{i\omega t}] \qquad F_D = -i\omega b Y$$

$$m\ddot{y}(t) = F_u(t) + F_L(t) + F_d(t)$$

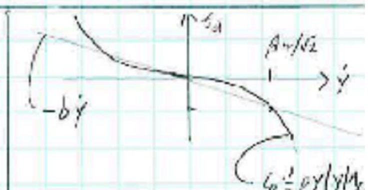
$$Re[-m\omega^2 Y e^{i\omega t}] = Re[(K_u(X - Y) - K_L Y - i\omega b Y)e^{i\omega t}]$$

$$0 = m\omega^2 Y + K_u X - (K_u + K_L)Y - i\omega b Y$$

$$-K_u X = [m\omega^2 - (K_u + K_L) - i\omega b]\,Y$$

$$Y = \frac{-K_u X}{(m\omega^2 - (K_u + K_L) - i\omega b)}$$

$$RAO = \left|\frac{Y}{A}\right| = \left|\frac{-K_u}{m\omega^2 - (K_u + K_L) - i\omega b}\right| = \frac{K_u}{\left[(m\omega^2 - (K_u + K_L))^2 + \omega^2 b^2\right]^{1/2}}$$



$$b = \frac{C_D \frac{1}{2}\rho A_c \left[\frac{A\omega}{\sqrt{2}}\right]^2}{A\omega/\sqrt{2}}$$

$$b = C_D \frac{1}{2}\rho A_c \frac{A\omega}{\sqrt{2}}$$

$\hookrightarrow$ LINEARIZED RELATIVE TO RMS EXCITATION VEL.

UNDAMPED RESONANCE PT:

$$\omega_R = \sqrt{\frac{(K_u + K_L)}{m}}$$

DAMPED RESONANCE PT:

$$\omega_R = \sqrt{\frac{(K_u + K_L)}{m} - \frac{b^2}{2m^2}}$$

WHEN SUFFICIENTLY UNDER DAMPED, IE $\zeta = \frac{b}{2\sqrt{m(K_u + K_L)}} < 1/\sqrt{2}$

Appendix C: Component Characterizations