Kinematics Study and Implementation of a Biomimetic Robotic-Fish Underwater Vehicle based on Lighthill Slender Body Model

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***Abstract*—Sir J. Lighthill mathematical slender body swimming model formulates the biological fish propulsion mechanism (undulation) in fluid environment. The present research has focused on the relevance of Lighthill (LH) based biomimetic robotic propulsion. The objective of this paper is to mimic the propulsion mechanism of the BCF mode carangiform swimming style to show the fish behavior navigating efficiently over large distances at impressive speeds and its exceptional characteristics. The robotic fish model (kinematics and dynamics) is integrated with the Lighthill (LH) mathematical model framework. Comparative studies are undertaken between a LH model based and a non-LH based model. A comprehensive propulsion mechanism study of the different parameters namely the tail-beat frequency (TBF), the propulsive wavelength, and the caudal amplitude are studied under this framework. Yaw angle study for the underwater robotic fish vehicle is also carried out as it describes the course of the robotic fish vehicle. Inverse kinematics based approach is incorporated for trajectory generation of the robotic fish vehicle motion. Analysis of these critical parameters affecting the kinematics study of the vehicle vis a vis the real fish kinematic study [8] is carried out for a given trajectory. TBF is found to be the effective controlling parameter for the forward speed of the vehicle over a wide operating conditions. Performances and comparative results of propulsive wavelength and amplitude variations are also shown and discussed.**

***Index Terms*—Biomimetic, Robotics, BCF, Kinematic Modeling, Lagrange-Euler equations, Lighthill Equation.**

I. Introduction

With the advent of the bio-inspired underwater vehicle/robots applications, finding an efficient propulsion mechanism is of the utmost importance. Sir J Lighthill (LH) mathematical slender body swimming model plays a vital role to envisage the propulsion mechanism of such a biomimetic robotic fish. Exploring the spatial locomotion of certain aquatic vertebrates that are able to travel at surprisingly remarkable speeds, has been the interest of researchers towards the contribution of the bio-inspired robotics or bio-mimetics [1] and has been continuously increasing through the last decade. Biomimetics reflect the features and capabilities of natural evolution of a system that could be efficiently replicated or mimicked in a human engineered system to design of new technologies and improvement of conventional ones. One of the focused technologies has been the development of autonomous underwater vehicles [2] as a greater part to the increasing interest in unmanned underwater surveillance and monitoring. Of particular interests are regions of the underwater environment, underwater detection, pollution source tracking, underwater archaeology, search and rescue, and so forth.

The propeller based locomotion [2] although rendered the initial answers to underwater locomotion but sets issues on high-maneuverability and efficiency. The scientific community and researchers also found that propeller-strikes produce greater amount of marine debris, marine creatures mortality and shallow waters ecosystem disturbances. Biomimicked or fish-like robots are expected to be quieter, more maneuverable (lesser accidents), and possibly more energy efficient (longer missions). Undulating-finned robot can preserve undisturbed condition of its surroundings for data acquisition and exploration (stealth). Movement of fish through water without creating ripples and eddies were more reasons to choose a bio-inspired design for underwater locomotion. Considering the propulsive features [3] of existing fish modes, a novel propulsive mechanism that integrates fish-like swimming with modular links and fin movements has been proposed [2]. The work in this paper specifically identifies the usefulness of present model for purposes of both speed and maneuverability.

The contributions in the present work are enumerated as following:

- Integrate the present robofish mathematical model (kinematics and dynamics) with the Lighthill mathematical model framework.
- Comparison between a LH and a non LH propulsion mechanism model is investigated for a given trajectory

traversal.

- Demonstrate the feasibility of real time flexible bodied Robotic Fish prototype (Robo-Fish) vehicular design propelled by a *n*-joint oscillating caudal fin under the proven Lighthill framework.
- Identify the different parameters affecting the kinematics study of the robotic fish vehicle.
- Undertake a comprehensive and comparative study of the different parameters under this framework affecting the kinematics and dynamics of the system vis a vis the real fish kinematic study [8].
- Proposes and solves system modeling problem as a *n*-joint manipulator based mobile vehicle, transformed from conventional propeller based single rigid body structure
- Inverse kinematics based approach is used for fish body motion as the independent variables are the joint variables and task is stated in terms of reference coordinate frame (inertial and earth).

The paper is organized as follows: Section II reviews the propulsive mechanism of the fish swimming. An overview of the *BCF* swimming mode [4] in fishes is presented. In Section III, the design characteristics and features of the prototype are explained with the the kinematics and dynamics modeling studies of a three-link robotic fish. Section IV discusses the Lighthills existing mathematical framework and its integration with the present prototype. This section also investigates in to the thorough study of the different kinematic parameters governing the fish undulatory motion. Section V presents the robotic fish body undulatory propulsion mechanism for a given trajectory,simulation results with, and without the Lighthill framework. Also a comprehensive kinematic analysis of the robotic fish based on the different parameters is simulated and verified with one of the earliest and foremost reported literature [8](used by Barrett [9]) for the real fish motion . In Section VI, the conclusions and directions for future work are discussed.

II. SYSTEM MODEL

The *BCF* mode carangiform style [4] of locomotion is approximated using a 3-link (including the pectorals attached to the head) mechanism with two actuated joints as shown in Fig.1. The first link as the "head", second link as the body is roughly two-thirds of the weight of the entire robot. The tail of the robot is formed by the third link connected to the caudal fin. Our mechanism is a reasonable approximation to *BCF* carangiform locomotion, and therefore with small modifications of this model, can be used in the analysis of carangiform swimming. Moreover, the caudal peduncle and fin have been rendered in to a *Thunniform* (shark)semi-crescent structure for an efficient thrust and therefore to allow high cruising speed for longer period of time [4].

While the kinematics study explains the geometry of the motion of robofish w.r.t a fixed reference coordinate frame as a function of time, the dynamics of any rigid body [6] can be completely described by the translation of the centroid and the rotation of the body about its centroid.This leads to the ability to derive the actuator torques necessary to produce the tail motion that is desired. The present research work investigates into the system modeling as a 2-joint, 3-link manipulator based mobile vehicle, with Denavit-Hartenberg (DH) Kinematics Model and Lagrangian Dynamics Model. The earth fixed frame has been defined w.r.t the fixed reference frame (body/inertial frame). When describing the kinematics and dynamics of the mode shown in Fig.2, the interlink actuator-shaft constitutes the inertial frame of reference.

A. Generalized Equations of Motion

The dynamic equations of the DOF (Degree of Freedom) base are obtained using, the Lagrange-Euler formulation [6] given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \left(\frac{\partial L}{\partial \dot{q}}\right) = \tau_i \tag{1}$$

Where, the Lagrange function (L) is defined as

$$L = K - P \tag{2}$$

Where, *K*: Total kinetic energy of robotic fish; *P*: Total potential energy of robotic fish; q_i: Joint variable of i_{th} joint; $\dot{q}$: First time derivative of joint variables; τ_i: Generalized force (torque) at i_{th} joint.

The right hand side of the equation is external forces or torques acting on the base such as thruster control jets or

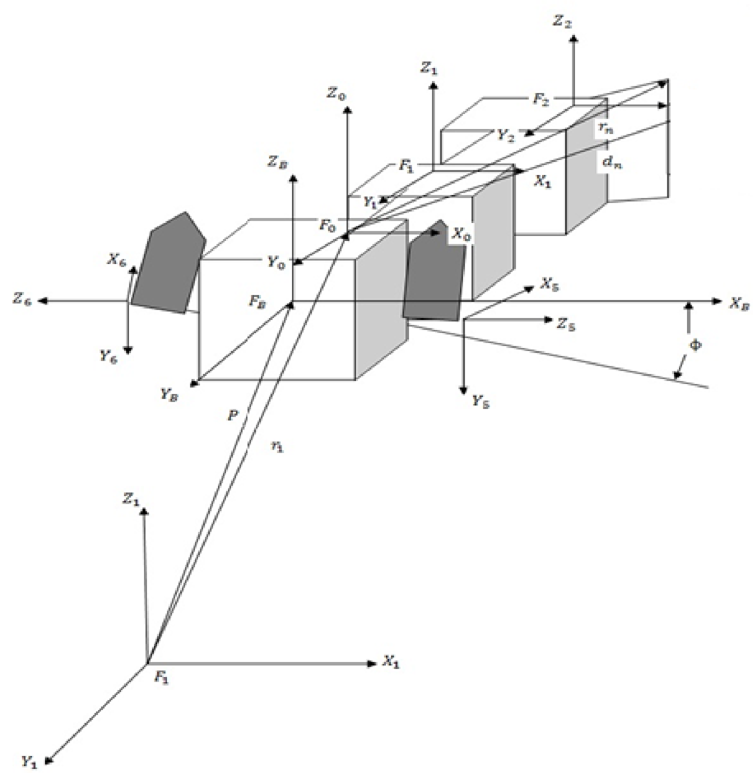
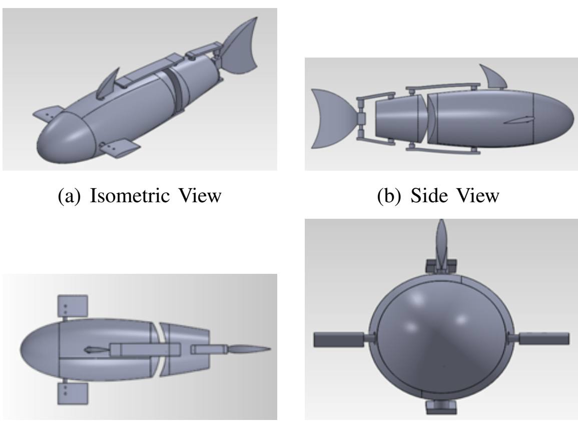


Fig. 1. Relative orientations and locations of local coordinate frames at the CM of the head and the Inertial reference frame

momentum gyros equations of motion.

$$\tau_i = \sum_{k=1}^{n} D_{ik} + \sum_{k=1}^{n}\sum_{m=1}^{n} h_{ikm}\dot{q_k}\dot{q_m} + c_i \qquad (3)$$

Where, *D* is the inertia matrix, *h* is the Coriolis-centripetal matrix, *c* is the gravity matrix, τ is the generalized force (torque) vector, and *q* is the generalized angular coordinate. The above manipulator dynamic equations have been developed in three dimensions for a *3* link manipulator on a 6 degree of freedom base, assuming that there is gravity acting on the system. As this research intends to develop *2*-joint manipulator based mobile vehicle, the above equation has been successfully used for investigating the dynamics of underwater vehicles [7] as well as robotic manipulators [6]. Following the analysis, the mechanical design in Solidworks is implemented shown in Fig. 2.



(a) Isometric View

(b) Side View

(c) Top View

(d) Front View

Fig. 2. Solidworks model of Robo-fish

III. LIGHTHILL MATHEMATICAL FRAMEWORK DESIGN

Lighthill [5] in his analysis of Grays work extended and formulated the *slender body theory*, which proposed the fact that the overall flow of the fluid around the body is compounded of firstly the steady flow around the straight body and secondly the flow due to the displacement *h*(*x*,*t*). The second flow component, *V* for a given cross section area, S_x with fluid velocity, *U* is given by

$$V(x,t) = \left(\frac{\partial h}{\partial t}\right) + U\left(\frac{\partial h}{\partial x}\right) \qquad (4)$$

The theory uses a set of partial differential equations to calculate the thrust and efficiency of swimming with definite rhythmic and symmetric body motions. The motion of a standing wave displacement vector *h(x,t)* was introduced as a empirical expression

$$h(x,t) = f(x) * g(t - x/c) \qquad (5)$$

where *f(x)* term indicates the amplitude of the wave, *g(x,t)* is an oscillatory frequency dependent wave function, and *c* is the fish body velocity. Lighthill was the first to apply the methods of slender body theory to an undulating body swimming in an inviscid fluid medium. Dewar's [8] practical work on swimming studies on the travelling wave, its relation to kinematics were investigated and postulated with a digitized record. It was later was used by Barrett et al[9] to validate a *Carangiform* swimming mode in the first of its pioneer work MIT Robotuna. In Barrett's work the fish body is represented by a planar spline curve due to the propulsive wave behavior as lateral curvature in spine and musculature.

$$y_{body}(x,t) = [(c_1 x + c_2 x^2)][sin(kx + \omega t)] \qquad (6)$$

where y_{body} is the transverse displacement of body, *x* is the displacement along main axis, *k* is the body wave number (k=$2\pi/\lambda$) , λ is the body wave length, c_1 is the linear wave amplitude envelope, c_2 is the quadratic wave amplitude envelope, ω is the body wave frequency. Once given this body-wave, the present research does kinematic analysis to determine the proper body-wave parameters (c_1, c_2, *k*, ω etc.) for a desired and efficient undulatory swimming motion.

IV. SIMULATION RESULTS AND DISCUSSION

The implementation in robotic fish prototype for the Lighthill model verification and kinematic analysis is carried out in MATLAB and Simulink environment. Simulation results including time histories of forward velocity as well as the corresponding trajectory are discussed. The undulatory motion of the fish body results in a propulsion mechanism validated through an envelope formation.

Simulation tests were conducted to show the effectiveness of the Lighthill's propulsion model. In Fig. 3 , two conditions were compared, where the fish is allowed to travel a path (using *inverse kinematics* algorithm [6]) from an initial point to a destination point, without the Lighthill undulatory action and with Lighthill mathematical model based implementation, fixed at three identified major parameters i.e. oscillating(tailbeat) frequency (ω), wavelength(λ) and amplitude. From the above simulation results and graphs, the performance

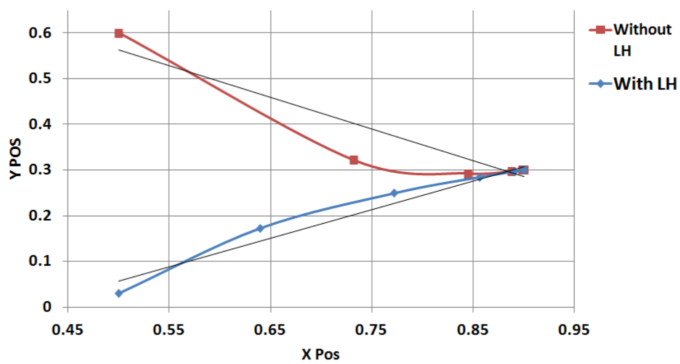


Fig. 3. Comparison graph in XY plane between with and without Light-Hill Implementation

factor of the robotic fish is calculated for a given (fixed distance) and it is found that with Lighthill implementation the motion performance improves by 14.68 %. Motion performance is defined by

$$\left(\frac{distance_{w/oLH} - distance_{withLH}}{totaldistance}\right) \quad (7)$$

It therefore validates the classical theory postulated by Sir James Lighthill for the present robotic fish model.

It is observed that fish in nature uses a combination of *frequency, amplitude* and *wavelength* for speed control. Oscillating frequency *f*, oscillatory amplitude of the body and length of the oscillatory part, are utilized to achieve different swimming speeds [4,8]. Substituting ω for 2πf, another form of body-wave equation is easily derived from the equation [9]:

$$y_{body}(x,t) = [(c_1 x + c_2 x^2)][sin(kx + 2\pi f * t)] \quad (8)$$

The control parameters of the robotic fish theoretically include kinematic (body wave) parameters *f*, λ , c_1 ,and c_2. An infinite number of parameter combinations can then be arranged to meet a wide variety of propulsive behaviors. It is assumed that the robotic fish is in an equilibrium condition in the *x* direction under the relative propulsion velocity, U_m . This velocity is obtained using the integration of Lighthill mathematical model with robotic fish inverse kinematics and dynamics model. Further, it is simulated and analysed under three different parameter variations.

A. Oscillatory frequency (Tail Beat Frequency TBF*) Based Control*

The literature [10] reports the increase of swimming speed with the oscillating frequency *f*, and it approaches a constant value i.e. a steady speed is achieved. Compared to it, the present paper insights in to the further details finding the variation, combinations of other parameters effecting the kinematics of robotic fish and comparing them with the real fish studied reports [8]. In Fig.4 , the variation of forward velocity (mass centre) is shown against the tail beat frequency (*TBF*). It is observed that the forward velocity fluctuates slightly around a positive mean value. It can also be recognized in Fig. 4 that the forward velocity (w.r.t time) increases with the raise of the oscillatory frequency. Further after reaching a peak velocity at the given frequency a descending pattern follows. The velocities have been measured at different tailbeat frequencies ranging from 0.05 hz to 0.8 Hz. Compared to the other simulated forward velocity[10] (u = 0.12 m/s at *f* = 0.1Hz) and (u = 0.115m/s at *f* = 0.64 Hz), present result seems evenly realistic and even reaches a maximum of 1.1 BL/s (body length per second), i.e., u_{max} = 0.138 m/s at *f* = 0.33 Hz, which may demonstrate the oscillatory caudal fin contributing more to thrust generation. The simulation results verify the real time kinematic studies reported by Dewar [8] in accordance with Lighthill theory.

In this paper, forward velocity, *u* results were obtained by varying the oscillating frequency or the tail-beat frequency parameter fixed at different values of wavelength and amplitude as shown in Fig. 4. Using (7) for the case: c_1 = 0.002, c_2 = 0.00835, *f* = 1 Hz, and *R* = 0.5, the instantaneous propulsive

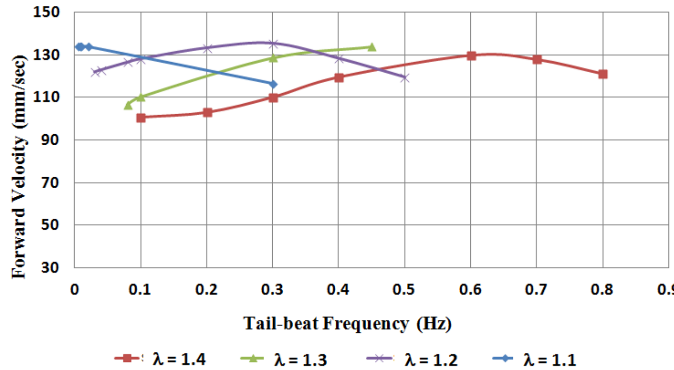


Fig. 4. Forward Velocity vs. Tail beat Frequency at different wavelength values (Operating Region)

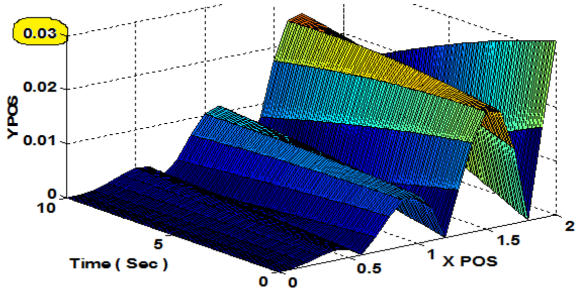
efficiency is obtained peaking a steady-state value of 0.68, which is less than that of real *Carangiform* fish as high as 0.9. This is accounted only due to the thrust from active deformations of the robotic fish i.e. the joint actuator motion. However, other important factors, e.g., passive deformation of the fish and producing vortexes in the field of wake, may also contribute to thrust production and would be investigated in the future work.

B. Amplitude Based Control

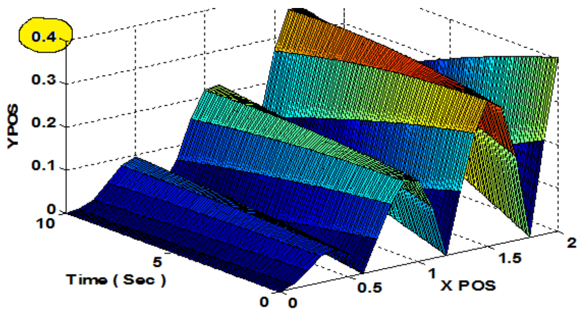
The existing second order amplitude envelop

$$y_{ampenvelop}(x,t) = [(c_1 x + c_2 x^2)] \quad (9)$$

is chosen to produce different body-waves by different values of c_1 and c_2. In practice, oscillatory-amplitude-based speed control method adjusts the transverse movement at a constant oscillating frequency. The thrust is hence changed and therefore the swimming speed. The variation of amplitude envelope which is caused by the change of parameter constants c_1 and c_2 is shown in Fig.5 (a) and (b). The values of these two coefficients are dynamically calculated for the present system in accordance with the Lighthill's model and kinematic study by Dewar[8]. Here, two different body length's 0.17 and 0.21 [8] are considered taking into account of the real fish(tuna). The undulatory envelope formation with these two values are simulated (at λ = 1.4m and *f* = 0.5) and shown in Fig 5. depicting the variation in amplitude due to the different body-lengths(BL) 0.17 pu BL and 0.21 pu BL [8] . Further, the variation of amplitude (best value chosen, 0.21 BL^{-1}) as a parameter affecting the forward velocity at a fixed wavelength is shown in the simulation work as Fig. 6 . The trend shows an increase in the forward velocity with an initial increase and subsequent decrease in the amplitude values. The choice of the amplitude between two speed limits also depend on the tailbeat frequency at a fixed wavelength. It is in agreement with the study/results of Dewar [8]. The similar results can also be seen with Low [10] but the present model provides a wider operating region with a non-linear trend in kinematic data obtained.



(a) c_1= .002, c_2 =.00835, 0.17 BL^{-1} u = 0.172 m/sec



(b) c_1= 0.2, c_2 =0.01736 ,0.21 BL^{-1} u = 0.215 m/sec

Fig. 5. Lighthill Amplitude Variation and Forward velocity (u)at different values of envelope coefficients c_1 and c_2.

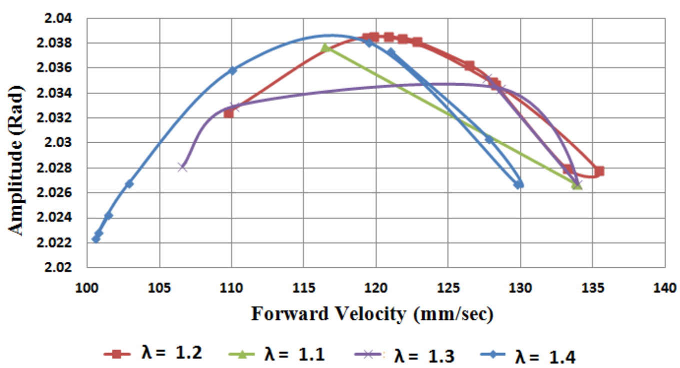


Fig. 6. Forward Velocity vs. Amplitude at different wavelength values (Operating Region)

C. Yaw Angle Variation

Yaw [7] (rotation about the vertical axis) describes the course of the robotic fish vehicle and is the primary mode of feedback control in underwater vehicles. In fish generally, yaw reduction is achieved through an increased body length [8] revisiting the anterior lateral motion resulting from recoil forces. It is also due to structural separation of the caudal fin and the rest of the body. The variation of yaw angle with tail-beat frequency is shown in Fig.7, for fixed values of propulsive wavelength and amplitude and is analogous to the forward velocity relation to TBF. For the present model, this is in partial agreement with Dewar's [8] work.

Over the same region of operation the plot of forward velocity versus yaw is also shown in Fig. 8 which is a linearly increasing function not in conjunction with the Dewar's result

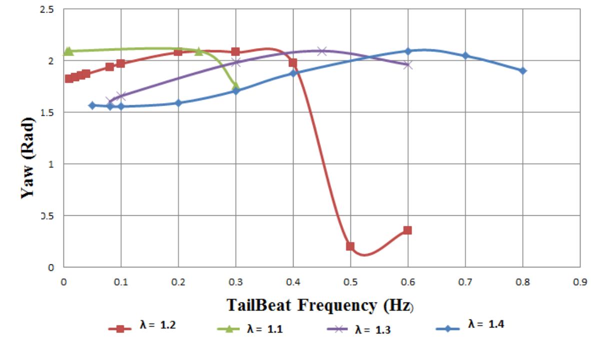


Fig. 7. Yaw Angle vs. Tail beat Frequency at different wavelength values (Operating Region)

[8] and but matches the kinematic mathematical model.

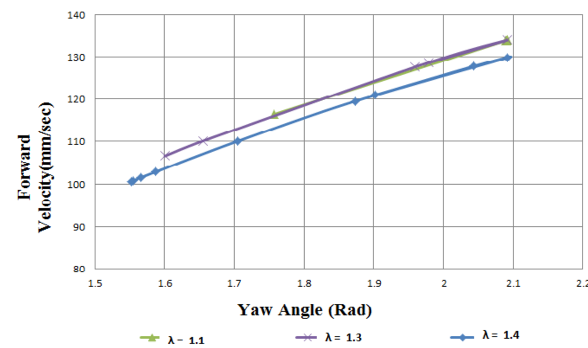


Fig. 8. Forward Velocity vs. Yaw Angle

D. Propulsive Wavelength Effects

The variation of forward velocity w.r.t the tailbeat frequency has been discussed(at a fixed propulsive wavelength). However, the variation with wavelength at a fixed frequency did not show a relationship/trend in the forward velocity. It was oscillating in the ranges of frequencies. Simulations were also performed to undertake the variation of fish forward velocity at different values of propulsive wavelength but it did not show a pattern over other values of frequency. Further, the simulation of forward velocity with respect to propulsive wavelength was undertaken for peak TBF frequencies (in this case TBF = 0.3) (where the highest forward velocity is attained) and it is observed that the velocity decreases with the increase in wavelength shown in Fig. 9 that is in agreement with Dewar's work [8].

The simulation results Fig.4, Fig.6 and Fig.7 are useful during the open-loop experimental verification and operation of the robotic fish prototype as we know now that the variation of forward velocity with tail-beat frequency, yaw angle and amplitude at different wavelength (and operating tailbeat freq.) values gives the operating region for experimental validation. Clearly, the kinematic studies relation of the four parameters (tailbeat frequency, wavelength, amplitude and yaw angle)for the present robotic fish underwater vehicle model is validated at par with Dewar's findings [8]. The simulation results also

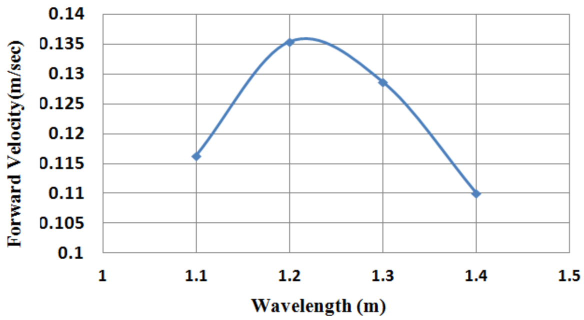


Fig. 9. Forward Velocity vs. Wavelength (Operating Region) at a given TBF = 0.3

denote that amplitude and wavelength tend out to be less chosen parameter over tailbeat frequency if swimming efficiency is considered. Also a compromise/ optimization of these three important parameters is required for the most efficient undulatory fish body motion.The project implementation is carried out in MATLAB and Simulink. The yaw-axis servomotor is a Hitec HS-5646WP (11.3Kg-cm/6 V) used in all the joints. Arduino-uno with ATMEGA328P serves as central processing unit for the robotic fish. The mechanical CAD design and animation has been implemented with the help of the Solidworks (mechanical model) and MATLAB SimMechanics, VRML (3D motion). The motivation of this paper is described by the remarkable swimming abilities of *BCF* mode *carangiform* style of swimming built as a robotic fish prototype. The NACA airfoil aerodynamic structure has been designed to boost the swimming efficiency by reducing the drag. Based on the steady simulation results the experiments validating the same were undertaken for different body-motion configurations to emulate the undulation of the Robotic-fish in fluid environment(Lab test) shown in Fig. 10.

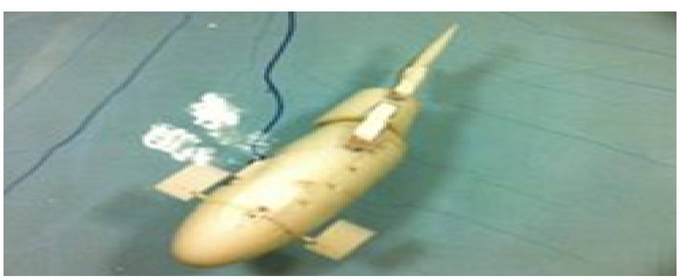


Fig. 10. Robotic-Fish underwater vehicle model prototype

V. Conclusion

The present paper deals with the study, implementation and kinematic analysis of Lighthill mathematical model based real time design of a biomimetic fish like underwater vehicle. The built prototype is a 74 cm. long, weighs 4.5 kg , *BCF* (Body Caudal Fin) mode 6 DOF fishlike underwater robot with a horizontal caudal fin (tail). The system uses DC servomotors as actuators and the controller bed is Arduino platform. A systematic and scientific approach of system modeling and integration has been presented with the robotic-fish kinematics, dynamics with real-time simulation in MATLAB and structural CAD design in Solidworks. Under this framework a comprehensive study of the different parameters like tail-beat frequency (TBF), amplitude, propulsive wavelength and yaw angle, affecting the kinematics and dynamics of the vehicular system vis a vis the real fish kinematic reports is reported. The paper also proposes the system modeling problem as a 2-joint, 3-link manipulator based mobile vehicle. The up-gradation of the servos and the optimization of the robotic fish has to be performed in the subsequent stages of the development of this project. The future work primarily focuses on the control solution algorithm and its implementation for energy efficient undulatory fish motion.

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