Undersea Acoustic Communication Maps for Collaborative Navigation

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***Abstract*—Communications play a key role in collaborative navigation algorithms. A better understanding of the ability to send and receive messages permits greater navigational flexibility and system robustness. This paper focuses on the building of an underwater acoustic communications map for collaborative navigation. The emphasis is in two areas - a local and global communications map. The local communications is defined with respect to a single destination reference point. Using a sample set of *a priori* signal to noise ratio acoustic modem data, Kriging techniques are used to create mean and variance map estimates. The global communications map is a compendium of local maps and is defined within a bounded survey space. Bayesian Inferencing is used for building the global map. It is based on REML parameter estimation of an anisotropic covariance function. The paper analyzes acoustic communication signal to noise datasets recently collected in Monterey Harbor, Monterey, CA and is used to demonstrate the above-described techniques.**

I. Introduction

Understanding the communication landscape of the operating environment can provide significant benefits for unmanned systems. This is especially true in cluttered, dynamic areas. Error bound analysis on collaborative navigation algorithms assume full network connectivity over a fixed time frame. A better understanding of the ability to communicate may increase this limited time frame which would result in (1) a more robust and effective system, (2) increased navigational flexibility (3) and improved energy efficiency.

For the underwater domain, there are a variety of applications where it is necessary to conduct a three-dimensional bathymetric survey of restricted waterway areas such as harbors, canals and rivers. While this may be accomplished by other means, a fleet of Autonomous Underwater Vehicles (AUVs) is a potentially better solution since the vehicles have greater access to survey the region.

For this to be accomplished, there are several hurdles to overcome. One is to maintain navigational accuracy for all AUVs so that the need for surface GPS position fixes is minimized. This can be achieved with a multi-AUV Simultaneous Localization and Mapping (SLAM) approach. A key to our approach is a partition of the survey space where each vehicle is responsible for a sub-region. In determining the partitions it makes sense to account for the feature and communication landscape, since these are the two critical components to bound navigational errors. This paper focuses on building of an underwater acoustic communications map for collaborative navigation. Specifically local and global communications maps are defined and techniques are described for building the maps.

Fig. 1. *The location of the acoustic modem SNR datasets, Monterey Harbor, Monterey CA.*

The Local Communications Map (LCM) is a measure of signal strength relative to a single fixed destination point, so that at each discrete position within a designated maximum range, there exists a signal to noise ratio (SNR) mean and variance estimate. Variogram analysis and Kriging techniques are employed to build the LCM. These techniques are especially appropriate for post-mission processing.

The Global Communications Map (GCM) provides an estimate of the ability to communicate between any two positions in the bounded survey region. The GCM is a compendium of LCMs such that at each position (at a designated resolution) there is an overall scalar measure of the ability to communicate and the uncertainty associated with the measurement. This is defined as the integral (over a bounded area) of the LCM mean estimate. It is demonstrated that the ability to communicate in a cluttered environment is not radially uniform and that the data is highly variable, (i.e. anisotropic). So, in addition to the scalar measurement that summarizes an overall ability to communicate from a position, there is a limited number of vector measurements indicating the ability to communicate along a particular bearing. Bayesian inference techniques are used to predict each random field based on estimated parameters of an anisotropic Matérn covariance function at all desired map positions.

In terms of contributions, this is the first known formulation of a global communication map for collaborative navigation. Second, this is the first known research into building a global communications map for collaborative undersea navigation. Third, the data collected represents the first known significant acoustic data collected in a noisy, cluttered, shallow water environment.

The paper is organized as follows: Section 2 describes related work. Section 3 defines and discusses the Local Communication Map. Section 4 summarizes the acoustic modem datasets and the LCMs built from these datasets. Section 5 defines and discusses the Global Communication Map. Section 6 presents a second series of acoustic modem datasets and the GCM from these datasets. Section 6 provides concluding remarks.

II. Related Work

A. Undersea Acoustic Communication

Undersea acoustic communication is characterized by low data rates, low link quality and high bit rate errors. In part, this is due to challenges of the ocean environment which include temperature and depth variability, surface waves and differing bottom types. An overview of undersea wireless networks include [1] and [2]. A summary of undersea localization techniques is available by Erol-Kantarci [3]. Another recent work in decentralized estimation for multiple underwater vehicles is documented in [4].

B. Kriging

Kriging is a well-established technique for estimating a spatial random field through a discrete number of measurements [5], [6]. Typically this is used in geo-statistics but it has recently been used for estimating signal-to-noise ratios in wireless radio networks [7]. The main advantage of the technique is that it can be used for interpolation to infer an ability to communicate at a specific location of interest. This includes a mean and variance prediction. Bayesian Kriging uses inferencing for estimation. It is superior to standard Kriging techniques when there is prior information available about the spatial covariance structure.

C. WiFi Localization

Wifi Localization is a related field where the goal is to determine the physical location of a mobile device from its wireless signal strengths [3]. Ferris et al [8], [9] have used a Gaussian Process Latent Variable Model in combination with a novel WiFi-SLAM technique for building wireless signal strength maps. The technique used by Ferris is conceptually similar to the one used in this paper for building LCMs (Kriging is identical to Gaussian Process Models when there is no prior knowledge of the covariance parameters).

III. The Local Communication Map

For some situations one may be interested in understanding the SNR relative to a fixed position. An example might be determining how to optimally deploy a number of anchored acoustic modems which serve as the basis for a Undersea Acoustic Network. It is assumed there are *a priori* SNR measurements that relative to a single fixed position. The objective is to predict the SNR mean and variance, between the fixed reference point and all desired positions of interest, taking into account the spatial relationship of the data.

A. Definition

The Local Communication Map $(\mathbf{\Omega_0})$ is a topological surface of a bounded survey space $(\mathbf{\Omega})$ in $\mathbb{R}^d$ such that $\mathbf{\Omega_0} \subseteq \mathbf{\Omega}$. Within $(\mathbf{\Omega_0})$ there exists a unique point $(\mathbf{x_c})$ that is the fixed communication node's position. Relative to $\mathbf{x_c}$ we seek SNR mean and variance estimates for a discrete number of points within $(\mathbf{\Omega_0})$. This corresponds to the map resolution where each point $(\mathbf{x_{1,1}} \cdots \mathbf{x_{n,m}})$ is the centroid of the region.

B. Kriging

The bounded survey space $(\mathbf{\Omega_0})$ is represented as a Random Field $\mathbf{Z}$ in $\mathbb{R}^d$. A model for the random field is

$$Z(\mathbf{x}) = \sum_{i=1}^{p} f(\mathbf{x_i})\beta_i + \epsilon(\mathbf{x}) \tag{1}$$

where $f(\mathbf{x_i})$ is a known spatial function, β is a vector of unknown coefficients and ϵ is a mean zero random field with known covariance structure $cov(Z(\mathbf{x}_i), Z(\mathbf{x}_j)) = K_{\boldsymbol{\theta}}(\mathbf{x}_i, \mathbf{x}_j)$, where $\boldsymbol{\theta}$ is a vector of covariance parameters. The Kriging predictor is the best linear unbiased predictor of the form $\hat{Z}(\mathbf{x}_0) = \lambda(\boldsymbol{\theta})Z$, where $\hat{Z}(\mathbf{x}_0)$ is the mean SNR prediction at the points of interest $(\mathbf{x_0})$, $\lambda(\boldsymbol{\theta})$ are the Kriging weights (based on covariance parameters $\boldsymbol{\theta}$ and Z are the SNR measurements.

There are five steps associated with Kriging: data transformation, variogram analysis, solving a linear system of equations for λ, calculating the mean prediction, and calculating the variance.

1) Data transformation: For most problems in geostatistics, a Euclidean distance metric is used for determining the spatial relationship. For building a LCM in an undersea environment we use a slightly different approach. This can be illustrate through two simple examples. In a deep ocean environment, along a flat bottom, the SNR may degrade uniformly over increasing distance relative to a fixed reference point. The result is that the SNR measurements may be highly correlated even if the Euclidean distance metric showed the points

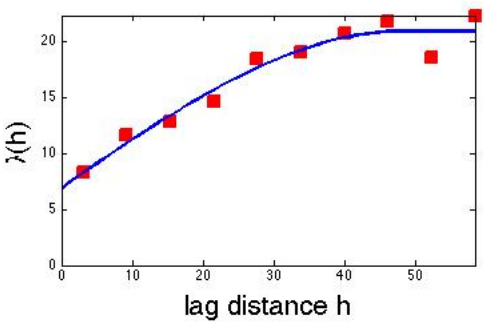

Fig. 2. *Example of an acoustic modem SNR measurements empirical semi-variogram (in red) with a best fit spherical model (in blue)*

were far apart. In a cluttered, shallow water environment, the SNR can degrade non-uniformly over increasing distance. In other words, the data is anisotropic. To determine the spatial relationship of SNR data, it is necessary to consider both range and bearing in relation to the fixed node. In both cases, a Euclidean metric is an insufficient distance metric. To compare two SNR measurements requires a transformation into range (r) and bearing (ψ) such that $[r, \psi] = s(\mathbf{x_1}, \mathbf{x_2})$ where,

$$\begin{bmatrix} r \\ \psi \end{bmatrix} = \begin{bmatrix} ||\mathbf{x}_1 - \mathbf{x}_c|| + ||\mathbf{x}_2 - \mathbf{x}_c|| \\ \text{atan2}(\mathbf{x}_1 - \mathbf{x}_c) - \text{atan2}(\mathbf{x}_2 - \mathbf{x}_c) \end{bmatrix} \quad (2)$$

2) Variogram Analysis: Variogram analysis determines the spatial relationship between data at different distances. There are two steps: The building of the empirical variogram and the modeling of the data with a parametric equation. The empirical semi-variogram ($\lambda^*(h)$) is a defined as:

$$2\lambda^*(h) = \frac{1}{N(h)} \sum_{N(h)} [Z(\mathbf{x}) - Z(\mathbf{x} + h)]^2, \quad (3)$$

With an anisotropic assumption variogram analysis is performed on a subset of the data that corresponds to a various distances within a bearing window. The second step is to define a function which fits the semi-variogram. The function is critical since it is used to solve the system of linear equations to produce an estimate for λ. Common isotropic model choices include Linear, Spherical, Exponential and Gaussian. Figure 2 shows an example of a semi-variogram with a best-fit spherical model (in blue).

3) Solving the System of Linear Equations: Given measurements $\{Z(\mathbf{x}_1), Z(\mathbf{x}_2), \ldots, Z(\mathbf{x}_n)\}$ of the spatial random field Z at locations $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$, the Kriging interpolator at point $\mathbf{x}_0$ is defined as:

$$\hat{Z}(\mathbf{x}_0) = \sum_{i=1}^{n} \lambda_i Z(\mathbf{x}_i) \quad (4)$$

such that $\hat{Z}(\cdot)$, is an estimator function for $\Omega_0((x))$ where the expected error is zero $E[\hat{Z}(\mathbf{x}_0) - Z(\mathbf{x}_0)] = 0$, and λ_i are weights that sum to one ($\sum_{i=1}^{n} \lambda_i = 1$). The optimal weights can be calculated from the following system of linear equations.

Destination	Number of SNR measurements
D_1	1113
D_2	1298
D_3	1275
D_4	948
D_5	1274

TABLE I
SNR MEASUREMENTS TO EACH DESTINATION (AUGUST 2011)

$$\begin{bmatrix} K(s(\mathbf{x}_1, \mathbf{x}_1)) & \ldots & K(s(\mathbf{x}_1, \mathbf{x}_n)) \\ \vdots & & \vdots \\ K(s(\mathbf{x}_n, \mathbf{x}_{\mathbf{q}_1})) & \ldots & K(s(\mathbf{x}_n, \mathbf{x}_n)) \\ 1 & \ldots & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ \mu \end{bmatrix} = \begin{bmatrix} K(s(\mathbf{x}_1, \mathbf{x}_0)) \\ \vdots \\ K(s(\mathbf{x}_n, \mathbf{x}_0)) \end{bmatrix} \quad (5)$$

The function $s(\mathbf{x}_i, \mathbf{x}_j)$ is the distance metric (equation 2) between points $\mathbf{x}_i$ and $\mathbf{x}_j$ and the K function calculates the covariance based on the selected variogram model. Once the weights $\lambda_1, ..., \lambda_n$ and Lagrange multiplier μ are calculated, the variance for the ordinary Kriging model is given by:

$$\sigma^2 = K(0) - \sum_{i=1}^{n} \lambda_i K(\mathbf{x}_i - \mathbf{x}) - \mu \quad (4)$$

IV. LCM RESULTS

A. Data collection

The acoustic modem datasets used to build the LCMs were collected in Monterey Harbor, Monterey, CA in August 2011 (Figure 1). The underwater acoustic modem used was the Woods Hole Oceanographic Institution (WHOI) Micro-Modem [10]. The system was configured at a nominal 27kHz with a set baud rate of 19.2 Kbps. There were two modems providing single-way communication estimates. An RF-Acoustic gateway buoy was positioned successively at five fixed positions (D_1, D_2, D_3, D_4, D_5) shown in Figure 3. This was the destination of the modem transmissions. A second mobile micro-modem was deployed from a small support craft maneuvering around the harbor. This was the source of the transmissions. Table 1 provides the number of acoustic modem transmissions for each destination.

Figures 4 and 5 show the mean and variance Kriging interpolation results on the SNR datasets with a map resolution set to 10 meters (D_4 has a 4 meter map resolution). Note that the measurements don't degrade uniformly as a function of distance. The D_4 dataset shows the degradation of the SNR Measurements with the pier occluding a direct transmission path when compared with measurements at approximately

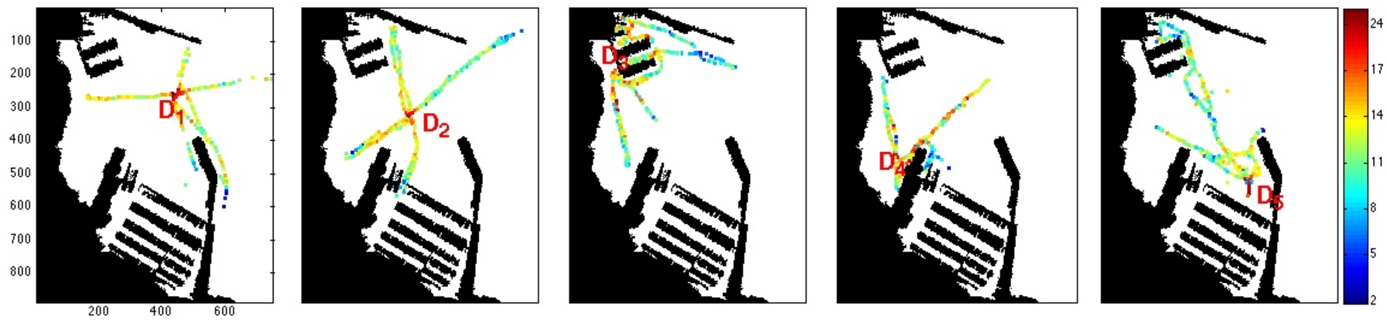

Fig. 3. *The five acoustic modem data sets collected in Monterey Harbor, CA in August 2011. Each data set consists of Signal to Noise Ratio (SNR) measurements from the surface boat as the source of the transmission to the destination (D_1 to D_5).*

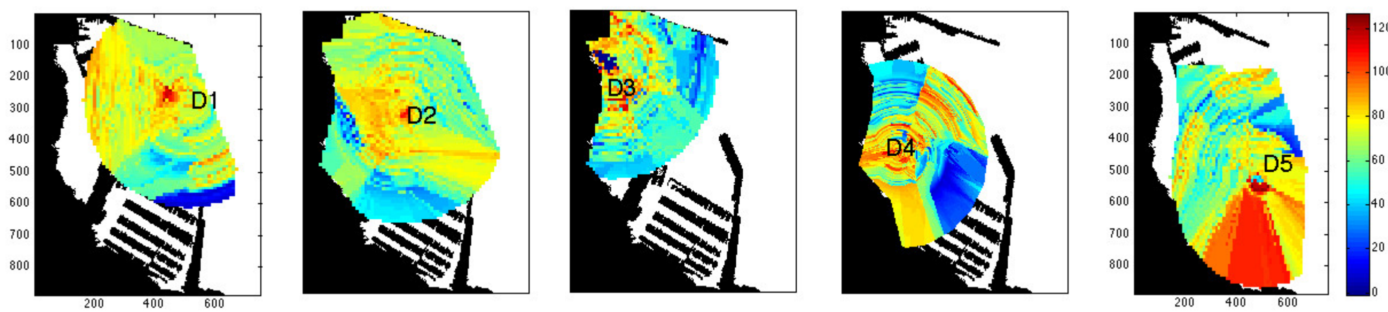

Fig. 4. *The Local Communication Map for the reference points (D_1 to D_5). The colors in the map represents a mean SNR prediction to the destination.*

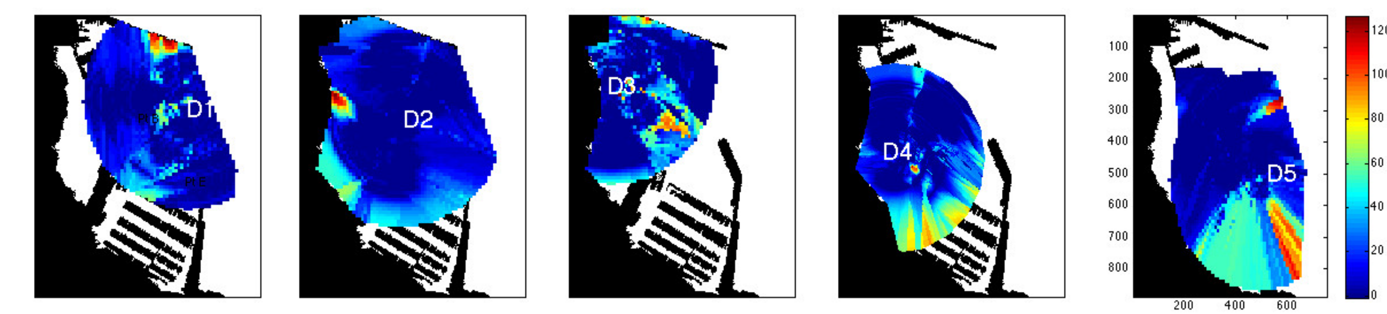

Fig. 5. *The Local Communication Map for the reference points (D_1 to D_5). The colors in the map represents variance of the SNR prediction to the destination.*

equal radial distances from the reference node at a roughly 30 degree bearing. This view is expanded in Figure 6.

Clearly, when estimating at point $Z(\mathbf{x})$, selecting data that has both a direct transmission path and an occluded path would result in an inaccurate estimate (this is represented by the circle in Figure 6). This highlights a strength of the Kriging technique - given *a priori* measurements one can be deliberate with determining a semi-variogram model that is appropriate for the data. It can handle the anisotropic nature of the data by being selective about the points to build the empirical semi-variogram. The mean SNR LCM also demonstrates a potential strength of the approach in that acoustic communications between the fixed node and position $\mathbf{x}$ are degraded but still possible through the pier and this would be difficult to predict ahead of time.

Second, in the D_5 dataset, the Kriging techniques predict a strong SNR to the south. This is a poor estimate since there is a solid concrete wall between the inner and outer harbor that prevents acoustic communications. The estimate was based on a small number of measurements taken to the south of the

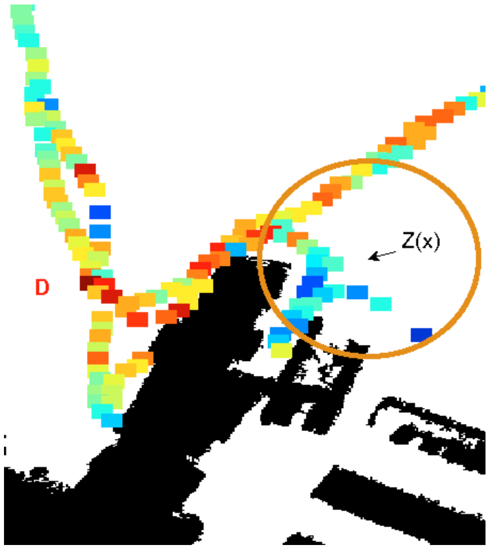

Fig. 6. *The expanded view of the D_4 acoustic modem SNR dataset. It highlights the difficulties in building an empirical semivariogram given the anisotropic nature of the data. In estimating the SNR at point x ($Z(\mathbf{x})$) using a range limited nearest search would produce an inaccurate mean estimate.*

fixed buoy. In the D_5 variance map it can be seen that the mean estimate has a high level of uncertainty in that southern area. As one might expect, the variance increases in the areas without measurements available. It suggests that these maps would be useful for determining where to sample to improve the mean SNR map.

There are several limitations that need to be addressed for a collaborative navigation implementation. First, the Kriging technique assumes complete *a priori* SNR measurements. In many circumstances this may not be available. Second, it is necessary to take the inverse of the covariance matrix (equation 5) to solve for λ. If A becomes large this could be computational expensive. Third, LCM provides an estimate from a single reference point, computing the LCM from each desired point would be inefficient.

V. The Global Communication Map (GCM)

The GCM is meant to address some of the issues previously stated. We seek a map representation of the communication landscape such that any AUV in the environment can determine where to communicate to improve the probability of successfully communicating to another AUV. This could also be used to determine where acoustic transmission is necessary to gain better map accuracy and resolution. Finally we seek calculations that are computationally efficient in the incremental building of the GCM.

A. GCM Definition

Consistent with the LCM, we define the GCM such that there exists a bounded survey space Ω in $\mathbb{R}^d$ where we partition Ω into a lattice (where $\Omega = \cup_{i,j=1}^{n,m}\Omega_{nm}$) and from the center of each grid cell Ω_{nm}^c we seek to predict the mean and variance of the random field $Z_{nm}(\mathbf{x})$. Each cell in the lattice has (1) an scalar estimate of the overall ability to communicate omnidirectionally and (2) a discrete number of vectors which estimate the ability to directionally communicate over increasing distances within a defined bearing window. This models the potential anisotropic behavior of the random field. The scalar estimate is mathematically defined as follows:

$$\Omega_{nm}(\mathbf{x}) = \int^{R} Z_{nm}(\mathbf{x})\mathrm{d}\mathbf{x} \tag{5}$$

where R is a defined maximum range. The bearing vectors provide SNR predictions and are defined with the parameters ψ from $1,\ldots,v$ that defines an area of coverage (e.g. ψ_1 = 0 to 30 degrees and $\cup_{i=1}^{v}\psi_i = 2\pi$) and a fixed number of increasing ranges $r_0,\ldots,r_u$.

$$\mathbf{\Omega_{nm}^{c,\psi_v}} = \{Z_{nm}(r_0,\psi), Z_{nm}(r_1,\psi),\ldots,Z_{nm}(r_u,\psi)\} \tag{6}$$

and the parameter ψ from $1,\ldots,v$ that defines an area of coverage (e.g. ψ_1 = 0 to 30 degrees) and $r_0,\ldots,r_p$ are the fixed number of increasing ranges.

With the LCM, Kriging doesn't take into consideration the uncertainty associated with the covariance function. For the GCM, we look to explicitly take this into account by introducing a Bayesian intrepretation. This is known as Bayesian Kriging [11]. We are interested in predictive estimate of the marginal posterior pdf ($p(Z(q_0)|Z)$) such that

$$p(Z(q_0)|Z) = \int_{\boldsymbol{\theta}} p(Z|\boldsymbol{\theta})p(\boldsymbol{\theta}|Z)\mathrm{d}\boldsymbol{\theta} \tag{7}$$

where the first probability on the right hand side of the equation is obtaining the measurements given covariance parameters $\boldsymbol{\theta}$. The second term ($p(\boldsymbol{\theta}|Z)$) is the probability of the parameters given the measurements. While there are approximations for the right-hand side of equation 5, it requires significant numerical computation. Instead inference is accomplished through estimation of the covariance parameters $\hat{\boldsymbol{\theta}}$ such that

$$p(Z(x_0)|\hat{\boldsymbol{\theta}}, Z) \propto N(\hat{Z}_{\hat{\boldsymbol{\theta}}}(x_0), V_{\hat{\boldsymbol{\theta}}}) \tag{8}$$

where N is the normal distribution and $\hat{Z}(x_0)$ is equal to

$$\hat{Z}(x_0) = k_{\hat{\boldsymbol{\theta}}} K_{\hat{\boldsymbol{\theta}}}^{-1} Z + b_{\hat{\boldsymbol{\theta}}} K_{\hat{\boldsymbol{\theta}}} \hat{\beta}(\hat{\boldsymbol{\theta}}) \tag{9}$$

where $k_{\hat{\boldsymbol{\theta}}} = K_{\hat{\boldsymbol{\theta}}}(\mathbf{x_0},\mathbf{x_i})$ is the covariance between the desired estimation points and the measurements and $b_{\hat{\boldsymbol{\theta}}} = f(\mathbf{x}_0) - F' K_{\hat{\boldsymbol{\theta}}}^{-1} k_{\hat{\boldsymbol{\theta}}}$ with $F = \{f_j(x_i)\}_{nxq}$ consistent with [12].

$$V_{\hat{\boldsymbol{\theta}}} = K_{\hat{\boldsymbol{\theta}}}(\mathbf{x_0},\mathbf{x_0}) - k'_{\hat{\boldsymbol{\theta}}} K_{\hat{\boldsymbol{\theta}}}^{-1} k'_{\hat{\boldsymbol{\theta}}} + b'_{\hat{\boldsymbol{\theta}}}(F' K_{\hat{\boldsymbol{\theta}}}^{-1} F) b_{\hat{\boldsymbol{\theta}}} \tag{10}$$

Destination	Number of SNR measurements
D_1	1851
D_2	1535
D_3	1023
D_4	950
D_5	1141

TABLE II
SNR MEASUREMENTS TO EACH DESTINATION (JULY 2012)

Estimation of the covariance parameters is normally accomplished through maximum likelihood estimation (MLE) methods. In this case Restricted maximum likelihood estimation (REML) was used since MLE tends to underestimate the variation in the process [6].

In summary, to build the GCM, Bayesian inferencing is used to predict random fields $Z_{1,1}, \ldots, Z_{n,m}$ according to a desired map resolution. In order to do so REML was used to estimate the covariance function parameters $\boldsymbol{\theta}$. The next step is the selection of a covariance function. The function should be flexible enough to provide a reasonable representation of the SNR measurements.

B. The Matérn class of correlation functions

The Matérn class of correlation functions has been identified as an appropriate function for spatial processes [6]. The equation is given by:

$$\rho(s;\beta,\nu) = \frac{1}{2^{\nu-1}\Gamma(\nu)}\left(\frac{2\nu^{1/2}s}{\xi}\right)H_\nu\left(\frac{2\nu^{1/2}s}{\xi}\right) \qquad (11)$$

where s is the distance metric between observations and H_ν is the modified Bessel function of the third kind of order ν. There are two function parameters, a range parameter $\xi = 2\nu^{1/2}\zeta$ and a smoothness parameter $\nu > 0$. The larger the values of ν correspond to a smoother spatial process [13]. While there are other parameterizations of the Matérn class this is attractive since ξ is approximately independent of ν.

The Matérn Model is isotropic. The SNR measurements in cluttered environments have been shown to be anisotropic. It is possible to transform the Matérn Model to handle anisotropic conditions through a linear transformation of the spatial coordinates. It introduces two additional parameters δ and α such that the correlation function is of the form $\rho(s(\mathbf{x}_1, \mathbf{x}_2, \delta, \alpha), \beta, \nu)$. α is an anisotropic rotation angle and δ is a scaling parameter. This is important since the derivative of the anisotropic Matérn correlation function is necessary for the REML estimation of the model parameters. More details are available in [14].

VI. RESULTS

A second acoustic modem dataset was collected in Monterey Harbor in July 2012. The same destination locations were used for the gateway buoy. This time the surface boat initiated transmission of acoustic messages at positions that more fully covered the outer harbor in a lattice pattern. Table 2 provides the number of measurements for each of the destinations points (D_1 to D_5). Figure 7 gives an example of the measurements and positions for one of the five datasets. Figure 8 shows the the results of the GCM taking into consideration all 5 data sets. One of the benefits of the methodology is that measurements can be used in the estimation of more than one random field; this greatly reduces the sampling requirement to accurately build the GCM.

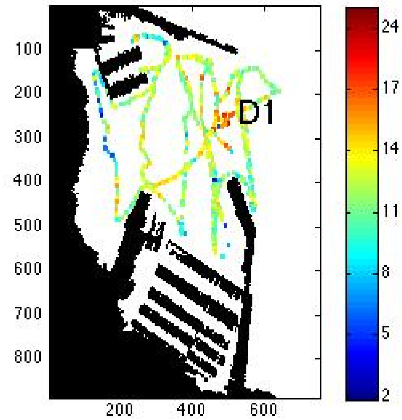

Fig. 7. *This is one example of the datasets collected in July 2012. The source of the modem transmissions is a surface boat maneuvering in the outer harbor. The destination is the fixed node* D_1.

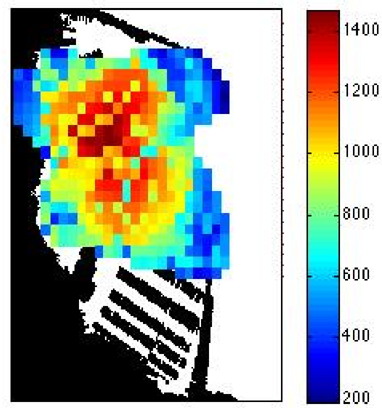

Fig. 8. *The GCM produced from the July 2012 dataset. The map resolution is 25 meters square.*

VII. CONCLUSION

This paper introduced the concept of undersea acoustic communication maps. The maps can be used in conjunction with navigation strategies for determining how to position vehicles for collaborative tasks. Kriging techniques were used with *a priori* SNR measurements to produce mean and variance estimates relative to a fixed node. Bayesian inference techniques were used to produce a more general map for estimating SNR between any two points in the survey region. The strength of the GCM approach is that it can be accomplished in an incremental fashion without prior knowledge of measurements and that each SNR measurement can be potentially used for estimating several random fields. One limitation is that Bayesian inference approach to estimate the random field uses only the SNR measurements. There are additional parameters that strongly influence acoustic communications, these include temperature and water depth. Future implementations may have stronger predictive value by considering these additional parameters. Future work will also include combining the communication map estimation techniques in a unifying SLAM framework.

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REFERENCES

[1] Stojanovic M. Zorzi M. Heidemann J. Underwater sensor networks: Applications, advances, and challenges. *Philosophical Transactions of the Royal Society (to appear)*, 2012.

[2] Kurose J. Partan J. and Levine B. A survey of practical issues in underwater networks. In *WUWNet06: Proceedings of the !st ACM international workshop on underwater networks*, pages 17–24, 2006.

[3] Oktug S. Erol-Kantarci M., Mouftah H. Localization techniques for underwater acoustic sensor networks, December 2010.

[4] A. Bahr, M. Walter, and J. J. Leonard. Consistent cooperative localization. In *IEEE International Conference on Robotics and Automation*, pages 3415–3422, May 2009.

[5] N.A.C. Cressie. *Statistics for Spatial Data (Wiley Series in Probability and Statistics)*. Wiley-Interscience, 1993.

[6] Michael L. Stein. *Interpolation of Spatial Data: Some Theory for Kriging*. Springer Series in Statistics, 1999.

[7] Muhammad Umer, Lars Kulik, and Egemen Tanin. Kriging for localized spatial interpolation in sensor networks. In *SSDBM*, pages 525–532, 2008.

[8] Hahnel D. Ferris B. and D. Fox. Gaussian processes for signal strength-based location estimation. *Proceedings of Robotics: Science and Systems*, 2006.

[9] Fox D. Ferris B. and Lawrence N. Wifi-slam using gaussian process latent variable models. *Proceedings of the International Joint Conference on Artificial Intelligence*, 2007.

[10] Sandipa S. Freitag L., Grund M. The whoi micro-modem: An acoustic communications and navigation system for multiple platforms. *Proc. OCEANS 2005, Washington, DC*, 2005.

[11] H. Omre. Bayesian kriging-merging observations and qualified guesses in kriging. *Mathematical Geology*, 19, 1987.

[12] Stein M.L. Handcock M.S. A bayesian analysis of kriging. *Technometrics*, 35, 1993.

[13] M.S. Hancock and J.R. Wallis. An approach to statistical spatial-temporal modeling of meteorological fields (with discussion). *Journal of the American Statistical Association*, 89:368–390, 1994.

[14] Kathryn Anne Haskard. *An anisotropic Matern spatial covariance model: REML estimation and properties*. PhD thesis, The University of Adelaide, School of Agriculture and Wine, Australia, 2008.