Approaching Maximum Power Conversion with Exergy-based Adaptive Wave-by-Wave Control of a Wave Energy Converter

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***Abstract*—This paper investigates wave-by-wave control of a single-mode wave energy converter driven to operate such that the oscillation velocity closely matches the hydrodynamically optimum velocity for best power absorption. Such control requires prediction of the wave profile using up-wave measurements. The goal of this research is to implement adaptive trajectory-tracking control using on-line estimation of mechanical and hydrodynamic parameters, where the hydrodynamically optimum velocity variation provides the reference trajectory. In a step-wise approach to this goal, this paper assumes that perfect predictions of the wave profile are available, and further that the impulse response functions representing the radiation and exciting forces are also fully known. The rest mass, infinite-frequency added mass, hydrostatic stiffness, and linearized viscous damping are estimated on line. The present method relies on feedback and feedforward forces derived based on a Lyapunov function comprised of a system Hamiltonian that combines the mechanical and information exergy functions.**

***Index Terms*—Wave energy, irregular waves, time domain, efficient conversion, adaptive control**

NOMENCLATURE

Λ_1, Λ_2, Λ_3 Gains associated with the parameter estimator
c_d Linearized viscous damping coefficient opposing heave velocity
$F_f(t)$ Exciting force in heave from incident + scattered waves
$F_r(t)$ Feedforward force based on latest parameter estimates and reference trajectory
F_{fb} Feedback force to provide trajectory tracking
$h_a(t)$, $h_b(t)$ Odd and even functions adding together to produce the causal function h_r.
$h_f(t)$ Impulse response function (kernel) for heave exciting force
$h_r(t)$ Impulse response function for the radiation force in heave
k Hydrostatic stiffness coefficient in heave
K_x, K_v, K_a Gains associated with the displacement, velocity, and acceleration errors
$M = m + \overline{a}(\infty)$ The mass of the device together with the infinite-frequency added mass in heave
$x(t)$ Heave displacement of the buoy
$x_r(t)$ Reference displacement trajectory to be tracked
$\hat{M}$, $\hat{c}_d$, $\hat{k}$ Estimates for values of the parameters M, c_d, k, respectively

I. INTRODUCTION

Wave energy converters utilizing relative oscillation between a wave-activated body and a reference have been studied for over 4 decades now (e.g. [1]). Because floating bodies in heave, pitch, or roll have distinct natural frequencies (due to hydrostatic restoring force/moments), they perform efficiently when the peak-frequencies in the incoming spectra are close to the body natural frequencies. For single-mode devices without viscous damping or oscillation constraints, efficient conversion requires that the device operate at resonance, and that the energy absorption rate match the energy radiation rate from body oscillation. Consequently, without hydrodynamic control, converters tend to be bulky and the annually average energy production frequently tends not to be cost-effective.

Early implementations of control (dating back to the mid 70's) involved adjustable resonant tuning via reactive (negative spring or positive inertia) loads [2], and latching-type switching control using braking forces to lock and release oscillation so as to force velocity when unlocked to be synchronous with exciting force [3]. Complex conjugate control consists

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of using the power take-off to apply a resistive load that matches the radiation damping for the floating body and in addition applying a reactive load that cancels the reactive part of the converter impedance (i.e. due to stiffness and inertia of the floating body). While such impedance matching results in maximum power absorption in regular waves, generalization of such control to irregular waves presents fundamental challenges ([4], [5]). Non-real time 'peak frequency tuning' type approaches have been attempted with success in practice [6], where simply matching impedances at the peak frequency of incoming spectra at regular intervals could lead to a three-fold improvement in annual energy production assuming that a power take-off capable of applying reactive power and drawing on on-board stored energy can be employed. 'Real-time application of complex conjugate control' could lead to still greater improvements, however, if the necessary reactive power can be supplied as required.

The challenges of implementing the frequency-domain notion of complex-conjugate control on a wave by wave basis for wave energy devices arise in part from the nature of the radiation force produced on the body by the waves generated by the body, and the fact that at any given instant, not only the waves created at that instant but also waves created at prior instants determine the total force. The impulse response function h_r describing this force is causal. Therefore, its Fourier transform is analytic in the upper half plane, which implies that the real and imaginary parts of its Fourier transform are constrained by the Kramers–Kronig relations [7]. Complex conjugate control requires that the real part be matched and the imaginary part be cancelled via the actuator load. For this reason the odd function part h_b and the negative of the even function part h_a of $h_r = h_a + h_b$ need to be synthesized separately in order to generate the real-time control force. Because both h_a and h_b are non-causal, velocity information from the future is required for complete synthesis of the control force. This situation is discussed in [4], [5]. In addition, because of the continuum nature of incident wave action and the body's geometric size, the body begins to 'feel' an exciting force somewhat before the incident wave arrives at its centroid. For these reasons, the impulse response function h_f describing the exciting force in the time domain is non-causal [see equation (1)] [5]. Application of complex conjugate control in real time therefore requires knowledge of the incident wave profile (and device oscillation), typically, 20-30 seconds in advance. Approximate approaches based on time-series analysis of past oscillations have been attempted (e.g. see [8], and the more recent, [9]). A number of approximate approaches have been developed in recent years (see [10]) for a comparative assessment). Direct use of incident wave profile measurement some distance up-wave for generating the control forces at the current instant has also been reported recently [11].

Considerable room for improvement exists in two areas: (1) more accurate approaches to utilize up-wave surface elevation for evaluating control forces, and (2) finding ways to deal with uncertainties in the physical and hydrodynamic parameters describing the converter dynamics. The work reported in this paper is relevant to the second area. First, although most converter models assume perfect knowledge of the in-air mass and moments of inertia, viscous damping, and hydrostatic stiffness, these may only be approximately known in practice and may even change as the device ages. A common example of model imprecision is the use of linearized stiffness coefficients where changes in water plane area with displacement are ignored, and linearized viscous damping. Second, the hydrodynamic quantities such as exciting force impulse response function, the radiation force impulse response functions, and the infinite-frequency added mass/inertias are generally only good approximations valid for small oscillations.

II. Formulation

The dynamics of a single-mode converter in irregular waves may be described as,

$$\begin{aligned}&[m+\overline{a}(\infty)]\ddot{x}+c_d\dot{x}+\int_0^\infty h_r(\tau)\dot{x}(t-\tau)d\tau\\&+kx=F_f+F_r+F_{fb}\end{aligned} \tag{1}$$

where the exciting force F_f due to the incident and scattered wave fields may be expressed as

$$F_f(t)=\int_{-\infty}^{\infty}h_f(\tau)\eta(x_G;t-\tau)d\tau \tag{2}$$

Here F_r is the feedforward force found using the current estimates of the parameters and the reference trajectory $\dot{x}_r$ meeting the impedance matching condition. F_{fb} denotes the feedback force to enable tracking, and $\eta(x_G;t)$ the incident wave elevation at the centroid. h_r is the radiation impulse response function. h_r is causal in that $h_r(t)=0, t<0$. h_f is the impulse response function representing the exciting force (due to the incident + scattered wave field with the body held fixed). c_d here represents a linearized viscous damping coefficient, and k denotes the linearized hydrostatic stiffness coefficient. m denotes the in-air mass of the body, while $\overline{a}(\infty)$ represents the infinite-frequency added mass for the body. In the following, these two masses are combined under a single symbol M. As mentioned in the previous section, there could be considerable uncertainty in the best estimates for all three quantities c_d, k, and M.

This paper investigates an adaptive control strategy which estimates and updates device parameters while computing control forces in real time. The approach is based on a technique used effectively with flexible robots, wind turbine blades, and micro-grid operation [12], [13], [14], [15], which begins with the synthesis of a convex Lyapunov function $\mathcal{H}$ which equals the total exergy formed by summing the mechanical and information exergies associated with the device dynamics and the parameters used in computing the feedforward and feedback forces. The term 'exergy' is here used in a more general sense than the traditional thermodynamic exergy (more details can be found in [16] and [17]). Response and parameter updates are then solved such that the time rate of change of

$\mathcal{H}$ is negative definite. The overall formulation is as outlined below. The total exergy is given by,

$$\mathcal{H} = \mathcal{H}_m + \mathcal{H}_i \tag{3}$$

where,

$$\begin{aligned}\mathcal{H}_m =& \frac{1}{2}M(\dot{x}-\dot{x}_r)^2 + \frac{1}{2}k(x-x_r)^2 \\ &+ \int_0^t \left[\int_{-\infty}^{\infty} h_a(\tau)(\dot{x}-\dot{x}_r)(t-\tau)d\tau\right](\dot{x}-\dot{x}_r)(\xi)d\xi\end{aligned} \tag{4}$$

represents the mechanical exergy. Further, the information exergy H_i is expressed as,

$$\begin{aligned}H_i =& \frac{1}{2}\{\phi\}^T[\Gamma]^{-1}\{\phi\} \\ & \frac{1}{2}K_x(x-x_r)^2 + \frac{1}{2}K_a(\dot{x}-\dot{x}_r)^2\end{aligned} \tag{5}$$

Next choosing the feedback force F_{fb} so as to provide a proportional-derivative-acceleration (PIA) control,

$$\begin{aligned}F_{fb}(t) =& K_x(x-x_r) + K_v(\dot{x}-\dot{x}_r) \\ &+ K_a(\ddot{x}-\ddot{x}_r)\end{aligned} \tag{6}$$

The feedforward force $F_r(t)$ is chosen to be,

$$\begin{aligned}F_r(t) =& \hat{M}\ddot{x}_r + \hat{k}x_r + \int_{-\infty}^{\infty} h_a(\tau)\dot{x}_r(t-\tau)d\tau \\ &- \int_{-\infty}^{\infty} h_b(\tau)\dot{x}_r(t-\tau)d\tau - \hat{c}_d\dot{x}_r\end{aligned} \tag{7}$$

Here $\hat{M}$, $\hat{c}_d$, and $\hat{k}$ are the estimates for the quantities M, c_d, and k respectively. In this work, perfect knowledge is assumed for h_r and h_f, as well as the incident wave profile $\eta(x_G;t)$ into the future. The overall controller thus consists of the feedback part F_{fb} and the feedforward part F_r. Equations (6) and (7) can be inserted into equation (1) to provide the dynamic constraint on the system states and the parameter estimates. This can be included directly in the total exergy $\mathcal{H}$. $\mathcal{H}$ can then be shown to be a stable manifold (positive definite function) with $dH/dt < 0$ if the following conditions are satisfied,

$$\begin{aligned}\dot{\hat{M}} &= -\gamma_1\ddot{x}_r(\dot{x}-\dot{x}_r) \\ \dot{\hat{c}}_d &= -\gamma_2\dot{x}(\dot{x}-\dot{x}_r) \\ \dot{\hat{k}} &= -\gamma_3\dot{x}_r(\dot{x}-\dot{x}_r)\end{aligned} \tag{8}$$

where γ_1, γ_2, and γ_3 are the adaptation gains and further, if the reference trajectory x_r is defined such that,

$$2\int_{-\infty}^{\infty} h_b(\tau)\dot{x}_r(t-\tau)d\tau + 2c_d\dot{x}_r = F_f \tag{9}$$

then,

$$\frac{d\mathcal{H}}{dt} = -(K_v + c_d)(\dot{x}-\dot{x}_r)^2 < 0 \tag{10}$$

where K_v and c_d are positive. Equation (10) is a negative semi-definite function which has a single equilibrium point. By applying LaSalle's Invariance Principle [14], it can be shown that equation (10) leads to asymptotic stability.

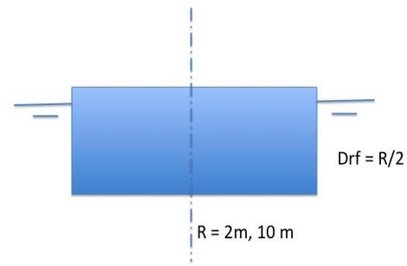


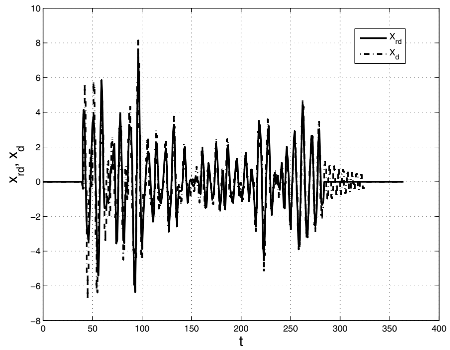
Fig. 1. A schematic view of the cylindrical buoy used in these calculations. Predominant small-amplitude heave oscillations with perfectly known radiation and excitation force kernels and wave surface elevation predictions are assumed. The rest mass + infinite frequency added mass, linearized viscous damping, and hydrostatic stiffness are assumed to contain nonlinearities.

III. Calculations

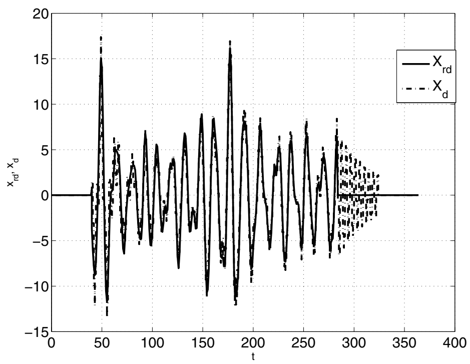
Simulations were carried out on a cylindrical buoy of radius $R = 10$ m, and draft $D_r = R/2$ m. A schematic view of this design is shown in Figure 1. A hydraulic ram or linear generator type power take-off is assumed, and a deeply submerged mass is thought to provide a reference for the heave oscillations, which for small-amplitude long waves are assumed to predominate. The hydrodynamic parameters and exciting force amplitude and phase were computed using the software HYDRAN [18] and provided by the University of Hawaii. The irregular wave time series were generated using Pierson-Moskowitz type 2-parameter wave spectra. For the results included here, the significant wave height H_s was set at $H_s = 1$ m. Calculations were carried out for two spectra computed for $T_e = 9$ s, and $T_e = 13$ s. The phases of the individual frequency components were chosen via a random number generator drawing over the interval $[0, 2\pi]$. Each simulation was run for 400 s. In cases where imperfect knowledge was assumed, both feedforward and feedback forces were updated at each time step using the most recent estimates for parameter values.

IV. Discussion of Results

Representative results from the simulation runs are discussed here. Simulations were carried out in two stages: (1) assuming perfect knowledge of the three parameters, i.e., $\hat{M} = M$, $\hat{c}_d = c_d$, and $\hat{k} = k$; and next (2) with parameter uncertainty in place and the estimates $\hat{M}$, and $\hat{c}_d$, and $\hat{k}$ updated at every time increment. Figure 2 plots the reference trajectory and actual trajectory for $T_e = 9$ s and $T_e = 13$ s. From Figure (2), it is observed that despite large initial deviations in the actual and desired velocity variations, the actual velocity converges rapidly to the reference trajectory. It should be noted that, the irregular waveform is regenerated at



(a) $T_e = 9$ s.



(b) $T_e 13$ s.

Fig. 2. Reference and actual trajectories for the heave velocity. Perfect knowledge of the parameters is assumed for $T_e = 9$ s. For $T_e = 13$ the parameter estimates are also updated at every iteration.

each run. Thus, with the phases of the individual frequency components are selected randomly, considerable differences could be expected in the reference trajectory over $t \in [0, 350]$ s from one run to the next. For the feedback and feedforward gains chosen to satisfy $\mathcal{H} > 0$, $d\mathcal{H}/dt < 0$, both the tracking and estimation errors should asymptotically $\to 0$. Nevertheless, swift convergence requires that the feedback gains be very high, as observed in the present calculations (with all three gains $\to 10^5$). High gains effectively reduce sensitivity to initial conditions. Note that each simulation run presents new initial conditions here, given random phase mixing of the frequency components making up each irregular incident wave input.

Figures 3–5 show the variation of the parameter estimates over $t \in [0, 350]$ s, for $T_e = 13$ s. There appears to be some room for improvement in the convergence properties for the estimates. Good trajectory tracking appears to be possible with imperfect estimates, which could be due to the fact that feedback terms dominate the $\mathcal{H}$ for the present gain selections.

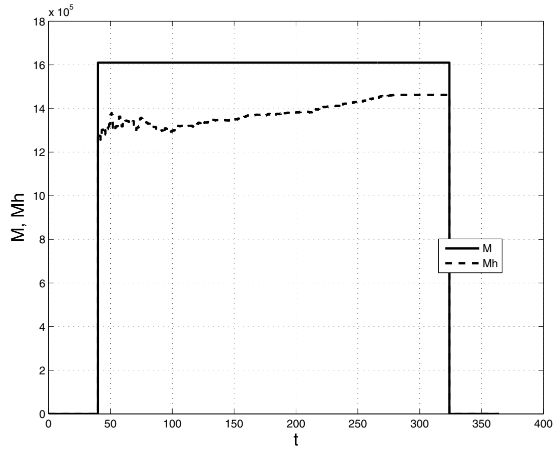


Fig. 3. True value of the mass parameter M and improving estimates.

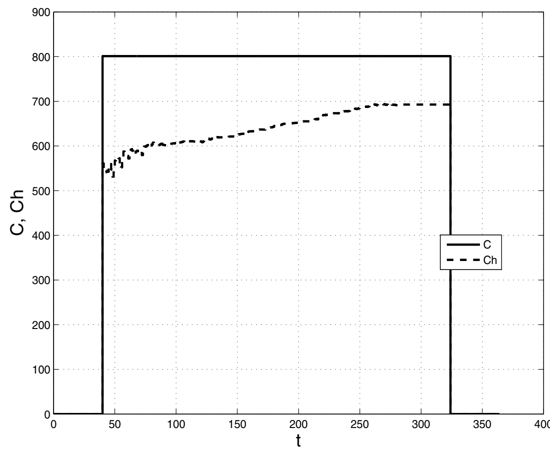


Fig. 4. True value of viscous damping c_d and improving estimates.

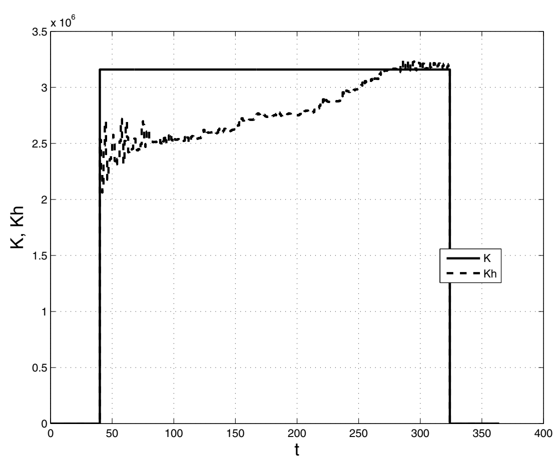


Fig. 5. True value of stiffness k and improving estimates.

V. Conclusions

The paper focuses on a ‘complex conjugate’ implementation in the time domain on a device with imperfectly known hydrostatic stiffness, viscous damping, and in-air mass. Perfect knowledge is assumed in the work reported in this paper, for the more challenging hydrodynamic parameters (the two impulse response functions h_r and h_f mentioned above) and the incident wave input into the future. However, approaches that would allow adaptive estimation of hydrodynamic parameters are currently being developed. Trajectory-tracking results incorporating position, velocity, and acceleration feedback (in addition to the feedforward force) show good convergence for large feedback gains. On-line estimates for the parameters M, c_d, and k show some steady-state errors, and further optimization of the estimator gains is possible.

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