Selection of Multi-angle Combination in Precise Turbidity Measurement

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***Abstract*—Turbidity is an obvious and direct indicator of water quality which embodies the existence and concentration of certain particles in water sample to some extent. Quantitative measurement of turbidity by methods with a light intensity sensor at a single angle has been studied adequately whereas the study is rare on methods involving multi-angle light intensity signals. In this paper, an optimization on angle selection of an existing four-angle turbidity measurement method has been discussed. Experiments have been conducted using an 850 nm laser and a power meter. Scattering light intensities at a series of angles are collected to estimate turbidity. Results show that the accuracy can be further improved by using proper angle combinations. Relative estimation error by using optimized angle combination is only about 36% of the existing method in average for the test samples.**

I. Introduction

Turbidity is "the reduction of transparency of a liquid caused by the presence of undissolved matter"[1]. It is a measure of the attenuation of light in water, especially scattering effect of the particles in water and is a direct and obvious indicator of water quality. Quantitative measurement of turbidity dates back to 1900[2] and the pursuit of accuracy promotes the improvement on measuring method. Nowadays, mainstream commercial turbidity meters mostly follow USEPA Method 180.1[3] or ISO 7027[1] standard.

An monochromatic radiation with the wavelength of 860 nm is required as the light source by ISO 7027 standard because light at the wavelength around 860 nm are less susceptible to sample's color[4] which has a negative impact on measuring.

In low turbidity condition, there is a well-defined linear relationship between 90° scattering light intensity and turbidity[5], thus turbidity estimated by this method can be accurate. A detecting angle at 90° between the incident light and photon detector is considered to be standard for particle scattering measurement and employed in most modern turbidity meters(Fig. 1).

However, when the turbidity is higher, for example the turbidity is more than 400 NTU, the method using light intensity signal at only one angle dose not preform well. Then, a so called Ratio method was used in a commercial turbidimeter[6] where four sensors measuring signals from $0°, 30°, 90°$ and $138°$, in order to deal with the dramatic variation of the light intensity in measuring a wide range of turbid samples. This idea is inspiring that more angles may bring in more information about the sample therefore improving the accuracy. However, there is still a lack of research in the selection of multi-angle combination so far.

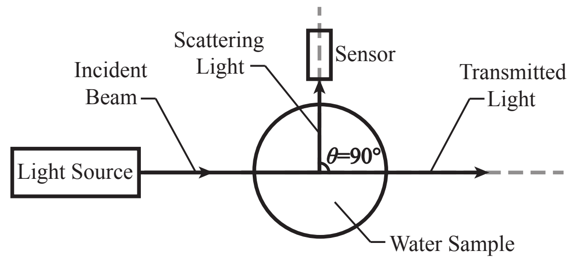

Fig. 1. Principle of a mainstream turbidimeter which detects the scattering light at 90°.

In this paper, optimized combinations of the four angles has been found which leads to further improvement of the accuracy. In Section 2, the methods used to estimate turbidity and to evaluate angle combinations are illustrated. The experiment is described in Section 3. In Section 4, the results are analyzed and conclusions are drawn in Section 5.

II. Methods

A. Ratio Method (Method 1)

The attenuation of light in water consists of absorption and scattering. Rayleigh scattering[7] and Mie scattering[8] are involved in turbidity measurement. With the turbidity increasing, more and more intensity of light is absorbed and multi-scattering aggravates, which leads to the non-linearity between 90° light intensity and turbidity. The approach of ratio method[6] is to divide the scattering light intensity at 90° by the weighted sum of four light intensity at angles of $\{0°, 30°, 90°, 138°\}$, which can be expressed as

$$T = \frac{I_{90}}{I_{90} \cdot k_1 + I_0 \cdot k_2 + I_{30} \cdot k_3 + I_{138} \cdot k_4} \tag{1}$$

where T represents turbidity, $I_0, I_{30}, I_{90}, I_{138}$ represents the light intensity at the angle of $0°, 30°, 90°, 138°$ respectively. This method gets rid of the influence of sample color and fluctuate of light source. In high turbidity cases, effects from light absorption and multi-scattering can be alleviated to some extent.

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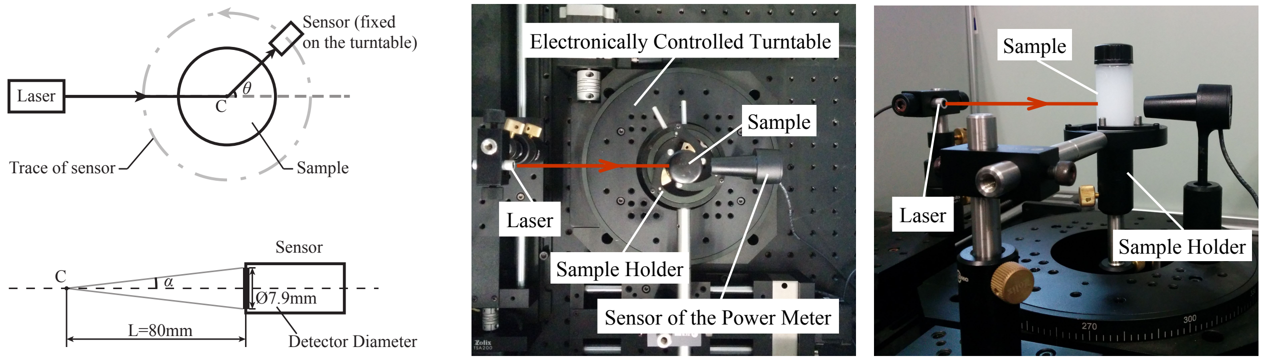

(a) Schematic of the experimental setup. (b) Top view of the experimental setup. (c) Side view of the experimental setup.

Fig. 2. An 850nm laser is used as light source. A power meter is fixed on a turntable to collect scattering light intensity at a series of angles. The turntable rotates at an interval of 5° and the intensity is collected every five degrees.

B. Estimation and Evaluation

In method 1, the four angles are fixed. In order to optimize the angle combination, a general form can be written as

$$T = \frac{I_{\theta 1}}{k_1 \cdot I_{\theta 1} + k_2 \cdot I_{\theta 2} + k_3 \cdot I_{\theta 3} + +k_4 \cdot I_{\theta 4}} \tag{2}$$

where $\{I_{\theta 1}, I_{\theta 2}, I_{\theta 3}, I_{\theta 4}\}$ are the scattering light intensities at $\{\theta_1, \theta_2, \theta_3, \theta_4\}$ degree, $\{k_1, k_2, k_3, k_4\}$ are parameters to be determined. In Eq. (2), any four distinct angles can be assigned to $\{\theta_1, \theta_2, \theta_3, \theta_4\}$. It is clear that $I_{\theta 2}, I_{\theta 3}, I_{\theta 4}$ are at the same position while $I_{\theta 1}$ is more special. Any change in the order of $\{\theta_2, \theta_3, \theta_4\}$ will not impact the result. For instance, $\{90°, 0°, 30°, 138°\}$ and $\{90°, 138°, 0°, 30°\}$ result in the same T for Eq. (2). However, $\{90°, 0°, 30°, 138°\}$ and $\{0°, 90°, 30°, 138°\}$ lead to different T.

In order to determine $\{k_1, k_2, k_3, k_4\}$, the linear least squares method is used. Rewrite Eq. (2) as

$$TI_{\theta 1}k_1 + TI_{\theta 2}k_2 + TI_{\theta 3}k_3 + TI_{\theta 4}k_4 = I_{\theta 1}, \tag{3}$$

which can be represented in a vector form of

$$\begin{bmatrix} TI_{\theta 1} & TI_{\theta 2} & TI_{\theta 3} & TI_{\theta 4} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = I_{\theta 1} \tag{4}$$

Suppose we have M sets of T and $\{I_{\theta 1}, I_{\theta 2}, \ldots, I_{\theta 4}\}$, which means M different samples are used for calibration. For the Mth sample, the turbidity is T_M and the scattering light intensities at angles of $\{\theta_1, \theta_2, \ldots, \theta_p\}$ are $I_{\theta 1,M}, I_{\theta 2,M}, \ldots, I_{\theta 4,M}$. Then, M equations can be formed and stacked in a matrix form as

$$\underbrace{\begin{bmatrix} T_1 I_{\theta 1,1} & T_1 I_{\theta 2,1} & \ldots & T_1 I_{\theta 4,1} \\ T_2 I_{\theta 1,2} & T_2 I_{\theta 2,2} & \ldots & T_2 I_{\theta 4,2} \\ \vdots & \vdots & & \vdots \\ T_M I_{\theta 1,M} & T_M I_{\theta 2,M} & \ldots & T_M I_{\theta 4,M} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_4 \end{bmatrix}}_{K} = \underbrace{\begin{bmatrix} I_{\theta 1,1} \\ I_{\theta 1,2} \\ \vdots \\ I_{\theta 1,M} \end{bmatrix}}_{B}$$

which can be expressed as a matrix equation as

$$AK = B \tag{5}$$

Where $A \in \mathbb{R}^{M \times 4}$ is a coefficient matrix with each element from the data of T and $\{I_{\theta 1}, I_{\theta 2}, \ldots, I_{\theta 4}\}$, $K \in \mathbb{R}^4$ is the unknown vector to be determined, $B \in \mathbb{R}^M$ is a coefficient vector built from T. If $M \geqslant 4$, i.e., the number of measurement data sets is more than the number of unknown parameters, then the columns of matrix A are linear independent in general. Then vector K can be estimated as

$$\hat{K} = (A^{\mathrm{T}} A)^{-1} A^{\mathrm{T}} B \tag{6}$$

where $\hat{K}$ is the estimate of K, with $\hat{K} = [\hat{k}_1, \hat{k}_2, \hat{k}_3, \hat{k}_4]^{\mathrm{T}}$. Thus the estimated turbidity $\hat{T}$ can be calculated as:

$$\hat{T} = \frac{I_{\theta 1}}{\hat{k}_1 \cdot I_{\theta 1} + \hat{k}_2 \cdot I_{\theta 2} + \hat{k}_3 \cdot I_{\theta 3} + +\hat{k}_4 \cdot I_{\theta 4}} \tag{7}$$

Mean relative error is used to evaluate the accuracy of different angle combinations. For each estimated $\hat{T}$, the relative error ϵ is

$$\epsilon = \frac{|\hat{T} - T|}{T} \tag{8}$$

Assume there are n samples for test, the mean relative error can be calculated by

$$\epsilon_{mean} = \frac{1}{n} \sum_{i=1}^{n} \frac{|\hat{T}_i - T_i|}{T_i}, \tag{9}$$

where ϵ_{mean} is the mean relative error, n is the number of samples in the test set, T_i and $\hat{T}_i$ are turbidity and the estimated turbidity of a certain sample, respectively.

Moreover, the maximum relative error (ϵ_{max}) among the n test samples is used as a subsidiary index of accuracy.

III. Experiment

An experimental setup has been built to estimate turbidity. As shown in Fig. 2, the setup mainly includes a turntable, a laser source and a power meter. The electronically controlled turntable (RSA200, Zolix, China) is fixed on a breadboard and is controlled by a computer with LabVIEW (NI, US). A power meter (FieldMaxII-TO, Coherent, US) matching with a silicon photoelectric diode sensor (OP-2 VIS, Coherent, US) is used to measure the scattering light intensity. The sensor of power meter is fixed on the turntable and rotates around the center of the turntable. An 850 nm laser (CPS850S, THORLABS, US) is used as the light source and a sample holder is fixed at the

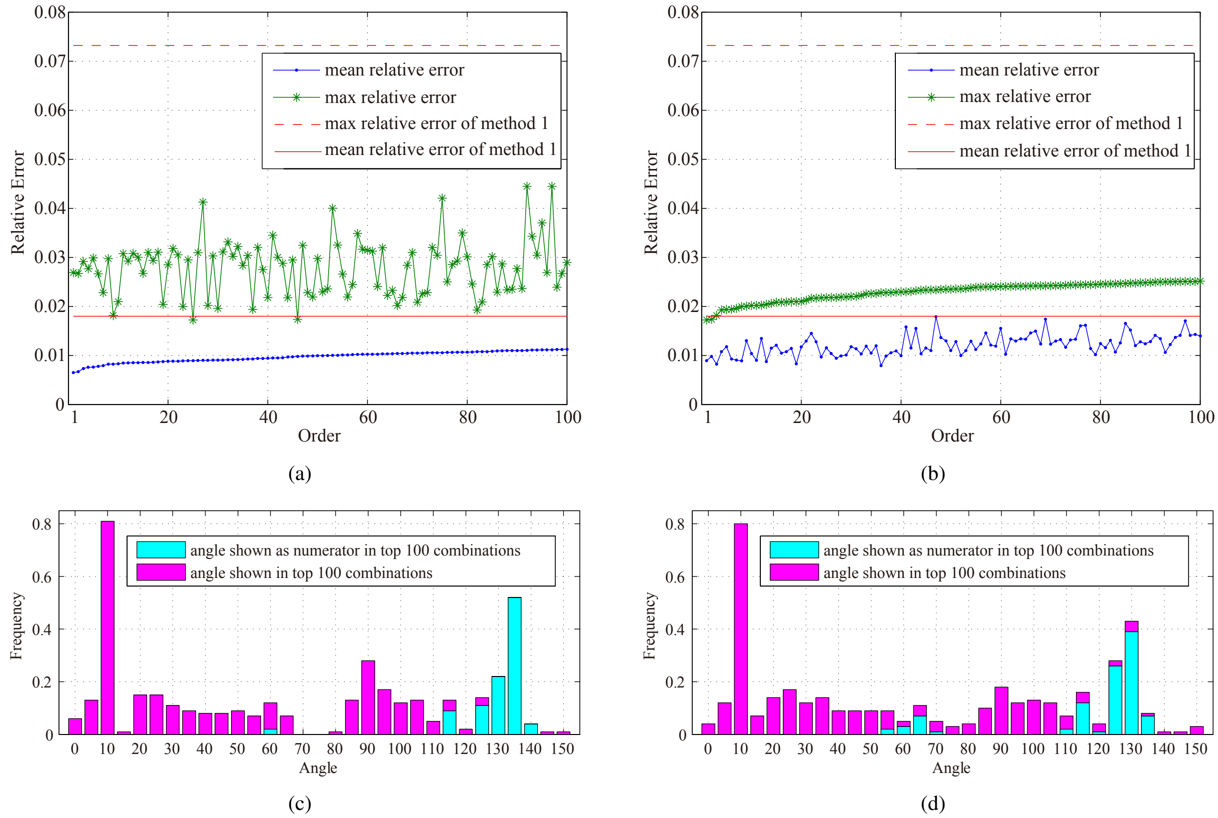

Fig. 3. (a) and (b) show the relative error of top 100 combinations sorted in ascending order of mean relative error and max relative error, separately. (c) and (d) show the frequency of angle appearance in top 100 combinations sorted in ascending order of mean relative error and max relative error, separately.

centre of the turntable. The laser and sensor are adjusted to be in the same horizontal plane.

In order to collect the light intensities at a series of angles, the turntable is rotated at an interval of $5°$ and thereby recording the intensity every five degrees. Because of the sensor shade, scattering light beyond $150°$ cannot be detected, therefore the detectable range is $0° \sim 150°$. Assume all scattering light detected by the sensor comes from the center of sample (point C in Fig. 2(a)). Since the distance from point C to the detector of sensor is 80 mm (denoted as L) and the diameter of detector (denoted as D) is 7.9 mm, the visual angle can be calculated by $2\alpha = 2tan^{-1}(D/2L) \approx 5.6°$. Thus the angle around $138°$ specified by Method 1 is replaced by $135°$ in this paper. Moreover, the interval of $5°$ is adequate to cover all angles from $0°$ to $150°$.

The measuring time for each sample is within two minutes so that the variation in turbidity can be ignored. The measurement of each sample has been repeated for three times and the average is taken to alleviate the influence of measurement noise.

Formazin is used as sample, which is specified in ISO 7027[1]. A bottle of standard formazin solution of 4000 NTU is diluted into several different turbidities with reverse osmosis purified water (resistivity larger than $18M\Omega \cdot cm$ at $25°C$) in lab, following the instruction by Barter's study[9]. The turbidity of the samples is listed in TABLE I, ranging from 160 NTU to 4000 NTU with an equal interval of 160 NTU. In total, 25 samples have been prepared for experiments, with 15 samples (in normal format, e.g., 160) for the calibration of parameters and 10 (in bold and underlined, e.g., **320**) for test.

TABLE I. TURBIDITY OF THE STANDARD FORMAZIN SOLUTIONS USED AS SAMPLES. THE BOLDFACED AND UNDERLINED VALUES (E.G., **320**) ARE TURBIDITY OF THE SAMPLES FOR TESTS AND OTHERS (E.G., 160) ARE FOR CALIBRATION.

Turbidity (NTU)									
160	**320**	480	**640**	800	960	**1120**	1280	**1440**	1600
1760	**1920**	2080	**2240**	2400	2560	**2720**	2880	**3040**	3200
3360	**3520**	3680	**3840**	4000					

IV. RESULTS AND DISCUSSION

During experiments, 25 samples have been used and for each of them, the scattering light intensity at 31 angles (every 5 degrees from $0°$ to $150°$) are recorded in a matrix $\mathbf{I} \in \mathbb{R}^{25\times31}$. The standard turbidity of the samples are recorded in a vector $T \in \mathbb{R}^{25}$. These data are divided into calibration set and test set according to TABLE I.

There are 31465 different combinations by choosing 4 angles from 31 angles. For each combination, the order of the four angles is changed (to make every one of the four angles been the first position once) to produce four different patterns. Then, there are 125860 different angle combinations in total, i.e., there are 125860 sets of $\{\theta_1, \theta_2, \theta_3, \theta_4\}$ and the goal is to find the appropriate set. For each combination, the T and corresponding $\{I_{\theta1}, I_{\theta2}, I_{\theta3}, I_{\theta4}\}$ of calibration set are substituted in Eq. (5) and Eq. (6) to estimate $\{k_1, k_2, k_3, k_4\}$. Then, $\{I_{\theta 1}, I_{\theta 2}, I_{\theta 3}, I_{\theta 4}\}$ from test set are substituted into Eq. (7) to calculate $\hat{T}$ for each sample in test set. After that, Eq. (9) is used to work out the mean relative error (ϵ_{mean}).

The red solid line and dashed line in both Fig. 3(d) and Fig. 3(b) illustrate the $\epsilon_{mean} = 1.80\%$ and $\epsilon_{max} = 7.32\%$ of method 1 with angle combination of $\{90°, 0°, 30°, 135°\}$, respectively.

The angle combinations are evaluated mainly by ϵ_{mean}. Mean relative errors of different angle combinations are sorted in ascending order and the top 100 are shown as the blue line with dots in Fig. 3(d). The minimal ϵ_{mean} is 0.65% with the angle combination of $\{135°, 10°, 35°, 90°\}$ and the 100th ϵ_{mean} is 1.13% with the angle combination of $\{130°, 10°, 30°, 95°\}$. The mean relative errors increase gently from the first to the 100th. The corresponding ϵ_{max} are shown as the green line with stars in Fig. 3(d) which ranges from 1.72% to 4.45%. It is clear that the mean relative error can be significantly decreased by choosing the proper angle combination. Fig. 3(c) shows the frequency of angle appearance in the top 100 combinations sorted in ascending order of mean relative error. For example, the angle $10°$ shows 81 times in the top 100 combinations and occupied 81% in Fig. 3(c). Especially, $135°$ shows 52 times and all 52 times are in the position of the first angle, thus the red bar is covered by the blue bar. It can be seen from Fig. 3(c) that angles around $10°, 135°$ and $90°$ shows much more than other angles. Additionally, larger angles around $135°$ are the most appropriate angles to be the numerator in Eq. (2).

Besides, ϵ_{max} is also used to evaluate angle combinations. Max relative errors of different angle combinations are sorted in ascending order and the top 100 are shown as the green line with stars in Fig. 3(b). The minimal ϵ_{max} is 1.72% with the angle combination of $\{125°, 10°, 30°, 90°\}$ and the 100th ϵ_{max} is 2.52% with the angle combination of $\{60°, 120°, 5°, 25°\}$. The max relative errors also increase gently from the first to the 100th. The corresponding ϵ_{mean} are shown as the blue line with dots in Fig. 3(b) which ranges from 0.79% to 1.79%. As to angle distribution, $10°, 90°$ and $125° \sim 130°$ are dominant. Again, using larger angles at around 130 as the first angle is more beneficial.

V. Conclusion

The optimization of angle selection in the existing four-angle turbidity measurement method has been discussed in this paper. An experiment has been conducted to estimate turbidity and evaluate different angle combinations. Results shows that the accuracy can be further improved by harnessing proper angles. The mean relative error of a set of sample by using appropriate angles is decreased to only $36°$ of the result by using the existing method. Involving angles around $10°, 90°$ and $130°$ tends to result in better turbidity estimation accuracy. Backward scattering light at larger angles around $125° \sim 135°$ are more proper to be the first angle in angle combinations.

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