Underwater Acoustic Localization Approach to an Arbitrary Distributed Multi-Element Array

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***Abstract*—For USBL (ultra-short baseline) positioning system, orthogonal receiving arrays with three elements distributed in 'L'-shape or four elements forming a cross are generally used. With this configuration, the accuracy of the localization results depend on the accuracy of the coordinates of array elements, and sometimes the quadrant of the results may be significantly changed. Using redundant data measured by multi-element array can make the localization results robust. But, the calculation procedure to the multi-element array is based on geometry projections, which is complicated. And extra localization errors are generated because of model approximation. In order to overcome these problems, this article designed a novel underwater acoustic localization approach to an arbitrary multi-element array, such as a three-dimensional or two-dimensional array with multiple elements. Using coordinate rotation theory of surveying and mapping field, this approach establishes a reference coordinate system, so that the object's bearings in the reference coordinate system and the measured data can be combined with the help of rotation matrix, and the bearings can be determined by least square estimation. Taking the approach developed in this article, not only the complicated calculation procedure is simplified, but also the localization robustness is improved. Computer simulation and tank test results show the novel approach can work well.**

Keywords—localization; coordinate rotation; multi-element array; arbitrary

I. Introduction

Nowadays, more and more attentions have been paid to exploring the ocean resources and environment. Underwater submersibles (such as ROVs, AUVs, etc.) play a very vital role in the activities for exploring oceans. Because of the small size and convenient deployment, USBL (ultra-short baseline) has been widely used as an underwater navigation and positioning equipment to underwater submersibles. During the past years, much effort are put on designing a localization algorithm which can make the localization results more robust and more accurate [1-5]. Although some methods exist, they are aimed at the USBL receiving array with simple structure, such as three-element or four-element orthogonal arrays [3, 6]. And the approach to these simple arrays is based on geometry projections, which lead complicated calculation procedures [3-5]. This paper designs a novel localization approach based on coordinate rotation theory, with the advantages: 1) can be applied to arbitrary multi-element USBL receiving array (such as the receiving array with four elements or more), no matter it is two-dimensional or three-dimensional; 2) simplifies the complicated calculation procedure; 3) improves the localization robustness.

This paper is organized as follows. Firstly, the coordinate rotation theory is introduced to show how to make coordinate rotation in the reference coordinate system. Then a specific two-dimensional pentagon array is considered to show the detailed calculation procedures of the approach. In the end, Computer simulation and tank test results is given to verify the conclusion that the novel approach can work well and the localization robustness is improved.

II. Positioning Theory

The positioning principle of USBL is described as follows. Firstly, calculating the object's bearings and slant range in the reference coordinate system defined by the receiving array, so that the object's coordinate in the reference coordinate system can be determined. Then, gathering the geodetic coordinate of the origin of the reference coordinate system by GPS receiver and the attitude information of the receiving array by gyro. The installation error, made up of position and angle error, is also needed to know. In the end, combining these data and use coordinate transformation theory, the geodetic coordinate of the object could be calculated.

In this paper, we only talk about the approach that how to determine the object's coordinate in the reference coordinate system.

A. Bearings Estimation

As shown in Fig. 1, the USBL receiving array is composed of five elements forming a three-dimensional arbitrary formation, and the reference coordinate system $\Psi = (O, x, y, z)$ is defined by the receiving array, which is a left-handed coordinate system. The origin of $\Psi = (O, x, y, z)$ coincides with the geometric center of the array. In Fig. 1, element 1, 2, 3 lies on the xoy plane, while element 4, 5 are located on the each side of the xoy plane respectively. We

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assume that the object's coordinate in $\Psi=(O,x,y,z)$ is $\mathbf{X}=(X,Y,Z)^{\mathrm{T}}$, with the slant range R and the bearing α, β, γ (α is the angle between the direction to the object and the positive direction of x-coordinate axis. β is the angle between the direction to the object and the positive direction of y-coordinate axis. γ is the angle between the direction to the object and the positive direction of z-coordinate axis). The coordinates X, Y and Z of the object in $\Psi=(O,x,y,z)$ can be expressed as follows: $X=R\cos\alpha$, $Y=R\cos\beta$, $Z=R\cos\gamma$.

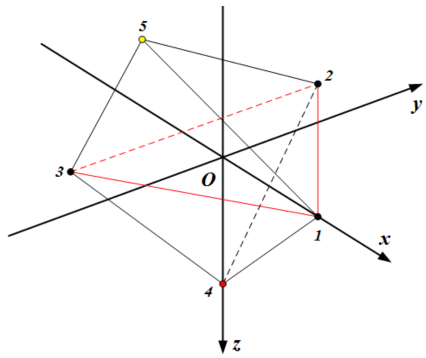

Fig. 1. USBL Receiving Array and the Reference Coordinate System

The coordinate system $\Psi_n=(O,x_n,y_n,z_n)$ is determined by $\Psi=(O,x,y,z)$ and element n. Fig. 2 shows the coordinate system $\Psi_n=(O,x_n,y_n,z_n)$: the origin of $\Psi_n=(O,x_n,y_n,z_n)$ coincides with the origin of $\Psi=(O,x,y,z)$, element n is located on the positive x_n-coordinate axis. That is, $\Psi_n=(O,x_n,y_n,z_n)$ can be generated when $\Psi=(O,x,y,z)$ rotates centering on the origin. Fig. 3 shows the rotating diagram from $\Psi=(O,x,y,z)$ to $\Psi_2=(O,x_2,y_2,z_2)$.

In Fig. 2, the coordinate of point P is $\mathbf{X}_n=(X_n,Y_n,Z_n)^{\mathrm{T}}$ in $\Psi_n=(O,x_n,y_n,z_n)$, α, β, γ are the corresponding bearings (α_n is the angle between the direction to the point P and the positive direction of x_n-coordinate axis. β_n is the angle between the direction to the point P and the positive direction of y_n-coordinate axis. γ_n is the angle between the direction to the point P and the positive direction of z_n-coordinate axis). The coordinate of point P in $\Psi_n=(O,x_n,y_n,z_n)$ can be written as: $X_n=R\cos\alpha_n$, $Y_n=R\cos\beta_n$, $Z_n=R\cos\gamma_n$. According to the coordinate rotation theory, there is

$$\mathbf{X}_n=\mathbf{R}_n\cdot\mathbf{X}, \tag{1}$$

where $\mathbf{R}_n$ denotes the rotation matrix from the coordinate $\mathbf{X}$ in $\Psi=(O,x,y,z)$ to the coordinate $\mathbf{X}_n$ in $\Psi_n=(O,x_n,y_n,z_n)$.

To show the detailed calculation procedure, a regular pentagon USBL receiving array is considered to derive the equation including the unknown bearings α and β. As mentioned above, $\Psi=(O,x,y,z)$ is the array reference coordinate system. The array elements lie on the xoy plane of $\Psi=(O,x,y,z)$, uniformly distributed on a circle with the radius r, element 1 is located on the positive x-coordinate axis, as shown in Fig. 4. The time measurements of the array elements are respectively t_1, t_2, t_3, t_4, t_5. For convenience narration, element 0 is introduced as a virtual element at the origin of $\Psi=(O,x,y,z)$ with the measurement t_0.

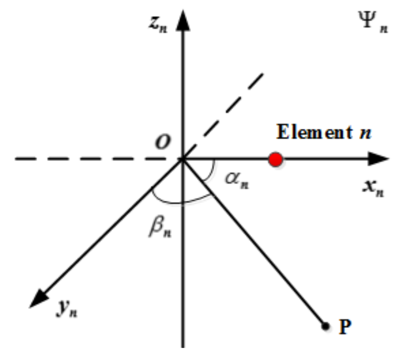

Fig. 2. Coordinate System $\Psi_n=(O,x_n,y_n,z_n)$

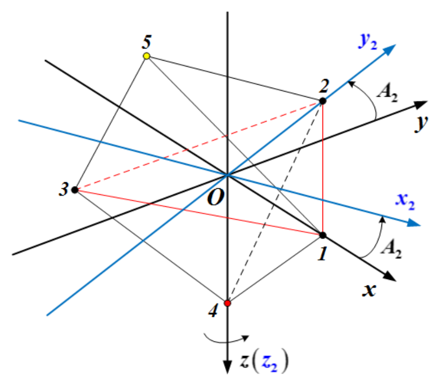

Fig. 3. The Diagram of Coordinate System Rotation

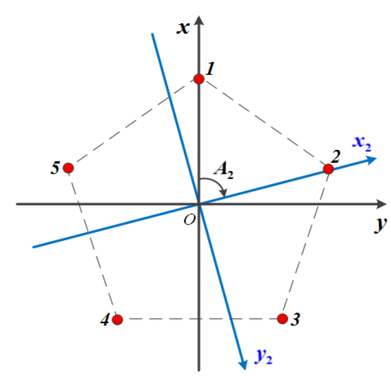

Fig. 4. The Diagram of Pentagon USBL Receiving Array

In $\Psi_n=(O,x_n,y_n,z_n)$, element *n* is located on the positive x_n-coordinate axis, and z_n-coordinate axis coincides with z-coordinate axis. A_n denotes the rotated angle from $\Psi=(O,x,y,z)$ to $\Psi_n=(O,x_n,y_n,z_n)$, and the positive direction of A_n is clockwise.

Supposing the coordinate of point P is $(X,Y,Z)^{\mathrm{T}}$ in $\Psi=(O,x,y,z)$, with the slant range *R*. In $\Psi_n=(O,x_n,y_n,z_n)$, the bearings of point P are α_n and β_n, as shown in Fig. 2. The time delay τ_{n0} between element *n* and the virtual element 0 (assuming that R>>r) can be expressed in the following way:

$$\tau_{n0}=\frac{L\cdot\cos\alpha_n}{c}, \tag{2}$$

where c is sound speed, L is the length of baseline from element *n* to the virtual element 0, equaling to the radius *r*, $\tau_{n0}=t_0-t_n$. According to (1), there is

$$\begin{bmatrix} R\cos\alpha_n \\ R\cos\beta_n \\ R\cos\gamma_n \end{bmatrix}=\begin{bmatrix} \cos A_n & \sin A_n & 0 \\ -\sin A_n & \cos A_n & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} R\cos\alpha \\ R\cos\beta \\ R\cos\gamma \end{bmatrix}, \tag{3}$$

choosing the first row of (3),

$$\cos\alpha_n=\cos A_n\cos\alpha+\sin A_n\cos\beta. \tag{4}$$

Combining (2) and (4), we can get the equation

$$\frac{c\cdot\tau_{n0}}{L}=\cos A_n\cos\alpha+\sin A_n\cos\beta, \tag{5}$$

then (5) is the equation corresponds to $\Psi_n=(O,x_n,y_n,z_n)$. So the equations respectively corresponding to $\Psi_1=(O,x_1,y_1,z_1)$ and $\Psi_2=(O,x_2,y_2,z_2)$ are

$$\frac{c\cdot\tau_{10}}{L}=\cos A_1\cos\alpha+\sin A_1\cos\beta, \tag{6}$$

$$\frac{c\cdot\tau_{20}}{L}=\cos A_2\cos\alpha+\sin A_2\cos\beta, \tag{7}$$

Since there is no measurement t_0 exist, we need to make a subtraction between (6) and (7) to remove t_0. The result of (6) minus (7) is

$$\begin{aligned}\frac{c}{L}(t_2-t_1)=&(\cos A_1-\cos A_2)\cos\alpha\\&+(\sin A_1-\sin A_2)\cos\beta\end{aligned}. \tag{8}$$

Equation (8) is the positioning equation based on element 1 and element 2, which can be used to calculate the bearings α and β.

According to combination knowledge, there are $\mathrm{C}_5^2=10$ positioning equations exist and every equation has the unknown factors α and β. So we can choose two of the ten equations to calculate the unknown factors α and β. For example, the equation group composed of the equation based on element 1 and element 2 and the equation based on element 1 and element 3 is

$$\begin{cases}\frac{c}{L}(t_2-t_1)=(\cos A_1-\cos A_2)\cos\alpha\\ \qquad\qquad+(\sin A_1-\sin A_2)\cos\beta\\ \frac{c}{L}(t_3-t_1)=(\cos A_1-\cos A_3)\cos\alpha\\ \qquad\qquad+(\sin A_1-\sin A_3)\cos\beta\end{cases}. \tag{9}$$

Equation (9) can determine a group of solutions about α and β.

It is worth noting that not all equation groups like (9) are resoluble, such as the equation group made up of the equation based on element 1 and element 2 and the equation based on the element 3 and element 5. After choosing the resoluble equation groups, there are only twenty different equation groups exist, any one of which is independent from the others. Also, we can use three or more of the ten equations like (5) to determine α and β with least square estimation, but only on the condition that the time measurements correspond to the chosen elements are unbiased estimated can the correct α and β be calculated. However, on most situations, there are one or two biased time measurements because of the terrible working environment or the bad performance of the device, which has slight effect on range estimation but has significant effect on bearings estimation on the situation that using all the ten equations to calculate α and β with least square estimation. Fortunately, we can remove the biased time measurements through analyzing the consistency of the results of the twenty independent equation groups like (9). Then using the left correct time measurements to calculate α and β with least square estimation. This analyzing process is simple but necessary to improve the localization robustness.

The value of the unknown bearing γ is determined by the values of α and β. The formula can be written as

$$\cos\gamma=\sqrt{1-(\cos^2\alpha+\cos^2\beta)}. \tag{10}$$

B. Range Estimation

The range estimation is also important to the position estimation. Because the USBL receiving array is small and the origin of the reference coordinate system coincides with the geometric center of the array, the propagation time from the object to the origin can be calculated by averaging the time measurements of all the elements. Therefore, the slant range of the object in array reference coordinate system can be calculated as follows:

$$R = c \times \frac{1}{N} \times \sum_{i=1}^{N} t_i \ , \qquad (11)$$

where N is the number of the receiving elements.

C. The Coordinate in the Reference Coordinate System

Based on the known bearings and slant range of the object in array reference coordinate system, the object's coordinate can be determined as follows,

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R \cdot \begin{pmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{pmatrix} . \qquad (12)$$

III. SIMULATION

In the simulation, we also consider a two-dimensional pentagon array (as shown in Fig. 4). The simulation parameters are listed as follows: the radius of the receiving array is 0.14m, sound speed is constant set as 1480m/s. The simulation is just used to verify the validity of the localization approach, so it is assumed that the propagation times from the object to the different elements can be measured accurately.

Fig. 5 shows the slant range error between the estimation array and the true value. The coordinate Z of the object is set as 2m, slant range varies from 2m to 100m. As we can see in Fig. 5, the slant range error decreases with the increase of slant distance, and the maximum error is less than 0.005m. The simulation result indicates that the slant range error generated by averaging the time measurements of the receiving elements has slight effect on the position determination.

Fig. 6 shows the positioning results of the object. In the simulation, the object moves along three different tracks, all of which are circular with different radius 17.23m, 10m, 5.78m. The coordinate Z of the object is set as 10m. In Fig. 6, the lines with different color named '60°True', '45°True', and '30°True' represent the different true tracks with the radius 17.23m, 10m, 5.78m respectively. And the points with different color named '60°Res', '45°Res', and '30°Res' denote the positioning results corresponding to the tracks with the radius 17.23m, 10m, 5.78m respectively. As we can see in Fig. 6, it is obvious that the positioning results and the true tracks are well matched, which demonstrates that this novel localization approach is valid.

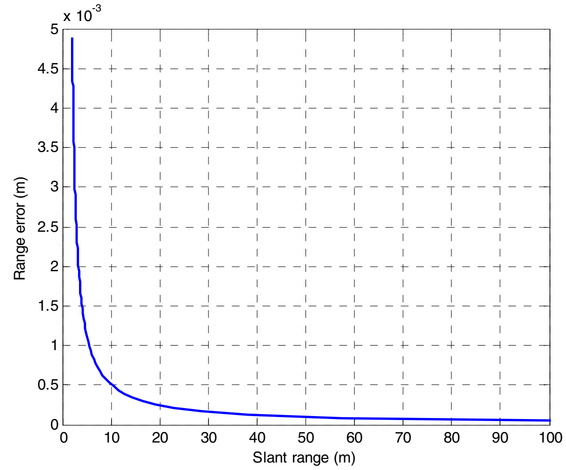

Fig. 5. The Slant Range Error Change Tendency

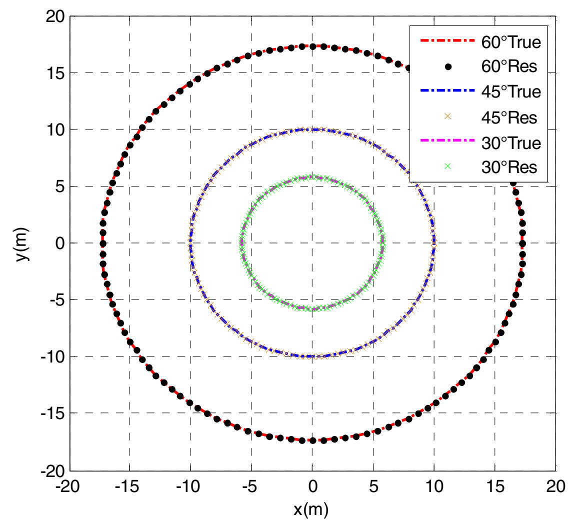

Fig. 6. Simulation Positioning Results

IV. EXPERIMENT

The tank test was carried out at the lab of the College of Underwater Acoustic Engineering, Harbin Engineering University, on November 3rd, 2014. The shape of the USBL receiving array is pentagon, whose elements are uniformly distributed on a circle with the radius 0.14m. The array was deployed on the bridge crane, and the sound source was fixed on the other side of the tank, as Fig. 7 shows. The transmitted signal is sinusoidal with the frequency 25kHz. The propagation times from the sound source to the different receiving elements are estimated based on cross-correlation detection technology. Sound speed in the tank was 1480 m/s.

Fig. 7 shows the diagram of tank test process. In the test, the array moves along three different tracks. Track 1 was a circle when the array turned clockwise. Track 2 was also a circle when the array turned anticlockwise. Track 3 was a line when the array made lateral movement along the bridge crane path.

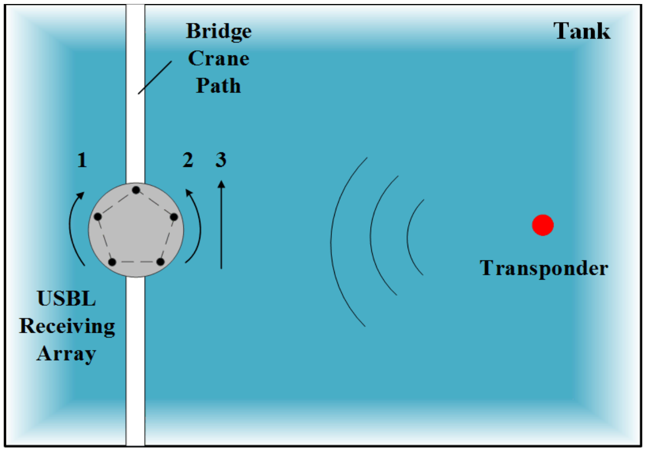

Fig. 7. The Schematic Diagram of Test Process

Fig. 8 shows the localization results to the different tracks. The left figure shows the results to track 1 and track 2. The black points are the localization results to track 1. The green points and the red points both are the results to track 2. The red results is gained by directly utilizing all the time measurements of the array to calculate by least square estimation. The green results is gained on the condition that the biased time measurements are removed. The right figure shows the results to track 3. It is obvious that the black trajectory and the green trajectory both are a circle with the same radius and they are well matched. And the blue trajectory in the right figure is a line, corresponding to track 3.

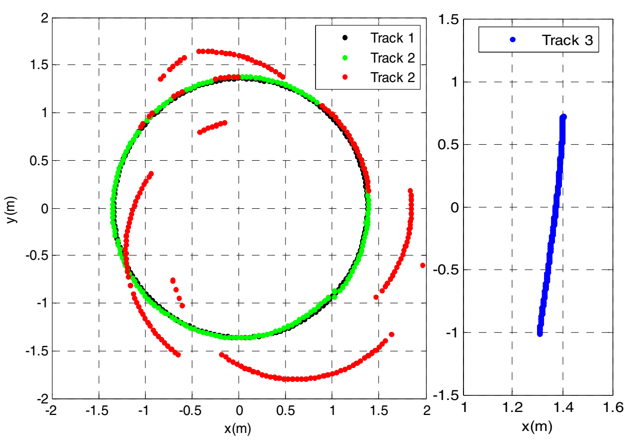

Fig. 8. The Localization Results to Different Tracks

Table 1 lists the statistic results to track 2 on the condition that making direct use of all the time measurements to calculate bearings without removing the biased ones. The number of the results to track 2 is 386. The column named 'Status' in table 1 represents that which element has the biased time measurement. If the status value is 1, it means that element 1 has the biased time measurement, value 2 corresponds to element 2, and so on. Notably, if the status value is 15, it means both element 1 and element 5 have the biased time measurements. And if the status value is 0, this is the best situation, which means that no element has the biased time measurement. Therefore, if the status value is not 0, a wrong localization results will be gained when directly utilizing all the time measurements to calculate bearings with LMS. From table 1, we can get that there are only 17.62% of the localization results are correct, as the red trajectory shows in Fig. 8. But after removing the biased time measurements by analyzing the consistency of every point, 100% of the results are correct only with random error, like the green trajectory shows in Fig. 8.

TABLE I. EXPERIMENTAL RESULTS TO TRACK 2

Status	Number	Percent
0	68	17.62%
1	76	19.69%
2	13	3.37%
3	40	10.35%
5	99	25.65%
15	90	23.32%
Sum	386	

Fig.8 and table I indicate that this approach can work well and improve the localization robustness.

V. CONCLUTION

This article designs a novel localization approach for USBL acoustic positioning system based on coordinate rotation theory, which can be applied to arbitrary receiving multi-element array. In this approach, a reference coordinate system is built, and the bearings of the object in the reference coordinate system and the time measurements of the USBL receiving array are combined to build the equation groups with the help of rotation matrix. Then the consistency of results to the different equation group is analyzed to remove the invalid time measurements, so that the correct bearings can be gained. The coordinate of the object can be determined based on the calculated bearings and the estimated slant range. Computer simulation and tank test results indicate this approach can work well and improve the robustness.

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