Moving Targets Detection and Tracking in Reverberation Environments

Ying Chen, Feng-Xiang Ge, Yishu Shi
College of Information Science and Technology, Beijing Normal University, Beijing, China
ying.ch@mail.bnu.edu.cn; ge@bnu.edu.cn

Suiling Ren
State Key Laboratory of Acoustics, Institute of Acoustics, Chinese Academy of Sciences, Beijing 100190, China
rsl@mail.ioa.ac.cn

***Abstract*—The moving target detection and tracking is always an important but challenging task with applications in radar and sonar, video surveillance, web search, bioinformatics, etc. An effective method for the moving target detection and tracking in reverberation environments is presented by utilizing spare and low-rank matrix decomposition. Due to the strong coherence between the beamspace data with the almost same background (reverberation structure), the matrix with beamspace data has potentially low-rank structure. Accordingly, the moving targets can be modeled as the sparse component, and separated from the reverberation.**

Keywords—target detection and tracking; reverberation; beamspace; low-rank.

I. INTRODUCTION

The effective detection and tracking of moving targets is always an important but challenging task with applications in radar and sonar, video surveillance, web search, bioinformatics, etc. On account of the various disturbances in these scenes, the spatial location and the pixel values of the moving targets are both uncertain. In addition, the illumination always changes. These will lead to a significant decline in the moving target detection accuracy.

Currently, various algorithms have been proposed to detect and track the moving targets, for different scenarios, regions and time points. Background subtraction method is commonly adopted. The background model [1] is established by observing each pixel of video sequence over time, and then the target model is created to locate the moving targets by employing a combination of shape analysis and tracking. A codebook model is built in [2] for each pixel according to the luminance information in the degree of distortion between successive frames, but it takes long time for training, moreover, the model cannot be updated in real-time. For further improving the performance of target tracking, subspace methods has been applied as [4] and [5]. Actually, principal component analysis (PCA) is an approach to calculate the linear subspace. However, the model in PCA requires that target observations and predictions must be with Gaussian distribution, but the condition is not always satisfied in practice.

In this paper, an effective method for the moving target detection and tracking in reverberation environments is presented by utilizing spare and low-rank matrix decomposition. Our approach is based on the strong coherence between the beamspace data with the almost same background (reverberation structure), thus the matrix with beamspace data has potentially low-rank structure. Correspondingly, the moving targets could damage the low-rank structure, and may be not Gaussian errors but have arbitrarily large amplitude and sparse distribution. Accordingly, the moving targets can be modeled as the sparse component, and separated from the reverberation. The robust principal component analysis (RPCA) [6] is a general method to decompose the matrix into low-rank and sparse components. However, the iterative thresholding programme principal component pursuit (PCP) in [6] has high computation complexity to converge for the large-sized matrix. Our method takes the accelerated proximal gradient (APG) algorithm presented in [15], which is faster than the programme in [6] on solving the low-rank and sparse matrix decomposition problem.

II. SPARSE AND LOW-RANK DECOMPOSITION

In this paper, we assume that the beamspace data matrix from an array processing output of the measurements is $\mathbf{M} = [\mathbf{m}_1, \mathbf{m}_2, ..., \mathbf{m}_n]$, where $\mathbf{m}_i$ is the ith frame of beamspace data. The beamspace data matrix $\mathbf{M}$ can be written by

$$\mathbf{M} = \mathbf{L} + \mathbf{S} \tag{1}$$

where $\mathbf{L}$ corresponds to the low-rank components, $\mathbf{S}$ corresponds to the sparse components, which are shown as Fig. 1, and both components are of arbitrary magnitude. Due to the strong coherence between the beamspace data with the almost same background (reverberation structure), $\mathbf{M}$ with these beamspace data has potentially low-rank structure, i.e., $\mathbf{L}$. Correspondingly, the moving targets can be modeled as the sparse matrix $\mathbf{S}$.

The beamspace data matrix has low inherent dimensionality, which lies on some low-dimensional subspaces. Then the beamspace data matrix $\mathbf{M}$ can be written by

$$\mathbf{M} = \mathbf{L} + \mathbf{N}, \tag{2}$$

978-0-933957-43-5 ©2015 MTS

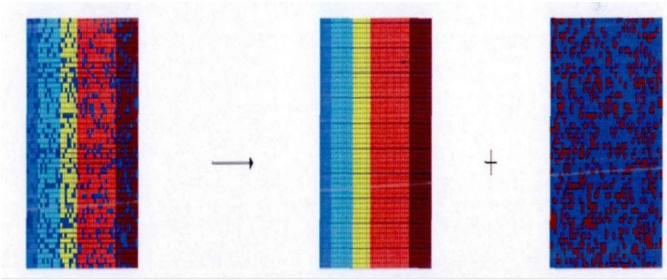

Fig. 1. The beamspace data and the experimental results

in principal component analysis (PCA) [7], **N** is assume to be a independent and identically distributed (i.i.d) Gaussian perturbation matrix. It seeks the best rank-k estimate of **L** by solving:

$$\min \|\mathbf{M} - \mathbf{L}\|_2 , \quad \text{s.t.} \quad \text{rank}(\mathbf{L}) \le k, \tag{3}$$

where $\|\bullet\|_2$ denotes the l_2-norm. This problem can be efficiently solved by the singular value decomposition (SVD), and the noise N is i.i.d Gaussian distribution.

In general, PCA is an effective tool for data analysis. But for the targets detection and tracking in reverberation environments, the moving targets in beamspace data may have arbitrarily large magnitude and dissatisfy the i.i.d. Gaussian distribution, that does not confirm to the assumption of PCA. Nevertheless, it accords with the model the low-rank and sparse decomposition regularized as following problem,

$$\min_{L,S} \text{rank}(\mathbf{L}) + \lambda \|\mathbf{S}\|_0 , \quad \text{s.t.}, \mathbf{L} + \mathbf{S} = \mathbf{M}, \tag{4}$$

where λ is a parameter and $\|\mathbf{S}\|_0$ indicates the l_0-norm (a count of nonzero elements in $\mathbf{S}$). However, due to the nonlinear and non-convex nature of the rank function $\|\mathbf{S}\|_0$, the above optimization problem is difficult to solve. In addition, the nuclear norm may be employed instead of the rank function by the rank minimization scheme [10]. The sparse approximation matrix [11], [12] can also be taken by minimizing $\|\mathbf{S}\|_1$ instead of $\|\mathbf{S}\|_0$.

The spare and low-rank matrix decomposition in (4) could be done by the Robust Principal Component Analysis (RPCA) [6] for the moving target detection and tracking in reverberation environments, and have the following optimization problem,

$$\min \|\mathbf{L}\|_* + \lambda \|\mathbf{S}\|_1 , \quad \text{s.t.} \quad \mathbf{L} + \mathbf{S} = \mathbf{M}, \tag{5}$$

where $\|\mathbf{L}\|_* = \sum_{k=1}^{n} \sigma_k(\mathbf{L})$ (the sum of all singular value of **L**) denotes the nuclear norm of **L**, and $\|\mathbf{S}\|_1$ is the l_1-norm of **S**. (5) is a general convex optimization problem to estimate the true **L** and **S**.

Nevertheless, the low-rank reverberant background **L** and sparse moving targets S can be figured out from the beamspace data, if following two conditions are met.

1) Assume the low-rank matrix $\mathbf{L} \in \Re^{n_1 \times n_2}$ ($n_1 \geq n_2$) satisfies the following incoherent condition [6] about parameters μ,

$$\max_i \left\|\mathbf{U}^T e_i\right\|^2 \le \frac{\mu\gamma}{n_1}, \quad \max_i \left\|\mathbf{V}^T e_i\right\|^2 \le \frac{\mu\gamma}{n_2}, \quad \left\|\mathbf{U}^T \mathbf{V}^T\right\|_\infty \le \sqrt{\frac{\mu\gamma}{n_1 n_2}}, \tag{6}$$

where e_i is a unit vector, **U** and **V** are from the singular value decomposition of $\mathbf{L} = \mathbf{U\Sigma V}^T = \sum_{i=1}^{r} \sigma_i u_i v_i^T$, γ is the rank of **L**, $\|\mathbf{M}\|_\infty = \max_{i,j} |m_{ij}|$.

2) Suppose **L** obeys (6), the support set of **S** is uniformly distributed among all sets of cardinality, as long as it meets the condition [6]:

$$\text{rank}(\mathbf{L}) \le \rho_\gamma n_2 \mu^{-1} (\log n_1)^{-2}, \qquad m \le \rho_s n_1 n_2, \tag{7}$$

where ρ_γ and ρ_s are positive numerical constants. Then there exists a numerical constant c such that the optimization problem (5) with $\lambda = 1/\sqrt{n_1}$ can recover the original matrix, i.e. $\hat{\mathbf{L}} = \mathbf{L}$ and $\hat{\mathbf{S}} = \mathbf{S}$. The probability of recovery is at least $1 - c n_1^{-10}$.

If singular vectors of the reverberant background matrix, i.e. **L** distribution is reasonable, and the nonzero elements of the moving targets matrix, i.e. **S** are evenly distributed, then the reverberant background matrix **L** can be recovered from arbitrarily large magnitude errors, like the moving targets data, with the probability nearly one. Worth noting that the rank γ of **L** is same order of $n / (\log n)^2$ when μ is not too large. By selecting the appropriate balance factor λ, (5) can obtain a high probability of correct recovery.

Furthermore, when the noise is considered in the measures and beamspace data, The optimization problem in (5) has the following relaxed version problem (5) can be computed by the accelerated proximal gradient method[15],

$$\min_{\mathbf{L},\mathbf{S}\in \mathsf{H}} \frac{1}{\mu}\|\mathbf{M}-\mathbf{L}-\mathbf{S}\|_F^2 + \|\mathbf{L}\|_* + \lambda\|\mathbf{S}\|_1 , \tag{8}$$

where μ is a small positive scalar with an initial value μ_0 and decreasing in each iteration, H is an a real Hilbert space. Consequently, the spare and low-rank matrix decomposition can be solved with noise eliminated.

III. Solution by Iterative Convex Optimization

The essence of matrix recovery (RPCA) [6] is to minimize a combination of the l_l-norm and the nuclear norm in (5). The iterative thresholding programme is a usual way to solve problem in (5). However, the iterative thresholding scheme requires high computation complexity for large-sized matrices. Lin et al. [15] presented a new approach to accelerate the convergence of the original method, which is an accelerated proximal gradient (APG) algorithm and faster than iterative thresholding method.

A. Accelerated Proximal Gradient (APG)

In [15], the accelerated proximal gradient (APG) method was employed to solve Principal Component Pursuit optimization (5) by solving the following unconstrained convex problem:

$$\min_{\mathbf{X}\in \mathsf{H}} F(\mathbf{X}) := f(\mathbf{X}) + p(\mathbf{X}), \tag{9}$$

where H is an a real Hilbert space given an inner product $<\mathbf{x},\mathbf{y}>$, and can be in accordance with $\|\mathbf{x}\| = \sqrt{<\mathbf{x},\mathbf{x}>}$ to indicate norm. Both f and p are convex, in addition, f is further Lipschitz continuous:

$$\|\nabla f(\mathbf{X}) - \nabla f(\mathbf{Y})\|_F \le L_f \|\mathbf{X}\text{-}\mathbf{Y}\|_F , \tag{10}$$

where $\nabla f(\mathbf{X})$ is the Frechet derivative of f, and L_f is the Lipschitz constant.

Proximal gradient algorithms minimize quadratic approximations to $\mathrm{F}(\bullet)$, which denotes as $Q(\mathbf{X},\mathbf{Y})$, supersede the convex problem (9) to update $\mathbf{X}$:

$$\begin{aligned}\mathbf{X}_{k+1} &= \arg\min_{\mathbf{X}\in \mathsf{H}} Q(\mathbf{X},\mathbf{Y}_k) \\ &= f(\mathbf{Y}_k) + \langle \nabla f(\mathbf{Y}_k), \mathbf{X}-\mathbf{Y}_k \rangle + \frac{L_f}{2}\|\mathbf{X}-\mathbf{Y}_k\|^2 + p(\mathrm{x})\end{aligned} \tag{11}$$

where $\mathbf{Y}_k$ can be set as [16]:

$$\mathbf{Y}_k = \mathbf{X}_k + \frac{t_{k-1}-1}{t_k}(\mathbf{X}_k - \mathbf{X}_{k-1}) , \tag{12}$$

t_k satisfying $t_{k+1}^2 - t_{k+1} \le t_k^2$. For smooth p, they promote the convergence rate to $O(k^{-2})$. Then Beck and Teboulle [17] showed that this scheme can also obtain a convergence rate of $O(k^{-2})$ for the non-smooth p.

Suppose $\mathbf{G}_k = \mathbf{Y}_k - \frac{1}{L_f}\nabla f(\mathbf{Y}_k)$, then the minimizer $\mathbf{X}_{k+1}$ is given by soft-thredholding the entries of $\mathbf{G}_k$. For $x \in \Re$ and $\varepsilon > 0$, let:

$$S_\varepsilon[x] = \begin{cases} x-\varepsilon, & if \quad x > \varepsilon, \\ x+\varepsilon, & if \quad x < -\varepsilon, \\ 0, & otherwise, \end{cases} \tag{13}$$

Apply the operator to vectors and matrices, when H is an Euclidean space and $p(\bullet)$ is the l_l-norm, $\mathbf{X}_{k+1}$ can be expressed as $\mathbf{X}_{k+1} = S_{\frac{\mu}{L_f}}[\mathbf{G}_k]$. If instead, H is a space given with Frobenius norm $\|\bullet\|_F$, and $p(\bullet)$ is the nuclear norm. If $\mathbf{G}_k = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$, $\mathbf{\Sigma} = Diag(\sigma)$, which is the SVD of $\mathbf{G}_k$, then $\mathbf{X}_{k+1}$ can be computed as:

$$\mathbf{X}_{k+1} = \mathbf{U} S_{\frac{\mu}{L_f}} \mathbf{V}^T . \tag{14}$$

The RPCA relaxed problem in (5) can be solved by accelerated proximal gradient method directly [15],

$$\mathbf{X} = (\mathbf{L},\mathbf{S}),\ f(\mathbf{X}) = \frac{1}{\mu}\|\mathbf{M}-\mathbf{L}-\mathbf{S}\|_F^2,\ \text{and}\ p(\mathbf{X}) = \|\mathbf{L}\|_* + \lambda\|\mathbf{S}\|_1 , \tag{15}$$

where μ is a small positive scalar, with an initial value μ_0 and decreasing in each iteration step, until it reaches the floor $\bar{\mu}$, The expression above is a special case of (9), which is an approach to solve problem in (8).

B. Eigendecomposition of the covariance

The iterative thresholding programme in [6] has high computation complexity to converge for the large-sized matrix. An accelerated proximal gradient (APG) algorithm is presented in [15]. Generally, the singular value decomposition (SVD) of matrix $\mathbf{M}$ is required for the optimization problem in (8). In this paper, we take the eigendecomposition of the covariance matrix $\mathbf{M}^{\mathrm{T}}\mathbf{M}$ and the following processing for the APG algorithm to estimate the low-rank component $\mathbf{L}$ and the sparse component $\mathbf{S}$,

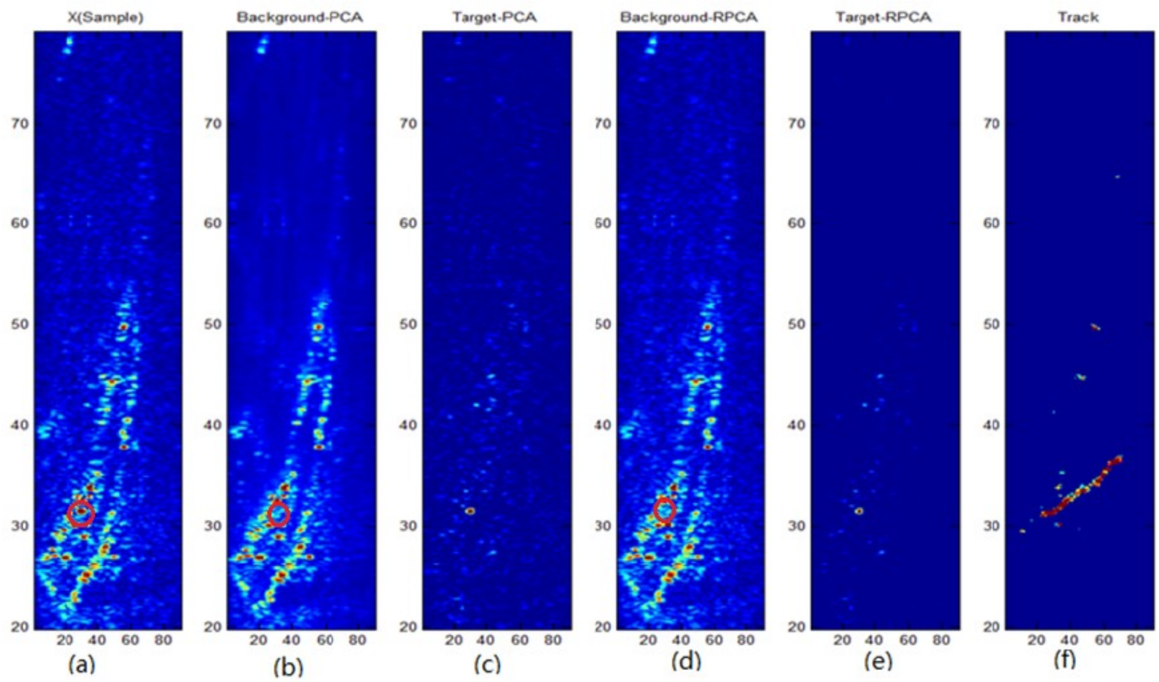

Fig. 2. The beamspace data and the experimental results.

$$
\begin{aligned}
&(\mathbf{M}^T\mathbf{M})v_i = \lambda_i v_i \\
&\sigma_i = \sqrt{\lambda_i} \\
&\mathbf{u}_i = \frac{1}{\sigma_i}\mathbf{M}\mathbf{v}_i \\
&\mathbf{M} \approx \mathbf{U\Sigma V}^T
\end{aligned} \tag{16}
$$

where v_i, σ_i, and u_i are the right singular vector, the singular value and the left singular vector, respectively.

C. Algorithm summary

Algorithm1 (RPCA via Accelerated Proximal Gradient)

Input: Observed data matrix $\mathbf{M} \in \Re^{n_1 \times n_2}$, λ

1. Initialization:

$\mathbf{L}_0 = \mathbf{L}_{-1} = 0$; $\mathbf{S}_0 = \mathbf{S}_{-1} = 0$; $t_0 = t_{-1} = 1$; $\bar{\mu} = \delta\mu_0$

2. while not converged do

3. $\mathbf{Y}_k^{\mathbf{L}} = \mathbf{L}_k + \frac{t_{k-1}-1}{t_k}\left(\mathbf{L}_k - \mathbf{L}_{k-1}\right)$,

   $\mathbf{Y}_k^{\mathbf{S}} = \mathbf{S}_k + \frac{t_{k-1}-1}{t_k}\left(\mathbf{S}_k - \mathbf{S}_{k-1}\right)$.

4. $\mathbf{G}_k^{\mathbf{L}} = \mathbf{Y}_k^{\mathbf{L}} - \frac{1}{2}\left(\mathbf{Y}_k^{\mathrm{L}} + \mathbf{Y}_k^{\mathrm{S}} - \mathbf{M}\right)$.

5. $(\mathbf{V}, \mathbf{D}) = \mathrm{eig}((\mathbf{G}_k^{\mathbf{L}})^T \mathbf{G}_k^{\mathbf{L}})$,

   $\mathbf{D} = diag(\lambda_1, \lambda_2, \cdots$, $\mathbf{V}^{\mathrm{T}} = [\mathbf{v}_1, \mathbf{v}_2, \ldots$,

   $\sigma_i = \sqrt{\lambda_i}$, $\mathbf{u}_i = \frac{1}{\sigma_i}\mathbf{M}\mathbf{v}_i$,

   $\mathbf{\Sigma} = diag(\sigma_1, \sigma_2, \cdots$, $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \ldots$,

   $\mathbf{U\Sigma V}^T \approx \mathbf{G}_k^{\mathbf{L}}$.

6. $\mathbf{L}_{k+1} = \mathbf{U}S_{\frac{\mu_k}{2}}[\mathbf{\Sigma}]\mathbf{V}^T$.

7. $\mathbf{G}_k^{\mathbf{S}} = \mathbf{Y}_k^{\mathbf{S}} - \frac{1}{2}\left(\mathbf{Y}_k^{\mathrm{L}} + \mathbf{Y}_k^{\mathrm{S}} - \mathbf{M}\right)$.

8. $\mathbf{S}_{k+1} = S_{\frac{\lambda\mu_k}{2}}[\mathbf{G}_k^{\mathbf{S}}]$.

9. $t_{k+1} = \frac{1+\sqrt{4t_k^{\,2}+1}}{2}$.

10. $\mu_{k+1} = \max(\eta\mu_k, \bar{\mu})$.

11. $k = k+1$.

8. end while

9. Output $\mathbf{L}, \mathbf{S}$.

In step 6 and 8, $S_\tau[x] = \mathrm{sgn}(x)\max(|x| - \tau, 0)$.

IV. Experiment

The measurements in reverberant environment have been taken to demonstrate the moving targets detection and tracking performance in Fig. 2. 60 frames of beamspace data from the array processing output are considered. Fig. 2 (a)-(e) are from 1 frame of beamspace data. Fig. 2 (a) is the original beamspace data corrupted by the reverberation and noise. The corresponding data matrix is decomposed to the low-rank and sparse components. The decomposed results by the proposed RPCA is shown in Fig. 2 (d)-(e), where Fig. 2 (e) is the sparse components, i.e., moving target. Since the separation of the moving target signal from the reverberation and noise highly depends on the data structure, but not the general signal-reverberation ratio, the experimental results and analysis show that the proposed method has the better ability to detect the moving target at a lower signal-reverberation ratio. Furthermore, we remove the elements whose values are less than 3 times of variance of all elements in Fig. 2 (e), and superimpose all these 60 frames, we have the moving target tracking results.

For the beamspace data in the experiment, the dimension of M is 71370×60. If we directly compute SVD of $\mathbf{M}$, the computational complexity is very high. However, In the presented method, the covariance matrix $\mathbf{M}^T\mathbf{M}$ has dimension of 60×60, and the computational complexity can be reduced significantly.

V. CONCLUSIONS

In this paper, an effective method for moving targets detection and tracking in reverberation environments is presented by using a robust subspace segmentation algorithm. Based on a low-rank matrix model, the covariance matrix of the beamspace data matrix is decomposed into the low-rank and sparse components. Then the moving targets from the sparse components are obtained with noise eliminated. The computational complexity has also been significantly reduced. The experimental results on the beamspace data have shown that the moving targets can be successfully detected and tracked by our presented method.

REFERENCES

[1] I. Haritaoglu, D. Harwood, L. S. Davis, "W4: Real-time surveillance of people and their activities," *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, vol.22, pp. 809-830, 2000.

[2] K. Kim, T. H. Chalidabhongse, D. Harwood, and L.Davis, "Real-time foreground-background segmentation using codebook model," in *Real-Time Imaging*, 2005, pp. 172-185

[3] D. Beymer, K. Konolige, "Real-time tracking of multiple people using continuous detection," in *International Conference on Computer Vision* pp. 56-60, 1999.

[4] G. D. Hager, P. N. Belhumeur, "Efficient region tracking with parametric models of geometry and illumination," *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, vol.20, pp. 1025-1039, 1998.

[5] J. Ho, K. C. Lee, M. H. Yang, and D. Kriegman, "Visual tracking using learned linear subspaces," in *Computer Society Conference on Computer Vision and Pattern Recognition, IEEE,* 2004, pp. I-782-789, vol.1.

[6] E. J. Candes, X. Li, Y. Ma, and J. Wright, "Robust principal component analysis?", *Journal of ACM*, vol. 58 (3), pp.1-37, Dec. 2011.

[7] I. Jolliffe, "Principal Component Analysis," *Springer-Verlag*, 1986.

[8] E. J. Candes and Y. Plan, "Matrix completion with noise," in *Proceedings of the IEEE*, 2009.

[9] R. H. Keshavan, A. Montanari, Andrea, and S. Oh, "Matrix completion from noisy entries," in *Journal of Machine Learning Research*, pp. 2057-2078, 2010.

[10] M. Fazel,"Matrix rank minimization with applications," *PhD thesis,* 2002.

[11] E. J. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information," *Information Theory, IEEE Transactions* on, vol.52, pp. 489−509, 2006.

[12] E. J. Candes, and T. Tao, "Decoding by linear programming," *Information Theory, IEEE Transactions* on, vol. 51, pp. 4203−4215, 2005.

[13] X. M. Yuan and J. F. Yang, "Sparse and low-rank matrix decomposition via alternating direction methods," preprint, 2009.

[14] W. C. Li, N. Subrahmanya, and F. Xu, "Online Subspace and Sparse Filtering for Target Tracking in Reverberant Environment," *IEEE 7th Sensor Array and Multichannel Signal Processing Workshop (SAM)*, pp. 329-332, 2012.

[15] Z. Lin, A. Ganesh, J. Wright, L. Wu, M. Chen, and Y. Ma ,"Fast Convex Optimization Algorithms for Exact Recovery of a Corrupted Low-Rank Matrix, ",UIUC Technical Report UILU-ENG-09-2214, August, 2009.

[16] Nesterov, Y.: A method of solving a convex programming problem with convergence rate O(1/k2). Soviet Mathematics Doklady 27(2), 372‑376, 1983.

[17] A. Beck and M. Teboulle, A fast iterative shrinkage-thresholding algorithm for linear inverse problems, SIAM Journal on Imaging Sciences, 2, pp. 183-202, 2009.