The Design and Analysis of UEP LT Codes with Correlation Chain Feedback in Underwater Acoustic Networks

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***Abstract*—To satisfy the performance requirement of LT codes with Unequal Erasure Protection (UEP) in underwater environment, an approach called Weighted Expanding Window Fountain (WEWF) codes is proposed in this paper. The WEWF codes can achieve strong UEP property by non-uniformly selecting input symbols within each window. Moreover, considering the significant price in terms of redundancy in the lower prioritized segments, we bring in Correlation Chain Feedback (CCFB), with which the transmitter can precisely adjust the encoding scheme. Therefore, the proposed approach achieves lower symbol error rate and overall redundancy in binary erasure channel. We analyze the proposed method by using asymptotic analysis and simulation results verify the performance improvement.**

***Keywords*—Fountain codes, unequal erasure protection, feedback, underwater acoustic communication**

I. Introduction

Recently, Digital fountain codes (DFC) become an efficient data transmission scheme for Underwater Acoustic Sensor Networks (UASNs)[1][2], and the most commonly used D-FC are LT codes and Raptor codes. LT codes are the first realization of DFC and have the characteristic of low computation complexity[3]. While Raptor codes achieve linear-time encoding and decoding by concatenating LT codes with other precoding schemes[4]. Traditional LT codes are designed to provide equal erasure protection (EEP) for all input symbols. However, for applications like underwater image transmission with MPEG or H.264 compress technology, a portion of data may need more protection than the rest. Such applications raise a requirement for having codes with unequal erasure protection (UEP), and the issue of how to satisfy this requirement has been addressed by several researchers.

Choosing a degree according to a degree distribution and uniformly selecting input symbols as neighbors are two core steps of LT encoding. Existing UEP LT coding methods can be divided into two categories, first of which is to change the selecting probability to guarantee that higher prioritized segments get more protection, such as Weighted LT (WLT) codes[5] and Expanding Window Fountain (EWF) codes[6]. And the second, such as the duplication method proposed in [7] and Duplicating-Expanding Window Fountain (D-EWF) codes[8], is based on the idea of virtually increase the source block size. The second class of methods are superior in terms of UEP performance, however they need a big transmission budget when the original source block size or repeat factors are large. Moreover, although the aforementioned methods successfully provide UEP, there is a significant price in terms of redundancy in the lower prioritized segments.

Several researchers provide a method to solve this problem by introducing an intermediate feedback into UEP LT codes [9][10], which shows a dramatic improvement on the decoding performance of the lower prioritized segments. Therefore, it is an efficient way to provide UEP for energy-restricted underwater environment with harsh channel conditions by utilizing the above mentioned method to improve the performance of the more important data and simultaneously, introducing feedback scheme to optimize the overall redundancy.

The paper is organized as follows. Section II describes the design and asymptotic analysis of the proposed UEP LT codes. The extraction process of Correlation Chain and the advanced encoding strategy with feedback will be outlined in Section III. Section IV presents the simulation results. Finally, we conclude the paper in Section V.

II. Proposed UEP LT Codes

In this section, we first explain how to improve the UEP performance of EWF codes by introducing weighted scheme into the lower prioritized windows. And then, the asymptotic analysis of the proposed method is given.

A. Weighted Expanding Window

We suppose that a source block of K symbols is partitioned into r adjacent classes $S_1, S_2, \cdots, S_r$ of size $\alpha_1 K, \alpha_2 K, \cdots, \alpha_r K$ such that $\sum_{m=1}^{r} \alpha_m = 1$ and it is useful to introduce $\mathrm{A}(x) = \sum_{m=1}^{r} \alpha_m x^m$ and $\Theta_i = \sum_{m=1}^{i} \alpha_m$. The important level of S_i decreases with the increase of i. Firstly we divide these classes into r windows $w_1, w_2, \cdots, w_r$, such that the window w_i, is defined as the concatenation of the classes $S_1, S_2, \cdots, S_i$, thus the size of the *i*-th window is $|w_i| = \sum_{m=1}^{i} \alpha_m K$. A degree distribution on the set $\{1, 2, \cdots, |w_i|\}$ is defined as $\Omega^{(i)}(x) = \sum_{m=1}^{|w_i|} \Omega_m^{(i)} x^m$,

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then an output symbol of our proposed Weighted Expanding Window Fountain (WEWF) codes is generated as follows:

1) Randomly select a window w_i according to a probability distribution $\Gamma(x) = \sum_{m=1}^{r} \Gamma_m x^m$ such that Γ_i is the probability that window i is chosen.
2) Randomly select a degree $d \in \{1, 2, \cdots, |w_i|\}$ according to the degree distribution $\Omega^{(i)}(x)$.
3) Suppose that the weighted vector of the *i*-th window is $\mathrm{B}^{(i)} = \sum_{m=1}^{i} k_m^{(i)} x^m$, where $\sum_{m=1}^{i} k_m^{(i)} \alpha_m = \Theta_i$. If $i = 1$, uniformly choose d distinct input symbols as neighbors. Otherwise, we select the adjacent input symbols non-uniformly at random. In every source block $S_m (1 \leq m \leq i-1)$, we choose $d_m = \min([\alpha_m d k_m^{(i)}], \alpha_m K)$ ($[x]$ means the nearest integer to x) distinct input symbols, and the last $d_i = d - \sum_{m=1}^{i-1} d_i$ neighbors are chosen from S_i.
4) Finally, the output symbol equals to the bitwise modulo-2 sum of the neighbors.

To summarize, WEWF code $\Psi_{WEW}(\mathrm{A}, \Gamma, \mathrm{B}^{(1)}, \cdots, \mathrm{B}^{(r)}, \Omega^{(1)}, \cdots, \Omega^{(r)})$ is a fountain code which assigns each output symbol to the *i*-th window with probability Γ_i and encodes the chosen window with distribution $\Omega^{(i)}(x)$ using WLT code with weighted vector $\mathrm{B}^{(i)}$. Fig.1. illustrates the process.

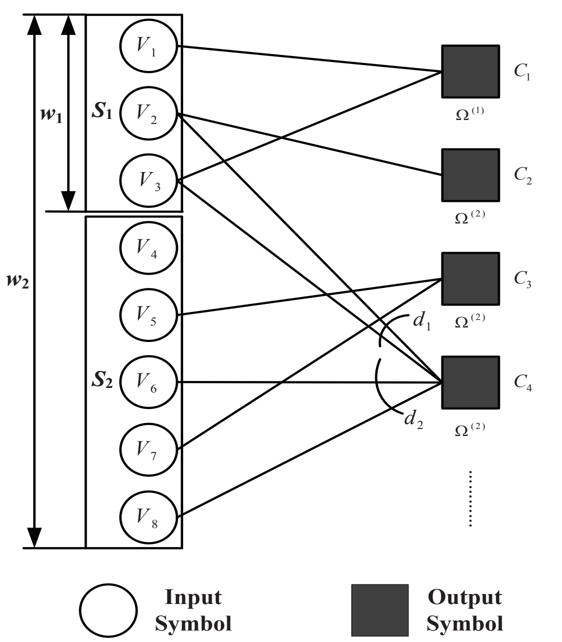

Fig. 1. Process of the Proposed Method

B. Asymptotic Analysis of W-EWF Codes

Denote ε as the reception overhead of WEWF codes and we derive the asymptotic erasure probabilities after l iterations of the iterative BP decoding algorithm for the input symbols of different classes. The original and-or tree analysis is already generalized for the weighted approach and extended for the expanding window scheme, where both different classes of OR nodes and AND nodes are introduced. Similarly, we further generalize the and-or tree construction by introducing different OR nodes selection probability within each selection window, and derive the and-or tree lemma suitable for analysis of WEWF codes.

As we know, the asymptotic degree distribution of the output symbols in the *j*-th class is $\Omega^{(j)}(x)$ and $\mu_j = {\Omega^{(j)}}'(1)$ denotes the average degree of the output symbols in the *j*-th class. For EWF, the set of the input symbol degree distributions is

$$\Lambda^{(j)}(x) = P\left[\sum\nolimits_{l=m}^{r} \mu_l \left(\Gamma_l (1+\varepsilon) K\right) \frac{1}{\Theta_l K}(x-1)\right] \quad (1)$$

where $P(x)$ is the Poisson distribution. The average size of the *l*-th class is $\Gamma_l(1+\varepsilon)K$, and $1/\Theta_l K$ is the probability that an OR node is chosen to be a children of an AND node from the *l*-th class. While for WEWF codes, let $\eta_j^{(m)} = k_j^{(m)}/\Theta_m K$ denotes the probability that an OR node from class is chosen to be a children of an AND node from the *m*-th class, then the input symbol degree distribution of the WEWF is

$$\begin{aligned}\Phi^{(j)}(x) &= P\left[\sum\nolimits_{l=m}^{r} \mu_l \left(\Gamma_l (1+\varepsilon) K\right) \eta_j^{(l)} (x-1)\right] \\ &= P\left[(1+\varepsilon) \sum\nolimits_{l=j}^{r} \frac{\mu_l \Gamma_l k_j^{(l)}}{\Theta_l} (x-1)\right]\end{aligned} \quad (2)$$

For EWF codes, the generalized and-or tree $GT_{l,j}$ constructed using r different classes of both AND and OR nodes. Suppose that the root node of $GT_{l,j}$ is an OR-node of the *m*-th class and the tree has a depth of $2l$. Each AND and OR node from the *m*-th class has *i* children with probabilities $\beta_{i,m}$ and $\delta_{i,m}$, respectively. Furthermore, an AND node from the *m*-th class can only have OR node children belonging to the classes $\{1, 2, \cdots, m\}$, with the associated probabilities of choosing a child from the different OR classes being $\{q_1^{(m)}, q_2^{(m)}, \cdots, q_m^{(m)}\}$, and an OR node from the *m*-th class can only have AND node children from classes $\{m, m+1, \cdots, r\}$ with the associated class probabilities $\{p_m^{(m)}, p_{m+1}^{(m)}, \cdots, p_r^{(m)}\}$. Let the nodes from the *m*-th class at the tree depth $2l$ be initialized as 0 with probability $y_{0,m}$ and 1 otherwise. We assume that OR nodes with no children have a value of 0, whereas AND nodes with no children have a value of 1. We quote the following generalized version of the and-or tree lemma from [6]:

***Lemma* 1.** *Let $y_{l,j}$ be the probability that the root of an and-or tree $GT_{l,j}$ evaluates to 0. Then*

$$y_{l,j} = \delta_j \left[1 - \sum_{i=j}^{r} p_i^{(j)} \beta_i \left(1 - \sum_{m=1}^{i} q_m^{(i)} y_{l-1,m}\right)\right] \quad (3)$$

Where $\delta_j(x) = \sum_i \delta_{i,j} x^i$ and $\beta_j(x) = \sum_i \beta_{i,j} x^i$. Similar to the derivation in [5], $\beta_{i,j}$ is the probability that the output symbol connected with a randomly selected edge has degree $i+1$ given that it belongs to the class j, equals $\beta_{i,j} = \frac{(i+1)\Omega_{i+1}^{(j)}}{{\Omega^{(j)}}'(1)}$, so that $\beta_j(x) = \frac{{\Omega^{(j)}}'(x)}{\mu_j}$. Similarly, $\delta_{i,j}$ is the probability that the input symbol connected with a randomly selected edge which belongs to class j and has degree $i+1$, equals $\delta_{i,j} = \frac{(i+1)\Phi^{(j)}_{i+1}}{(1+\varepsilon)\sum_{l=j}^{r}(\mu_l\Gamma_l k_j^{(l)}/\Theta_l)}$, so we achieve $\delta_j(x) = e^{(1+\varepsilon)\sum_{l=j}^{r}\frac{\mu_l\Gamma_l k_j^{(l)}}{\Theta_l}(x-1)}$. For the class m input symbols, the probability of having class output symbol as a children, $m \leq j \leq r$, equals $p_j^{(m)} = \frac{\mu_j\Gamma_j k_m^{(j)}/\Theta_j}{\sum_{l=m}^{r}(\mu_l\Gamma_l k_m^{(l)}/\Theta_l)}$. Similarly, the class m output symbols have the class j input symbol child, $1 \leq j \leq m$, with probability $q_j^{(m)} = \frac{k_j^{(m)}\alpha_j}{\Theta_m}$.

Lemma 2. *For a WEWF code* $\Psi_{WEW}(\mathrm{A},\Gamma,\mathrm{B}^{(2)},\cdots,\mathrm{B}^{(r)},\Omega^{(1)},\cdots,\Omega^{(r)})$*, the probability* $y_{l,j}$ *that the input symbol of class* j *is not recovered after* l *iterations of BP algorithm at the reception overhead* ε *is*

$$\begin{aligned} &y_{0,j} = 1; \\ &y_{l,j} = e^{-(1+\varepsilon)\sum\limits_{i=j}^{r}\frac{k_j^{(i)}\Gamma_i}{\Theta_i}\Omega^{(i)\prime}(1-\sum\limits_{m=1}^{i}\frac{k_m^{(i)}\alpha_m}{\Theta_i}y_{l-1,m})} \end{aligned} \quad (4)$$

C. WEWF Codes with Two Prioritized Segments

For a WEWF code $\Psi_{WEW}(\alpha_1 x+\alpha_2 x^2, \Gamma_1 x+\Gamma_2 x^2, k_1^{(2)}x+k_2^{(2)}x^2, \Omega^{(1)}, \Omega^{(2)})$, the expressions for erasure probabilities of the More Important Symbols (MIS) and the Less Important Symbols (LIS) after l iterations, $y_{l,MIS}$ and $y_{l,LIS}$, are respectively obtained in (5) and (6), for $l \geq 1$ and $y_{0,MIS} = y_{0,LIS} = 1$.

In UASNs, the block size of the input symbols is limited by the data transmission delay, therefore, in order to compare our results with the results in [5], we set the block size $K = 1000$, $\alpha_1 = 0.1$, thus the number of MIS and LIS are 100 and 900, respectively. We applied Robust Soliton Distribution (RSD) $\Omega_{rs}(\alpha_1 K, c, \delta)$ on the MIS window and $\Omega_{rs}(K, c, \delta)$ on the LIS window, where $c = 0.03$, $\delta = 0.5$ are chosen as the RSD parameters for further discussion.

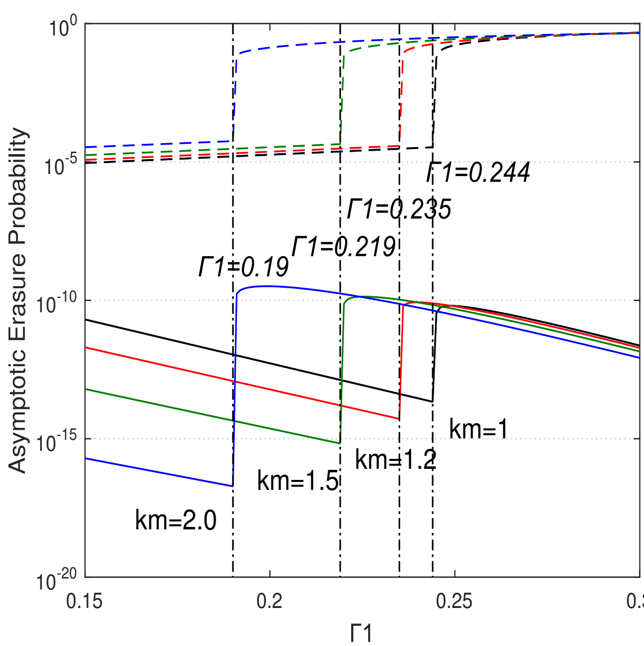

Fig. 2. Applicable scope on Γ_1 for various k_m

Fig.2 depicts the variations of the local minima of $y_{200,MIS}$ and $y_{200,LIS}$, when the reception overhead is set as $\varepsilon = 0.3$, for the values of the MIS weighted factors equal to k_m=1, 1.2, 1.5, 2.0 (when k_m=1, WEWF codes is equivalent to EWF codes). We can see that each value of k_m has an applicable scope on the selection probability of the MIS window Γ_1. When $\Gamma_1 \leq 0.19$, the MIS performance notably improves along with the increase of k_m, while in other interval, such as $\Gamma_1 \in (0.219, 0.235]$, the local minima of $y_{200,MIS}$ is achieved when $k_m = 1.2$.

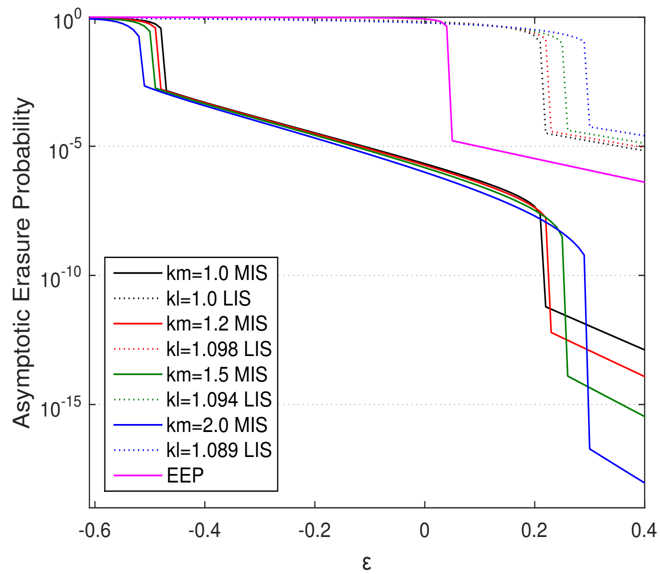

Fig. 3. Asymptotic analysis of SER versus the overhead ε

Fig.3 illustrates the asymptotic erasure probability curves of MIS and LIS classes as a function of the reception overhead ε. We compare WEWF codes of different weighted factors with EWF codes when the selection probability of the MIS window is Γ_1=0.19 which is optimized for the reception overhead ε=0.3 and $k_m = 2.0$. Fig.3 clearly shows that the WEWF codes' Unequal Recovery Time (URT) and UEP properties grow stronger with the increase of the weighted factor, except for that the overall redundancy of both MIS and LIS increases too in comparison with the EWF codes.

III. Optimized Feedback Scheme

Although the performance improvement of MIS is notable, it is at the cost of loss of performance on LIS compared with the EWF codes. Therefore, we bring in feedback scheme to solve this problem.

A. Correlation Chain Extraction Process

Traditional feedback message contains the number of input symbols that have already been recovered while ignoring the information remained in the updated generation matrix $\mathbf{G}'$. So we define Correlation Chain (CC) to describe this information:

$$y_{l,MIS} = \exp\left[-(1+\varepsilon)\left(\frac{\Gamma_1}{\alpha_1}\Omega^{(1)'}\left(1-y_{l-1,MIS}\right)+k_1^{(2)}\Gamma_2\Omega^{(2)'}\left(1-k_1^{(2)}\alpha_1 y_{l-1,MIS}-k_2^{(2)}\alpha_2 y_{l-1,LIS}\right)\right)\right] \tag{5}$$

$$y_{l,LIS} = \exp\left[-(1+\varepsilon)k_2^{(2)}\Gamma_2\Omega^{(2)'}\left(1-k_1^{(2)}\alpha_1 y_{l-1,MIS}-k_2^{(2)}\alpha_2 y_{l-1,LIS}\right)\right] \tag{6}$$

Definition 1. *(Correlation Pair) For* $\mathbf{G}'$*, a correlation Pair,* $CP(j)$*, is defined as a sequence,* $\{c_1^{(j)}, c_2^{(j)}\}$*, consisting of locations of the rows where the value is 1 in the* $j-$th *column whose degree is 2.*

	1	2	3	4	5	6
1	1	0	0	0	0	0
2	1	1	0	0	0	0
3	0	0	1	0	0	0
4	0	1	0	1	0	0
5	0	0	1	1	1	0
6	0	1	0	0	1	0

Fig. 4. Generation Matrix at the Decoder

If the correlation pairs contain the same element, they can be combined into a CC. Take the generation matrix in fig.4 for example, there are 4 correlation pairs, $CP(1) = \{1,2\}, CP(2) = \{3,5\}, CP(3) = \{4,5\}, CP(4) = \{5,6\}$, where the pairs $CP(2), CP(3), CP(4)$ have the same element 5, so they can constitute a CC $\{3,4,5,6\}$ and the correlation chains for the whole matrix are $\{1,2\}$ and $\{3,4,5,6\}$.

B. Encoding Strategy with Feedback

Through the extraction process of CC, it is clear that once any one of the input symbols corresponding to the elements in CC is recovered, other corresponding symbols can be decoded too. Traditional feedback schemes only pay attention to the number and order of the decoded symbols, however the generation matrix at the decoder contains plenty information about the correlation chain which can be used to adjust the encoding strategy.

Therefore, our feedback information, which called Correlation Chain Feedback (CCFB), contains not only the traditional feedback messages, but also CC of the generation matrix. We transport CCFB to the encoder when the MIS are fully recovered or the decoding process is interrupted before successful decoding.After obtaining the CCFB, the input symbols at the transmitter can be divided into following categories, whose priorities decrease in turn: 1) symbols corresponding to the first element of CC; 2) uncovered symbols of MIS; 3) other uncovered symbols; 4) symbols corresponding to other elements of the CC; 5) symbols that have been recovered. Utilizing a non-uniformly selecting scheme, the higher prioritized symbols will participate in the encoding process with higher probability. So the decoding success rate of the elements of CC and uncovered symbols will increase, which will reduce the overall transmission redundancy.

IV. Simulation Results

The number of input symbols is $K = 1000$, and the input symbols are partitioned into two prioritized segments. The MIS block consists of the first 10% input symbols. In the following simulations, we use RSD with parameters $c = 0.03$, $\delta = 0.5$ as degree distribution and set the selection probability of the MIS window as $\Gamma_1 = 0.23$.

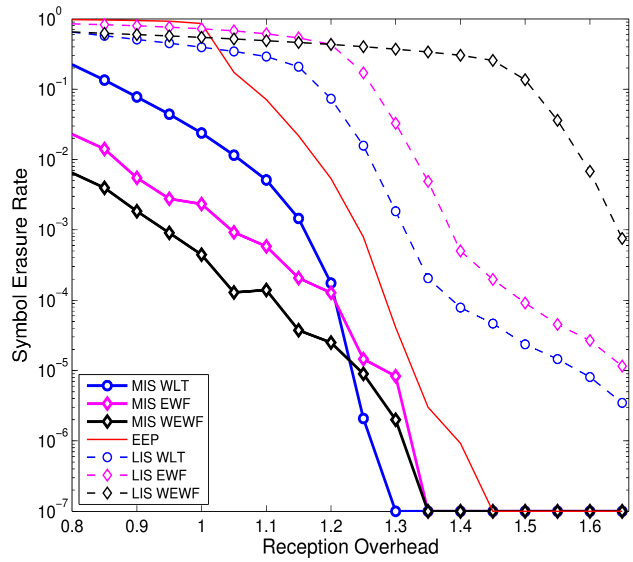

Fig. 5. Comparison of SER varied with overhead

We first compare the performances of three kinds of UEP LT codes, the proposed WEWF codes with weighted factor $k_m = 1.2$, EWF codes and WLT codes with parameter $k_m = 1.86$. The channel was lossless. Fig. 5 shows comparison of Symbol Erasure Rate (SER) of both MIS and LIS varied with overhead for all three kinds of UEP LT codes. We can see that for MIS, WEWF codes can achieve SER of 10^{-3} magnitude when the reception overhead $\varepsilon = 0.95$, while EWF and WLT codes need 1.05 and 1.17, respectively. However, when $\varepsilon = 1.25$, the SER of WLT codes is the lowest which equals to 2.083×10^{-6}, and WEWF codes has a SER 9×10^{-6} which performs better than EWF codes whose SER equals to 1.458×10^{-5}. So our method provide better performance for MIS compared to EWF and WLT codes when the reception overhead $\varepsilon < 1.23$.

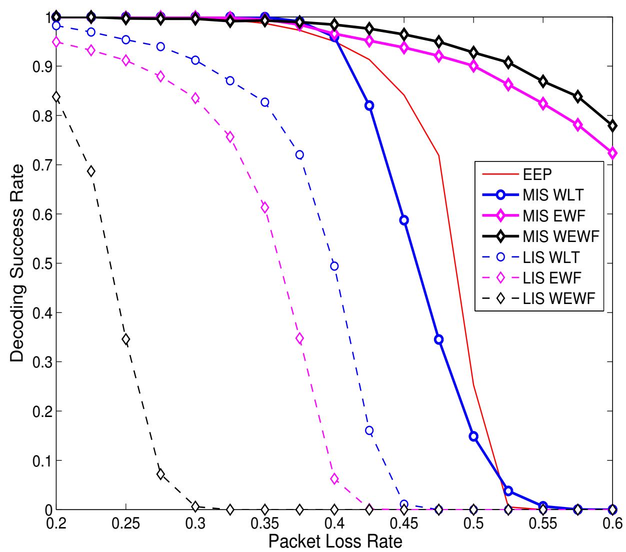

Fig. 6. Comparison of DSR varied with Packet Loss Rate

Considering the harsh underwater communication conditions, the influence of Packet Loss Rate (PLR) on the performance of UEP LT codes can not be ignored. We use Decoding Success Rate (DSR), which defined as the ratio of the number of successful decoding and the total number of simulations, as the performance metrics to compare the MIS and LIS performance of the three UEP LT codes under various PLR. For each UEP scheme, we transmit $K = 1000$ input symbols 5000 times with a transmission overhead $\mathrm{E} = 2.0$ and then count the number of successful decoding to calculate DSR.

Fig.6 depicts the DSR of three kinds of UEP LT codes varied with PLR. For MIS, their DSR are all close to 1 when the PLR is smaller than 0.4. With the increase of PLR, the DSR of WLT codes declines rapidly and reduce to 0.038 when the PLR is 0.525. While at the same condition, the DSR of EWF and WEWF codes are 0.8628 and 0.9076 respectively, from which we can see that our proposed method is more suitable for the underwater communication environment.

From Fig.5 and Fig.6 we can conclude that our method performs better in terms of UEP property than WLT and EWF codes. However, for LIS, WEWF codes achieves SER of 10^{-3} of magnitude until the reception overhead $\varepsilon = 1.65$ and its DSR declines to 0.006 when the PLR equals to 0.3, i.e., the improvement of MIS performance is at a expense of the lower prioritized segments. Therefore, bringing in feedback scheme to optimize our method will make it more practical.

Fig.7 shows the overall redundancy distribution of WEWF codes under different feedback schemes. The overall redundancy distribution is a statistical results of the transmission overhead when all input symbols are successfully recovered for 5000 simulations. WEWF codes without feedback need an

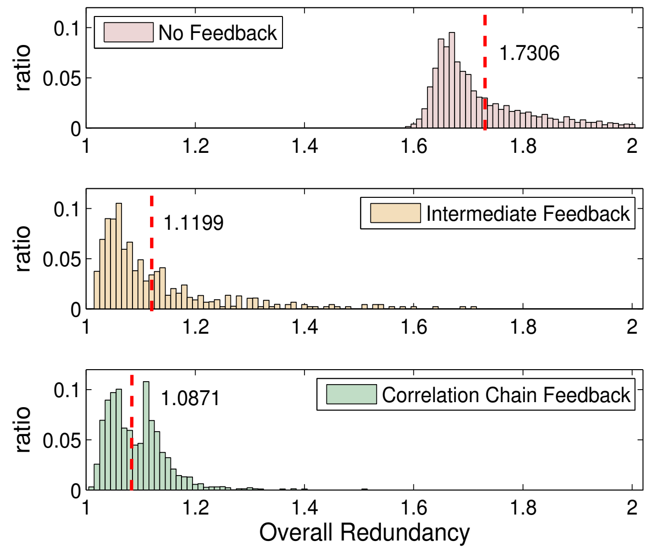

Fig. 7. Comparison of Overall Redundancy for different feedback schemes

average overhead 1.7306 to achieve successful decoding ,while with the intermediate feedback scheme in [9], the required overhead reduces to 1.1199 and with our CCFB scheme it further decreases to 1.0871. This confirms that the redundancy in lower prioritized segments reduces significantly by bringing in feedback scheme.

V. Conclusion

In order to achieve image or video transmission in underwater environment, a novel UEP scheme, which is a combination of WLT and EWF codes, is proposed in this paper. The proposed method can improve the performance of UEP property effectively, and a feedback scheme called CCFB is also brought in to optimize the performance of the lower prioritized segments. Several simulation results show that our proposed method achieve better performance than previous schemes.

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