Quantum Sensing in the Maritime Environment

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***Abstract*—Quantum sensors exploit quantum phenomena to achieve performance advantages over their classical counterparts. Some examples of quantum sensing devices that we will briefly describe in this paper include quantum photo-detectors, radar, lidar, magnetometers, and gravimeters. We will highlight their novel features and discuss their potential applications to problems related to naval operations in the maritime environment.**

I. Introduction

A major scientific thrust from recent years has been to try to harness quantum phenomena to increase the performance of a wide variety of information processing devices. Quantum Information Science (QIS) has emerged as a scientific undertaking that may revolutionize our information infrastructure [1], [2]. In particular, QIS offers the possibility of quantum sensors [3], [4], [5], [6], [7], [8], [9]. That is, sensing devices that exploit quantum phenomena in such a way that they perform much better than their classical counterparts. Some examples of quantum sensing devices include photo-detectors [10], radar [11], [12], magnetometers [13], and gravimeters [14].

In the first section of this paper, we will briefly review the basic physical phenomena which are necessary to understand how quantum sensors work. Subsequent sections will describe some of the most promising quantum sensing devices. Emphasis will be made to highlight the novel features of these devices when compared to their traditional counterparts. We will also discuss their potential application to solve a variety of problems related to naval operations in the maritime environment.

II. Quantum Sensor Physics

The two most important quantum physical effects that are harnessed by quantum sensors are *superposition* and *entanglement*. Let us assume a simple quantum system with one degree of freedom which can take on only two possible values. To each of these values we associate a quantum state that describes the behavior of the system, and these states will be arbitrarily denoted by $|0\rangle$ and $|1\rangle$. The superposition principle indicates that the most general state of the quantum system is given by a superposition of these two elementary states with complex amplitudes. That is:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad \alpha, \beta \in \mathbb{C} \qquad |\alpha|^2 + |\beta|^2 = 1 \tag{1}$$

where $|\alpha|^2$ is interpreted as the probability that, after measurement or observation of the state $|\psi\rangle$, it is determined that the system is in the $|0\rangle$ state. And similarly, $|\beta|^2$ is interpreted as the probability that, after measurement or observation of the state $|\psi\rangle$, it is determined that the system is in the $|0\rangle$ state.

Therefore, the state of a quantum system is probabilistically determined when measured, causing the superposition to *collapse* to one of the elementary states. After measurement the system state becomes essentially classical, and therefore all subsequent measurements will produce the same result.

A state such as $|\psi\rangle$ is often referred as a *qubit* and corresponds to a unit of quantum information (in contrast to the bit, which is the unit of classical information). Clearly, bits and qubits are radically different. Bits can assume the value of 0 or 1, but once fixed, its value is unique, deterministic, and unambiguous. On the other hand, qubits can simultaneously take the value of 0 and 1 in a probabilistic mixture of complex amplitudes, and as a consequence, its value is not deterministic.

But even more important, a quantum system of n qubits lives in an exponentially large space, as it can be used to represent 2^n elementary states. For instance, a system made of two qubits is described by:

$$|\Psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle \tag{2}$$

where:

$$\alpha, \beta, \gamma, \delta \in \mathbb{C} \qquad |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1 \tag{3}$$

and as before, $|\alpha|^2$ is interpreted as the probability that, after measurement or observation of the state $|\Psi\rangle$, it is determined that the system is in the $|00\rangle$ state. And similarly for the other three parameters β, γ, and δ. The elementary quantum states denoted with two digits represent the tensor product of the elementary states of each subsystem. That is:

$$|00\rangle \equiv |0\rangle_a \otimes |0\rangle_b, \quad |01\rangle \equiv |0\rangle_a \otimes |1\rangle_b, \quad \ldots \tag{4}$$

for the joint state of two qubits arbitrarily denoted "a" and "b".

It is important to remark that a superposition of elementary states such as $|\Psi\rangle$ is completely different from a classical system of probability distributions. The difference being that the elementary states in $|\Psi\rangle$ are not weighted by probabilities (e.g. $|\alpha|^2$, $|\beta|^2$, ...) but by *complex probability amplitudes* (e.g. α, β, ...). The fact that quantum superpositions are weighted by probability amplitudes imply that these systems can form interference patterns when they interact with each

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other (something that cannot be accomplished using a classical system).

On the other hand, *quantum entanglement* refers to systems that are found in a very specific type of superposition that cannot be factorized into the product of its elementary states. For example, consider the case of system made of two entangled qubits denoted by a and b given by:

$$|\Phi\rangle_{ab} = \frac{|0\rangle_a|0\rangle_b + |1\rangle_a|1\rangle_b}{\sqrt{2}} \neq |\psi\rangle_a \otimes |\phi\rangle_b \tag{5}$$

where $|\psi\rangle_a$ and $|\psi\rangle_b$ are one qubit states of the form:

$$|\psi\rangle_a = \alpha_a|0\rangle_a + \beta_a|1\rangle_a \qquad |\psi\rangle_b = \alpha_b|0\rangle_b + \beta_b|1\rangle_b \tag{6}$$

Indeed, it is impossible to find probability amplitudes for the one qubit states $|\psi\rangle_a$ and $|\psi\rangle_b$ that produce the joint state $|\Phi\rangle_{ab}$.

As a consequence of entanglement, the measurement of $|\Phi\rangle_{ab}$ will be highly correlated (with equal probability we obtain $|0\rangle$ for both sub-systems, or we obtain $|1\rangle$ for both sub-systems). Furthermore, it can be shown that the maximum degree of correlation observed in entangled systems is greater than the maximum degree of correlation found in a classical system. For as amazing as it may sound, while the maximum mutual information that can be found between two perfectly correlated classical sub-systems is equal to 1, it is equal to 2 in the quantum scenario. This is a truly fascinating quantum-mechanical effect. And as we will see, the high degree of correlation in entangled systems is of great value for the design of quantum sensors able to outperform their classical counterparts.

Unfortunately, quantum information is very “fragile”, in the sense that it is very susceptible to the effects of environmental noise. That being said, quantum error correction is an important area of research, and many protocols that protect the integrity of quantum information have been developed in the past few years [1]. These protocols have allowed a variety of experimental demonstrations of “stable” quantum information devices. For example, an experiment performed in 2007 showed that entanglement-based free-space quantum communications are possible at a distance of 144km [15]. Arguably, this result also suggests that entanglement-based free-space quantum sensing is feasible, even in the presence of an attenuating and noisy environment.

To summarize, we point out the basic design principles for the development of quantum sensing devices:

- **Superposition:** exploit the exponentially large state space before the superposition is destroyed by measurement or by the noisy environment.
- **Entanglement:** inject entangled states into the sensing system and try to exploit their non-classical correlations.
- **Noise:** mitigate the effects of noise and attenuation.

In what follows we will briefly describe how these design principles can be applied towards the development of novel quantum sensors such as photo-sensors, radar, lidar, magnetometers, and gravimeters. In addition, we will discuss the potential applications of these devices to naval operations in the maritime environment.

III. Biologically-Inspired Quantum Photosensors

It is a well known fact that, in spite of the attenuation of electromagnetic radiation produced by sea water, photosynthetic organisms thrive in the underwater ecosystem [18]. But it is rather surprising that bacterial photosynthesis has been observed to take place deep within the Pacific Ocean, at depths in excess of 2,000 meters [19]. This biological process is carried out by green-sulfur bacteria that are obligated photosynthetic organisms. That is, these organisms are required to conduct photosynthesis in order to survive. Even though at this depth the ocean is in total darkness to the human eye, the bacteria is able to efficiently absorb and process the dim light that comes from the sun or nearby hydrothermal vents. In a sense, the problem of highly efficient underwater photosensors has already been solved by nature through the evolution over millions of years of these underwater photosynthetic organisms.

Let us recall that photosynthetic organisms posses molecular antenna systems that capture solar light and transport the energy to a metabolically expensive reaction center where the biochemical processes of photosynthesis begins [20]. For many years it was conjectured that the transport of energy to the reaction center was due to classical energy transport mechanisms [21].

However, it has recently been observed that photosynthetic proteins appear to use quantum coherence to transport energy in an efficient manner (nearly perfect quantum efficiency of the photo collection capture and transport processes). Indeed, quantum effects in photosynthetic light harvesting systems have been experimentally observed at cryogenic and at room temperature [22], [23]. These experiments required of sophisticated setups requiring ultra fast optics and 2D spectrography to detect the characteristic quantum signatures. In addition, a variety of theoretical efforts have proposed viable physical mechanisms that explain how quantum phenomena can be relevant at room temperature [24], [25].

There are compelling reasons to attempt to construct devices that mimic photosynthetic energy transfer. For instance, current photosensors such as PIN, APD, and photomultiplicators use semi-classical charge transport processes based on the photoelectric effect. That is, light removes electric charge from a semiconductor's energy band and the resulting current is used as a signature of light. However, the big disadvantage of bulk semiconductors is that they require highly ordered crystalline phases that are rigid, expensive, and fragile. Furthermore, because of the strong absorption in their diffused surface layer, semiconductor devices are not optimal for the detection of light in the green-blue regime. In addition, these devices are not optimal because it is known that quantum transport is more efficient than classical transport.

Theoretically, biologically-inspired quantum photodetectors are made of new materials that attempt to synthesize the highly efficient quantum transport phenomena observed in

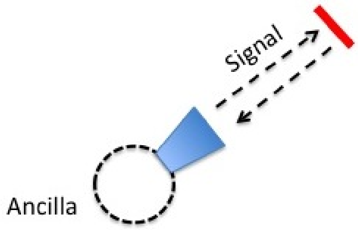

Fig. 1. Conceptual description of an entanglement-based monostatic standoff quantum sensor.

photosynthesis. It is expected that these materials will allow the construction of photodetectors with a quantum efficiency of nearly 90% in the 460-480 nm range, surpassing the current capabilities provided by APD and PIN photodetectors.

One such candidate involves the use of *J-Aggregates* [26]. These are molecules that self-assemble and exhibit quantum transport over hundreds of chromophores at room temperature [27], [28]. As a consequence, J-aggregates appear to closely mimic the behavior of chlorosome antenna in green sulfur bacteria.

The manufacturing of biologically-inspired materials that exhibit quantum transport is a work in progress. However, these materials appear to be feasible to design and engineer.

The maritime environment offers many potential applications of highly sensitive photodetectors in the blue-green regime. Perhaps the most important ones are found in the context of underwater communications and sensing. For instance, these devices could allow highly-secure communications between underwater vehicles, or between underwater vehicles and airborne platforms [10], [29]. Furthermore, as we will discuss in the following section, these quantum sensors could also be used to implement underwater imaging systems [30], [31].

IV. Quantum Radar & Lidar

The fundamental idea behind an entanglement-based standoff monostatic electromagnetic quantum sensor, either radar or lidar, is shown in Figure 1 and it is based on the concept of *quantum illumination* [8]. However, while quantum illumination thrives for increased sensing performance (how to detect better), our approach is more concerned with stealthy sensing operations (how to detect a target with some error probability, using the smallest possible number of photons) [16], [30], [31].

Monostatic sensing means that the detector and the emitter are in the same physical location. In the proposed sensing device, an entangled pair of photons is created. These are called the signal and the idler (or ancilla) photons. The idler photon is kept within the system while the signal photon is sent through a medium towards a potential target. With certain probability, the signal photon may, or may not encounter the target. If the signal photon does not encounter the target, then it will continue to propagate in space. In such a case all measurements performed by the detector will be of noise photons. On the other hand, if the target is present and the signal photon is "bounced back" towards the detector, then it will be detected with certain probability. In a sense, the signal photon is "tagged" because of the quantum correlations due to the entanglement, and therefore it will be "easier" to correctly identify it as a signal photon rather than misidentify it as a noise photon [11], [16], [30], [31].

Electromagnetic quantum sensors are formally described using quantum field theory. As a consequence, the exact same theory applies to the physical description of quantum radar and quantum lidar. Of course, while the equations are the same, the frequency and frequency-dependent parameters will have different values in the microwave (radar) and optical (lidar) regimes. Also, the actual physical implementation of these devices will be different according to the frequency. However, the general features and performance advantages offered by electromagnetic quantum sensors are similar in both regimes.

In what follows we will describe how electromagnetic quantum sensors are characterized by a target signature amplification and an improved detection error probability. In addition, we will discuss the application of a theoretical quantum imaging system to the navigation of underwater vehicles traveling under the Arctic ice shelf.

A. *Target Signature Amplification*

Similar to the radar cross section widely used in classical radar theory, one can define a *quantum radar cross section* to quantify the *visibility* of a target [11], [12], [32]. This quantity, denoted by σ_Q, depends strongly on the target geometry and composition. On the other hand, σ_Q depends weakly on the range to the target and the specific radar parameters such as the emitting power and field of view. As such, the radar cross section, either classical or quantum, characterizes a specific target when observed with electromagnetic radiation of a given frequency.

Let us consider the particular case of a rectangular target. Figure 2 shows the plots of σ_Q vs. θ ($\phi = 0$) for signal packets made of $n_\gamma = 1$, 2, and 3 photons. As a reference, the maximal value of the classical radar cross section for the same target looked at in the specular direction ($\theta = 0$) corresponds to the maximal value of the quantum cross section for one photon looked at in the specular direction. It can be observed that as the number of photons increases, so does the peak of the maximum value of σ_Q at the specular orientation. That is, a target near the specular direction appears to look bigger when illuminated with a quantum sensor. This is a pure quantum-mechanical effect due to the interference terms that arise from the correlations in the photons that make the signal packets. Thus, only in the case of beams of classical light we will expect to see the same cross sections without regard to the number of photons per beam. However, as the number of photons increases, the sidelobe structure decreases dramatically and the width of the specular orientation peak becomes more narrow (the sidelobe structure is not shown in the figure).

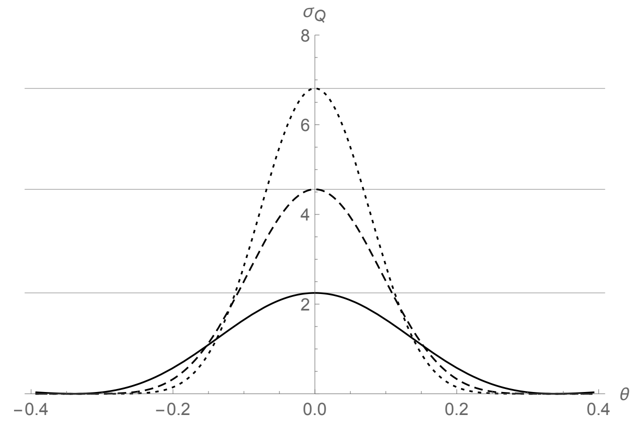

Fig. 2. Detail of the main lobe of the quantum cross section (σ_Q) as a function of the azimuthal angle (θ) with a fixed polar angle ($\phi = 0$) for 1 incident photon, (solid line), 2 incident photons (dashed line), and 3 incident photons (dotted line). The horizontal grid lines are located at $\sigma_Q = 2.25$, 4.56, and 6.81.

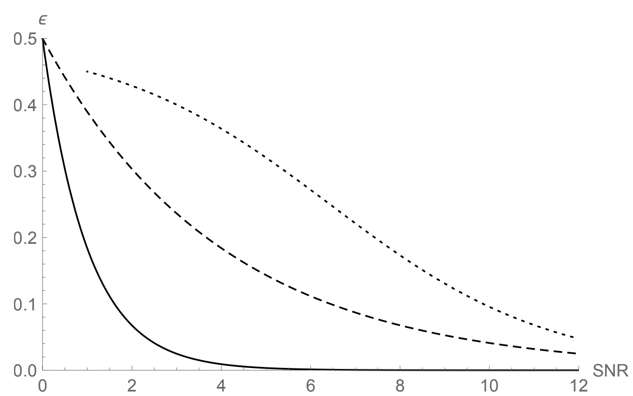

Fig. 3. Detection error probability upper bound ϵ for 10^6 entangled photon states (solid line), 10^6 coherent unentangled photon states (dashed line), and a coherent integration of 10^6 classical radar pulses (dotted line), with respect to the signal-to-noise ratio (SNR) in units of 10^{-6}.

B. Detection Error Probability

Let us now compare the theoretical performance of the following three types of standoff sensors:

- A quantum sensor that uses 2-photon entangled states (a signal and an ancilla);
- A coherent light sensor (such as a lidar) that uses non-entangled states of light; and
- A traditional radar system.

We can choose the error detection probability ϵ as a measure of performance. That is, we define ϵ as the probability of making an error when we try to determine if a target is present or not. Thus, if $\epsilon = 0.5$, then the sensor is not better than a coin flip. On the other hand, if $\epsilon = 0.0$, then the sensor has perfect certainty. Clearly, improved sensor performance implies smaller ϵ.

We will limit our analysis to the extremely low signal-to-noise ratio: $SNR \leq 12 \times 10^{-6}$. We also assume that the sensing is carried out by sending 10^6 pulses. That is, it requires the quantum sensor to create 10^6 2-photon entangled states, the coherent sensor sends 10^6 unentangled photons, and we perform coherent integration of 10^6 classical radar pulses (this is known to be the most optimal way to combine information from several classical radar pulses [33], [34]).

Figure 3 shows the comparative performance of the three sensors based on the detection error probability upper bound [12]. It can be observed that, in the low signal-to-noise ratio and low brightness regime, the quantum sensor is much better than the coherent and the classical sensors. However, if the SNR is too small, then all sensors have a detection error probability of nearly 0.5, which means that they are not better than the flip of a fair coin. Also, if the SNR is large enough, then all sensors have a detection error probability that asymptotically approaches zero. In such a case, the quantum sensor does not offer any advantage over the coherent or the classical system. However, there is a range of SNR, at around 3×10^{-6}, for which the entangled system outperforms the other two sensing devices.

C. Expected Performance in Clear Arctic Waters

The precise navigation of underwater vehicles is a difficult task due to the challenges imposed by the variable oceanic environment. It is particularly difficult if the underwater vehicle is trying to navigate under the Arctic ice shelf [30]. Let us recall that, in general, an underwater vehicle does not have access to radio-navigation aids, GPS and astronomical observations. Also, an underwater vehicle traveling in the Arctic is relatively close to the geomagnetic pole and the Earth's rotation axis, and as a consequence, common navigation tools such as compass and gyroscopes are plagued with large amounts of error. In addition, the shape and thickness of the ice shelf is variable throughout the year. The ice itself is a risk to navigation, as Arctic ice has the same strength as poor-grade concrete and may severely damage the hull of an underwater vehicle.

To overcome these strict limitations, most underwater Arctic vehicles rely on active sonar arrays that determine the position of the ice, terrain, and other obstacles present at the front, top, and below the vehicle. However, this strategy is not stealthy and can compromise the position of the submarine in a combat scenario. We believe that quantum sensors are the best solution to the problem of underwater Arctic navigation in combat and stealthy reconnaissance operations [30], [31].

Our proposed system operates with entangled photons in the blue-green regime. The entanglement correlations are used to distinguish between noise and signal photons bounced back by the target. The system is designed to operate with a low-brightness signal and immersed in a noisy environment. As we described before, the signal photons are "hidden" in the environment noise and entanglement correlations are the key to retrieve them.

In what follows, let us ignore *all* possible sources of noise, except for the optical attenuation produced by clear Arctic

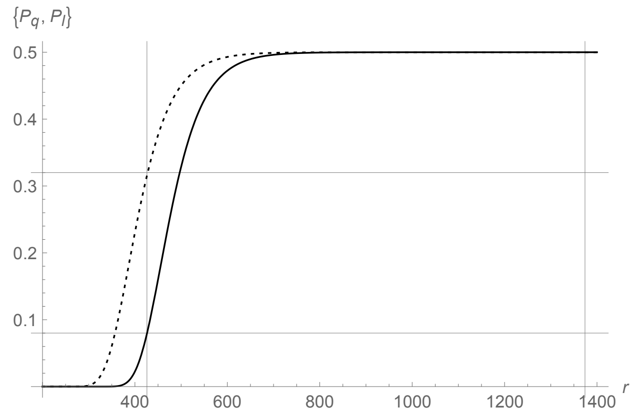

Fig. 4. Detection error probability bounds P_q (solid line) and P_l (dashed line) with respect to the distance to the target r.

seawater[1]. We also assume that the target is a perfect reflecting surface. These assumptions will allows us to establish upper bounds for the theoretical performance that characterizes the best case scenario for the sensing devices.

Let us denote P_l as the upper bound of the detection error probability for an optical coherent sensor (e.g. lidar), and P_q as the upper bound of the detection error probability for a sensor that uses entangled photons. Figure 4 shows the detection error probability bounds P_q (solid line) and P_l (dashed line) with respect to the distance to the target r in clear ocean waters (Jerlov Type I) using $\tilde{M} = 10^6$ photons. Notice that after about 800 m, the detection error probability bounds become very close to 0.5 (at which point, the detection systems are not better than the flip of a fair coin). Also, below 300 m there is very little difference in the performance of the entangled and coherent imaging systems. The first vertical grid locates the optimal distance of 426 m, in which the entangled imaging system has a detection error probability of 0.08, while the coherent system has a detection error probability of 0.32.

It can be shown that the entanglement between the signal and idler photons is completely annihilated by the noisy environment, but nonetheless some of the original quantum correlations persist. These "surviving" quantum correlations are the reason for the expected advantage of the entangled imaging system over the coherent imaging system.

The figure also makes evident that the plot of P_q is the same as P_l, but shifted to the left. In other words, with the same number of photons, the entangled system gives the same performance as the unentangled system, but with a 69 m improvement in range to target. Naturally, as the attenuation coefficient grows, this range advantage decreases.

Regarding the stealthy operation of the system, it is easy to check that, at a distance of $R_e \approx 1374$ m, less than one out of the $\tilde{M}$ photon states will survive the effects of attenuation: $\tilde{M}e^{-\chi R_e} \approx 0.9967 < 1$, where χ is the appropriate light attenuation coefficient. Therefore, it is extremely unlikely that any other sensor outside a range of 1374 m from the underwater vehicle will be able to measure any signal photon states. That is, the system is practically invisible outside a range of 1374 m, but manages to give a detection error probability of less than 0.1 for a target located 426 m away from the vehicle.

If the adversaries place sensors inside the stealthy range of operations of the quantum imaging system (i.e. within 1374 m), their detection systems may be able to measure at least one out of the $\tilde{M}$ photon states. This could be done, for example, if the adversary deploys several UUV's (Unmanned Underwater Vehicles) across the entire area of operations. In this case, depending on the number of measured photons, the adversaries will suspect with some probability the presence of our quantum imaging system. However, they will have a considerable amount of error in trying to determine if a given measured photon state is signal or noise, because they would not have access to the entanglement correlations. Indeed, as we argued before, the stealth strategy of the proposed quantum imaging system is to "hide" the signal photons in the environment noise.

In a rather arbitrary manner, we can define a *stealthy imaging system* if the detection error probability of an adversary that uses the radiation of the imaging system to localize the underwater vehicle is at least 0.4. Notice, however, that this detection error probability bound is completely arbitrary, and one could choose a different set of numbers given the risk constrains of the actual naval operation.

It is convenient to define the interception error probability π, which is the detection error probability of an adversary measuring the sensing photons. Thus, if $\pi = 0.5$, then the adversary has the same chance of determining the presence of the sensor as a coin flip. On the other hand, if $\pi = 0.0$, then the adversary has complete certainty about the presence of the sensor. In this case, stealthier sensor performance requires larger π.

Figure 5 shows the interception error probability for an adversarial sensor located at a distance r from the entangled photon imaging system (π_q, solid line) and the coherent imaging system (π_l, dashed line). As expected, if the adversarial sensors are located very close to the underwater vehicle (i.e. within a 600 m range), these will be able to detect with very high probability the signal photon states, entangled or not. It can also be observed that the entangled photon imaging system satisfies the stealthy imaging bound at 925 m. On the other hand, the coherent photon imaging system satisfies the stealthy imaging bound at 1063 m. Thus, the entangled photon imaging system is stealthier over an extra 138 m in comparison to the coherent imaging system.

Furthermore, one can observe that the plot of π_q is the same as π_l, but shifted to the right. In other words, with the same number of photons, the entangled system gives the same performance as the unentangled system, but with a 138 m improvement in stealthiness. Naturally, as the attenuation coefficient grows, this stealthiness advantage decreases.

[1]Other sources of noise produced by the environment and imperfect technological implementations of the quantum imaging system can be introduced using a link budget similar to the one proposed by the authors for underwater quantum communications [10].

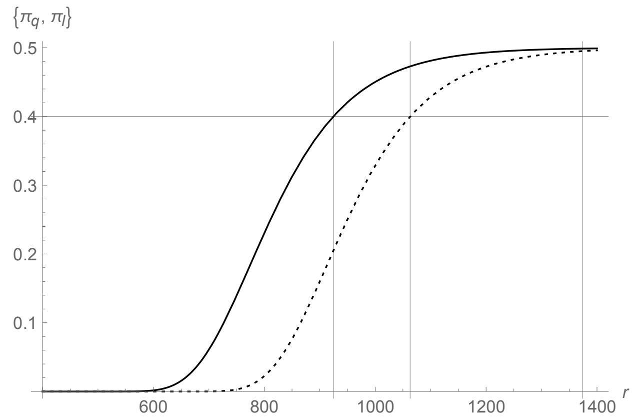

Fig. 5. Interception error probability for an adversarial sensor located at a distance r from the entangled photon imaging system (π_q, solid line) and the coherent imaging system (π_l, dashed line).

Even though water attenuation severely limits the operational range of the imaging system, the range is large enough to be used as a terrain estimation tool for underwater Arctic navigation. Indeed, strong attenuation is not a bad feature if we are exclusively interested in stealth sensing over relatively short distances. In this regard, it seems that "the operational requirements of stealthy underwater Arctic navigation are tailor-made for entangled qubits" [17].

V. Quantum Magnetometry

Our proposal for quantum magnetometry is another example of biologically-inspired quantum sensors. While the quantum photo-detectors described above are inspired by the quantum effects observed in photosynthesis, these quantum magnetometers are inspired by the quantum processes that appear to enable the navigation of migratory birds.

Let us recall that many species of migratory birds follow intricate routes across large distances. This is one of the reasons as to why the avian navigation mechanism has intrigued scientists for many years.

There is little doubt that Earth's magnetic field is the principal enabler of bird navigation and orientation [39]. Thus, migratory birds appear to posses bio-magnetometers able to detect magnetic anomalies (local variations of Earth's geomagnetic field due to the differences in the chemistry, composition, and magnetism of the environment). In addition, mature birds may also rely on visual landmarks, olfactory cues, and even magnetic maps stored in the trigeminal system.

It is important to note that the avian compass is fundamentally different from a traditional man-made magnetic compass [39]. Indeed, it has been observed that:

- The avian magnetic compass does not use the polarity of the magnetic field for the detection of the north direction.
- The avian system is an *inclination compass*. That is, birds derive the north direction from interpreting the inclination of the axial direction of the magnetic field lines in space.
- The avian compass depends on ambient light conditions, but it does not depend on the sun's position for orientation.

There is a biochemical reaction that satisfies these constrains and it is called the *radical pair biochemical reaction mechanism* (RPM) [40], [37]. In RPM, green light excites a caretonoid-porphyrin-fullerene (CPF) molecule into a singlet state, and sequential intramolecular electron transfer processes produce long-lived states (≈ 200 ns). These states are sensitive to the local magnetic field, as unpaired electrons undergo magnetic interactions. The result is that the decay rate of CPF to the ground state, via a singlet or triplet state, depends on the magnitude and direction of the external magnetic field. However, the reaction yield is independent of the polarity of the field.

Thus, it is conjectured that migratory birds posses special photopigments in their eyes that perform RPM [40], [37]. It is important to notice that, even though this process requires sunlight to excite the CPF molecules, it does not require of the sun's position for orientation. It is expected that about 10^8 radical pairs over a volume of 0.4 mm^3 are required to reliably detect a magnetic anomaly of 2%.

Quite amazingly, it has been shown that the initial excited singlet state is entangled, which leads to maximum sensitivity to the external magnetic field [36]. In addition, entanglement appears to be protected within the avian compass. Indeed, it has been shown that the RPM process has the right reaction time-scale: it is long enough to allow the external magnetic field to alter the molecular correlations, but short enough to avoid dissipation and decoherence due to the environment noise [41].

However, the question if birds rely, or not, on RPM for navigation is irrelevant for this discussion. Indeed, the important point is that biological studies inspire the development and synthesis of novel nanomagnets using CPF molecules undergoing RPM processes [35], [38]. In this case, low energy excitation of this type of nanomagnet, through biasing or proper choice of dielectric environment, provides means for sensing field strength and orientation. For instance, it has been shown that these nanomagnets are able to detect weak magnetic fields of about 50 μT. However, it may be possible to extend their sensitivity into the nano-Tesla regime.

Our proposed design for quantum magnetometers is based on CPF nanomagnets enabled by RPM. These devices are characterized by their μT sensitivity, small size, small weight, low cost, and negligible power requirements [13]. These quantum sensors could be of great value for a wide variety of maritime applications, especially in the area of magnetic anomaly detection (MAD) for anti-submarine warfare [13].

It is important to mention that recent years have witnessed the development of extremely sensitive magnetometers. For instance, the Spin-Exchange Relaxation Free (SERF) and Superconducting Quantum Interference Device (SQUID) have a sensitivity in the regime of pico-Teslas. However, these magnetometers have considerable requirements: SERF requires of a heated vapor cell and SQUID needs cryogenic temperatures.

As a consequence, only a handful of these devices can be deployed into the maritime environment during a MAD operation. That is, the current approach to MAD employs few, but very sensitive magnetometers.

On the other hand, our proposed quantum magnetometers could be mass-produced and mass-deployed into the maritime environment. Specifically, we have proposed the use of sensor data fusion on a large number of low-cost, low-power, compact, medium sensitivity quantum magnetometers [13]. Some of the advantages to this approach include: (1) the feasibility of an unattended, robust, flexible, scalable and easy to deploy sensor network; and (2) the feasibility of a network of sensor networks, turning existing airborne and maritime assets into sensors. We are currently conducting research to determine which approach is better given a specific set of operational conditions.

VI. Quantum Gravimetry

It is important to note that most devices proposed by quantum information science are spin-based. That is, the qubits are implemented using a quantum property that closely resembles an "internal angular momentum". Formally, however, spin can only be defined as those physical degrees of freedom that have a non-trivial transformation under a Lorentz transformation according to the rules of special relativity and relativistic quantum field theory [42]. As a consequence, most physical implementations of qubits are *intrinsically relativistic* and are *directly coupled* to classical gravitational fields described by Einstein's General Theory of Relativity [14].

The effect of gravity is best described as a qubit transformation known as the *Wigner Rotation* [14]. Thus, if we have a single qubit written in vector notation as:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \tag{7}$$

then, under the effects of spacetime curvature due to the presence of gravity, the qubit is rotated into:

$$|\tilde{\psi}\rangle = \begin{pmatrix} \cos\Omega/2 & \sin\Omega/2 \\ -\sin\Omega/2 & \cos\Omega/2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \tag{8}$$

where Ω is known as the *Wigner angle*. Notice that all the complexity of the gravitational field is encapsulated into Ω, which in general is very difficult to calculate.

It is important to remark that the Wigner rotation is very small. For instance, for a qubit aboard a satellite in circular orbit around Earth, $\Omega \approx 10^{-9}$ radians per day. However, this gravitational effect is cummulative, and in this sense, it is rather similar to the relativistic corrections applied to GPS satellites.

Clearly, in contrast to the much more abstract concept of a classical bit of information, *quantum information invariably gravitates*. Besides the philosophical implications of this observation, the gravitational effects on qubits has important consequences in the development of quantum technologies.

One of the most interesting consequences of these gravitational effects is the way in which gravity affects computational complexity [14]. More specifically, assume the computational complexity of a quantum algorithm in the absence of gravity is $\mathcal{O}(r)$ (i.e. the number of computational steps required in the worst case scenario is r). Then, under the effects of gravity, the algorithmic complexity scales as:

$$\mathcal{O}\left(r \times \left(\frac{1}{1-n\epsilon}\right)^r\right) \tag{9}$$

where n is the number of qubits involved in the computation, and ϵ is the computational error due to gravity ($\epsilon \approx \Omega$). That is, the complexity of a quantum algorithm may become exponential due solely to small relativistic effects that accumulate over time.

Let us consider, for example, the case of *Grover's algorithm*. This is a well known quantum algorithm designed to search an unstructured and unsorted database with a quadratic improvement in performance when compared to classical database search techniques. That is, if a database has N elements and a noiseless quantum computer has $n = \log_2 N$ qubits, then the classical search requires $\mathcal{O}(N)$ computational steps, while the quantum search algorithm simply requires $\mathcal{O}(\sqrt{N})$ computational steps. Therefore, under the influence of gravity, Grover's algorithm becomes exponential when the list size N is sufficiently large (as $r = \sqrt{N}$), implying that the presumed complexity speedup of a quantum algorithm over its classical counterpart can be entirely eliminated by gravity alone.

However, these gravitational effects on qubits could in principle be harnessed to design highly sensitive *Wigner gravimeters*. That is, it seems feasible to design gravimeters that rely on quantum information processing techniques that *amplify* the effects of the Wigner rotation [14].

Let us recall that the best gravimeters currently available are based on *atom interferometry* [43], [44]. In a nutshell, this technique uses interferometric techniques to measure the falling time of heavy atoms dropped in a gravitational field. The performance of these devices is limited by the *de Broglie's wavelength* of the atoms $\lambda = h/mv$, where h is Planck's constant, m the mass, and v the velocity of the atom. As a consequence, the resolution of this technique is expected to be limited to about $\delta g/g \lesssim 10^{-9}$. It is important to remark that, even though this technique uses sophisticated atom interferometric techniques, it does not use the Wigner rotation and does not exploit any entanglement correlations.

Our radical approach to gravimetry consists of using the gravity-induced Wigner rotation on a quantum system to measure the local gravitational field [14]. A possible implementation uses a quantum computer to run an algorithm that amplifies the effect of the Wigner rotation. This algorithm is somewhat similar to Grover's algorithm, where the computational "oracle" that checks if an item satisfies a search query is replaced by the gravitational field. In principle, it appears that these gravimetric techniques would only be bounded by quantum gravity effects if implemented in a fault tolerant universal quantum computer.

To give an idea of the potential applications of a Wigner gravimeter in the maritime environment, let us consider a manned, nuclear-powered underwater vehicle of $M = 8,000$ tons and a length of $L = 115$ m. In general, these vehicles tend to be dense towards the stern (because of the reactor and engines compartments) and the bow (because of the torpedo and sonar compartments). On the other hand, the mid-ship section has a much lower density (because this is the location of the living quarters which are mostly made of air). Thus, we could roughly approximate the gravitational field of this vehicle as a system of two uniform, speherical objects of mass $m_1 = 0.6M$ and $m_2 = 0.4M$ separated by a distance $L/2$.

If the vehicle is neutrally buoyant, by virtue of the Archimedes principle, the water generates an upward force that exactly counterbalances the vehicle's weight. This balance of forces keeps the vehicle from rising to the surface or sinking to the bottom. Then, at first sight, it would appear that the underwater vehicle has a zero gravitational signature, as its gravitational mass is being masked by the water that it displaces.

However, because the density of the vehicle is not uniform and its shape is not symmetric, we are required to use a multi-polar expansion to properly study the gravitational field that this vehicle generates, as well as its interaction with any other external gravitational fields [45]. Thus, it can be shown that the monopolar field contribution generated by the mass of the vehicle is cancelled exactly by virtue of Archimedes' principle. In addition, the dipole moment is zero because the force provided by Archimedes' principle acts on the center of mass of the neutrally buoyant vehicle.

On the other hand, the gravitational quadrupole moment of the underwater vehicle is different from zero:

$$\Phi(r, \theta) = G \left(\frac{L^2}{8} \right) \left(\frac{m_1 m_2}{m_1 + m_2} \right) \frac{1 - 3\cos^2\theta}{r^3} \qquad (10)$$

In principle, this field could be detected by a sensitive gravimeter. Furthermore, the exact quadrupole field of the underwater vehicle has a very specific configuration that distinguishes its gravitational signature from the gravitational signature of any other objects present in the environment.

Figure 6 shows the strength of the gravitational quadrupole field for the described approximation of the underwater vehicle (solid line). As seen in the graphic, the quadrupole field is much weaker than the monopole field (dashed line). It can be observed that the best atom interferometry gravimeter could detect the gravitational quadrupole of the vehicle up to a distance of about 50m ($\delta g \approx 10^{-8}$ kg m/s^2). However, a quantum Wigner gravimeter operating with 100 qubits in a noiseless quantum computer could in principle detect the vehicle at a range of 200 m. That is, a surface ship equipped with a Wigner gravimeter could be able to detect the underwater vehicle, even if it is traveling under the acoustic barrier provided by the thermocline (located at about 100 m deep).

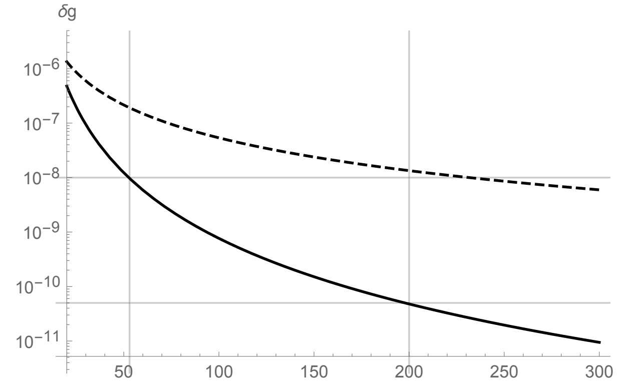

Fig. 6. Strength of the gravitational quadrupole (solid line) and monopole (dashed line) fields for a hypothetical underwater vehicle in kg m/s^2.

VII. Conclusions

In order to take full advantage of the many opportunities offered by our oceans, we need to develop improved sensing technology. We believe quantum sensors are the best alternative currently offered by modern science. Therefore, the development of quantum sensors is crucial to ensure the correct, lawful, secure, and efficient use of our oceans.

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