Maritime Applications of Quantum Computation

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I. Introduction

Computer technology is ubiquitous in every aspect of modern society. From scientific number-crunching algorithms and Artificial Intelligence algorithms for social networks to digital simulation of fabric texture for the fashion industry, we find hardware and software everywhere we glimpse at.

In spite of the tremendous success of computer science and computer engineering, it is well known that many scientific and engineering problems cannot be *efficiently* solved by current computers due to the amount of resources (e.g. time, memory, computer power) required to compute exact (or precise enough, at least) solutions. Hence, cutting-edge research in computer science and computer technology includes novel (mathematical, physical and architectural) models of computation (e.g., [1], [2], [3], [4], [5], [6], [7], [8], [9]), new materials and methods for manufacturing computer hardware (e.g. [11]), and novel methods for speeding-up algorithms (e.g., [79], [12]).

Theoretical computer science is a branch of mathematics that does not take into account the physical properties of those devices used for executing algorithms. Since the behavior of any physical device used for computation or information processing must ultimately be described by the laws of physics, several research approaches have therefore concentrated on thinking of computation within a physical context (e.g [13], [16], [15], [14], [53], [19], [17], [20], [21], [22].) Quantum computation can be defined as the interdisciplinary scientific field devoted to build quantum computers and quantum information processing systems, i.e. computers and information processing systems that use the quantum mechanical properties of Nature. Research on quantum computation heavily focuses on building robust and scalable quantum hardware as well as on designing and executing algorithms which exploit the physical properties of quantum computers. Among the theoretical discoveries and promising conjectures that have positioned quantum computation as a key element in modern science, we find:

1) The development of novel and powerful methods of computation that may allow us to significantly increase our processing power for solving certain problems (e.g. [23], [24], [25], [26], [27], [28].)
2) An increasing number of quantum computing applications in several branches of science and technology like image processing and computational geometry [30], [31], [29], [35], [36], [37], [39], [38], [40], [41], [42], [43], pattern recognition and machine learning [44], [46], [45], [47], [48], [49], [50], and warfare [51], [52].
3) The simulation of complex physical systems and mathematical problems for which we do not know any classical digital computer algorithm that could efficiently simulate them [53], [55], [57], [54], [56], [58].

Building efficient quantum algorithms is challenging, especially if they are to outperform classical alternatives that have been optimized and refined over many decades. Examples of successful results in quantum computation can be found in [60], [58], [59], [61], [47], [55], [57], [54], [56]. Good introductions and reviews of quantum algorithms can be found in [24], [62], [23], [63], [64], [67], [66], [68], [65], [69], [75], [76], [77], [78], [79], [80], [81], [82], [83].

Quantum computation is now an established research field in science and engineering. In addition to further advance the mathematical and physical foundations of this discipline, current research efforts are focused on identifying and developing cutting-edge quantum algorithms and relevant applications of quantum computing in fields like artificial intelligence [84], military technology [51], machine learning [48], [50], and image processing [85].

In this paper we will discuss the potential applications of quantum computation for signal analysis and image processing in examples relevant to maritime problems. In particular, we will talk about how plankton analysis and sonar signal analysis could receive a great benefit from the use of quantum information processing technologies.

In the following sections, we will succinctly review the fundamentals of quantum computing as well as introduce the field of quantum image processing with emphasis on existing protocols to store and manipulate visual information. We will then proceed to explore how quantum computing can be used for solving maritime problems.

II. A succinct introduction to quantum computing

In quantum mechanics, we associate a Hilbert space $\mathcal{H}$ to each closed (i.e. isolated) physical system. $\mathcal{H}$ is known as the **state space** of the system and, for our purposes, it suffices to think of it as a finite complex vector space. The physical system is completely described by its **state vector**, which is a unit vector $|\psi\rangle \in \mathcal{H}$. Please note that, in quantum mechanics, vectors are denoted as $|\psi\rangle$ instead of $\vec{\psi}$.

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The above description of physical systems implies that a linear combination of state vectors is a state vector. This is known as the **superposition principle**. Hence, any vector state $|\psi\rangle$ may be described as a superposition of basis states $\{|e_i\rangle\}$ in $\mathcal{H}$, i.e. $|\psi\rangle = \sum_i c_i|e_i\rangle$, $c_i \in \mathbb{C}$.

1) The qubit: In classical computation, information is stored and manipulated in the form of bits. The mathematical structure of a classical bit is rather simple. It suffices to define two 'logical' values, traditionally labelled as $\{0, 1\}$, and to relate these values to two different outcomes of a classical measurement. So, a classical bit 'lives' in a scalar space.

In quantum computation, information is stored, manipulated and measured using *qubits*. A qubit is the mathematical description of a physical entity described by the laws of quantum mechanics. Simple examples of physical implementations of qubits include two orthogonal polarizations of a photon (e.g. horizontal and vertical), the alignment of a (spin-1/2) nuclear spin in a magnetic field or two states of an electron orbiting an atom. A qubit may be mathematically represented as a unit vector in a two-dimensional Hilbert $|\psi\rangle \in \mathcal{H}^2$. A qubit $|\psi\rangle$ may be written in general form as

$$|\psi\rangle = \alpha|p\rangle + \beta|q\rangle \tag{1}$$

where $\alpha, \beta \in \mathbb{C}$, $|\alpha|^2 + |\beta|^2 = 1$ and $\{|p\rangle, |q\rangle\}$ is an arbitrary basis spanning $\mathcal{H}^2$. The choice of $\{|p\rangle, |q\rangle\}$ is often $\{|0\rangle, |1\rangle\}$, the so-called computational basis states which form an orthonormal basis for $\mathcal{H}^2$. In general $|\psi\rangle$ is a coherent superposition of the basis states $|p\rangle$ and $|q\rangle$ and can be prepared in an infinite number of ways simply by varying the values of the complex coefficients α and β subject to the normalization constraint.

We can rewrite Eq. (1) as

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle \tag{2}$$

where θ and $\varphi \in \mathbb{R}$. The numbers θ and φ define a point on the unit 3-dimensional sphere known as **Bloch sphere** (Fig. (1)).

Any model of computation must specify three stages: a) the initial conditions to be accomplished for starting algorithm execution, b) how information is processed (i.e., transformed) as algorithms are executed, and c) how information is extracted out from the computing device.

So, as for stage a), i.e. initial conditions for the execution of quantum algorithms, it is customary to set qubits as $|\psi\rangle = |0\rangle$.

2) Evolution of a closed quantum system: We now describe how to design quantum algorithms, i.e. how to use quantum mechanics to build a computational process.

The time evolution of a closed quantum system with state vector $|\psi\rangle$ may be described by a *unitary* transformation $\hat{U}$. The state of a system at time t_2 according to its state at time t_1 is given by

$$|\psi(t_2)\rangle = \hat{U}|\psi(t_1)\rangle. \tag{3}$$

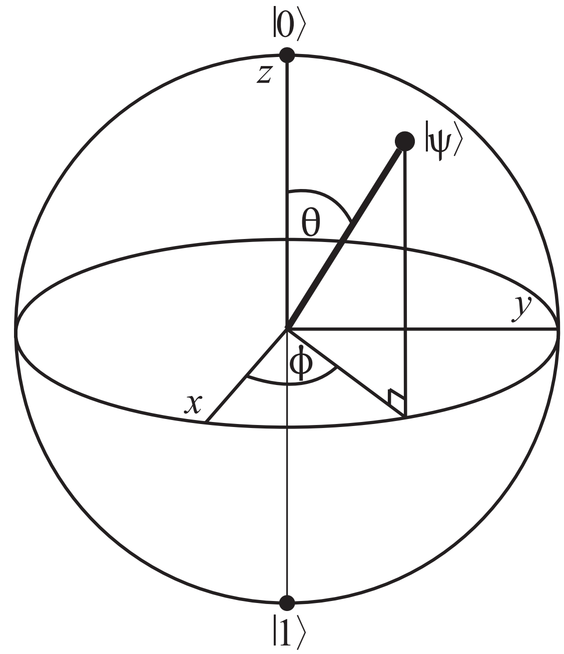

Fig. 1. Bloch sphere representation of a qubit $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$.

Eq. (3) only describes the mathematical properties that an evolution operator must have. The specific evolution operator required to describe the behavior of a particular quantum system depends on the system itself.

In quantum computer science parlance, a unitary operator is best known as a quantum gate, in full resemblance to the corresponding classical computer science concept. Moreover and as in classical computer science, in quantum computing we have sets of quantum gates for universal quantum computation [97], [98].

Hence, a quantum algorithm can be built as a sequence of quantum gates, i.e. the execution of a quantum algorithm is equivalent to transforming initialized qubits $|\psi_i\rangle$ by applying a chain of quantum gates to those qubits [stage b) of a model of computation].

3) Quantum measurements: In quantum mechanics, measurement is a non-trivial and highly counter-intuitive process. Firstly, because measurement outcomes are probabilistic, i.e. regardless of the carefulness in the preparation of a measurement procedure, the possible outcomes of such measurement will be distributed according to a certain probability distribution. Secondly, once a measurement has been performed, a quantum system is unavoidably altered due to the interaction with the measurement apparatus. Consequently, for an arbitrary quantum system, pre-measurement and post-measurement quantum states are different in general.

How is information extracted out from a quantum computing device? By measuring the qubits in which information is stored, after the execution of a quantum algorithm. In order to perform a measurement on a quantum system (e.g., qubits), it is needed to define a set of measurement operators. This set of operators must fulfill a number of rules that allows one to compute the actual probability distribution as well as post-measurement quantum states. Therefore, quantum computing is a probabilistic computational model [75].

4) Mathematical representation of an array of qubits: A composite qubit system $|\Phi\rangle$, often also called a qubit array, is a quantum system made up of several qubits, where each qubit is associated with a two-dimensional Hilbert space $\mathcal{H}^2$. The postulates of quantum mechanics establish that the vector space where a composite qubit system lives is the tensor product of the vector spaces of the component physical systems. Thus, if component qubits are $|\psi\rangle_1, |\psi\rangle_2, \ldots, |\psi\rangle_n$ (where each qubit resides in a two-dimensional Hilbert space $\mathcal{H}^2$) then $|\Phi\rangle$ lives in a Hilbert space $\mathcal{H}^{2^n} = \mathcal{H}^2 \otimes \mathcal{H}^2 \otimes \ldots \otimes \mathcal{H}^2$.

A simple classical data storage device can be considered as an array made up of n classical bits and capable of storing one of 2^n different classical bit strings. A simple quantum memory device is a qubit array. Such a data storage device consists of n qubits. Each is prepared in some quantum state determined by the desired information to be stored in that particular qubit. Thus, in principle, both classical and quantum arrays can store up to 2^n different values. However, in stark contrast to classical data storage, an array of n qubits is capable of storing in a coherent superposition 2^n different bit strings *simultaneously* [23].

5) Quantum mechanical properties as computational resources: The basic properties of quantum hardware and quantum algorithms, as well as the potential power of quantum computation, are inherited from the physical properties and mathematical descriptions of quantum mechanical systems. Firstly, the fact that a qubit is defined in a complex vector space provides us with vector state superposition, a key computational resource for developing quantum algorithms. Moreover, the mathematical operation for combining Hilbert spaces in quantum mechanics, the tensor product, allows for an exponential increase in dimensionality that, together with superposition, makes quantum parallelism another important resource for computation. Quantum entanglement, a special kind of correlation between quantum particles that has no exact counterpart in classical physics, is a central resource for teleporting information [23].

One of the key research thrusts in quantum computing is to determine the extent to which quantum mechanical properties can be exploited to solve complex algorithmic problems. To do so, a via regia is to cross-fertilize other fields of science and engineering to solve complex algorithmic problems more efficiently than is possible with classical computing.

III. A CONCISE INTRODUCTION TO QUANTUM IMAGE PROCESSING

Either manually or by automatic means, extracting information out of images and video is an important activity in modern society. Consequently, computer image processing is a pervasive discipline in science and engineering (e.g., computer vision [86], astrophysics [87], pattern recognition [88], and medicine [89].) Because of the restricted architecture of classical computers and the computational complexity of state-of-the-art classical algorithms in image processing and its applications, finding (more) efficient algorithms to manipulate visual information is a most important research area.

Quantum Image Processing, a blend of quantum computation and image processing, is a discipline focused on designing novel quantum algorithms for storing, processing and retrieving visual information. The field of quantum image processing was born with the publication of [90], [91], [30], [31]. Since then, a vigorous community has focused on several key topics like quantum methods for image storage and retrieval (e.g. [92]), image encryption/decryption (e.g. [93]), image segmentation (e.g., [94]), mathematical morphology, and image filtering (e.g. [96]), among many others.

Quantum Image Processing is a field full of exciting open problems for physicists, mathematicians, computer scientists and engineers. Since quantum image processing is an emergent discipline, the efforts of the community devoted to working in this area have been focused on developing the foundations and basic tools required to process images in quantum mechanical devices and, consequently, many areas of cross-fertilization and mutual benefit are to be explored yet. As examples of open questions in this field, we would include the pertinence of designing quantum algorithms for universal quantum computers or, alternatively, the development of quantum algorithms for specific applications running on specialized quantum hardware (e.g., applications within the realm of military or biological sciences).

The first method for storing and retrieving images using quantum systems was proposed in [30], [31], followed by several other proposals including FRQI (Flexible Representation of Quantum Images) presented by Le et al [92]. Let us briefly introduce Qubit Lattice and FRQI methods on the following lines.

Qubit Lattice.

Human color perception is a physio-psychological phenomenon based on the human eye's responsiveness to specific frequency ranges of the electromagnetic spectrum. Color models, like the RGB (Red, Green and Blue) and HSI (Hue, Saturation and Intensity) models, are used to specify colors in a standard way that makes sense under the theoretical and technological assumptions of classical computers and/or printing systems.

The mathematical definition of a qubit presented in Eq. (2) shows that θ and φ are real parameters in which color can be stored using the actual values of its physical parameter (frequency), rather than a representation of it (e.g., a linear combination of RGB).

Machine A, as is defined in [30], [31], is capable of detecting electromagnetic waves and, depending on the frequency of the detected wave, it outputs an initialized qubit. A acts like an injective function $A : F \rightarrow \Psi$, where F is the set of monochromatic electromagnetic waves whose frequencies can be detected by A and Ψ is the set of quantum states of the form $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$.

Regardless of the frequency value of a particular monochromatic electromagnetic wave, it is always possible to find a value for the real parameter θ in Eq. (2) such that A can initialize qubits in different states when different waves are detected. Moreover, this storage protocol allows us to store frequencies from all the spectrum. Thus, this quantum image processing method could be used for applications within non-visible ranges. Also, due to the continuous nature of θ, it is easy to accommodate the prospect of recording a new color with frequency lying anywhere in a given domain without readjusting our storage protocol as opposed to digital storage protocols, where an adjustment on the number of bits required to record color is needed once the storage capacity limit is reached.

In [30], [31], storing a full image in a quantum multipartite system is done as follows. Let us now define Q as a *lattice* of qubits, that is,

$$Q = \{|q\rangle_{i,j}\}, i \in \{1, 2, \ldots, n_1\}, j \in \{1, 2, \ldots, n_2\}$$

A set of qubit lattices Z is defined as

$$Z = \{Q_k\}, k \in \{1, 2, \ldots, n_3\} \tag{4}$$

Thus, $Z = \{|q\rangle_{i,j,k}\}$ is a set of $n_1 \times n_2 \times n_3$ qubits.

The goal of storing visual information in Z is achieved by using machine A on each qubit lattice Q_k (please see Fig. (2 for a visualization of Z)).

Each lattice $Q_k \in Z$ will be used to store a copy of the image so that, by the end of the recording procedure, Z will be a set of n_3 lattices all of which have been prepared identically. We must prepare several identical copies of the same image because, as exposed in subsection II-3, retrieving information from a quantum system is a probabilistic process, hence statistical techniques should be used to estimate the frequency value originally stored in Z. In [31], the authors present a statistical information retrieval protocol based on the Central Limit Theorem but any other quantum state estimation technique could be used instead.

The procedure to store an image in a qubit lattice is summarized in Algorithm 1. In this algorithm it is assumed that a lattice L made out of machines A is available. We will refer to the element of lattice L as $A_{i,j}$, $i \in \{1, 2, \ldots, n_1\}$ and $j \in \{1, 2, \ldots, n_2\}$.

Algorithm 1. Storing an Image in a Qubit Lattice set Z

1) **Indices initialization**. Set $i = 0$ and $j = 0$
2) **Prepare qubits as a frequency is detected**. For a given frequency $\nu_{i,j}$ use machine $A_{i,j}$ from lattice L in order to prepare qubits $|q\rangle_{i,j,k}$, $k \in \{1, 2, \ldots, n_3\}$, in the same quantum state corresponding to frequency $\nu_{i,j}$
3) **Update indices**. Update i, j values according to the way visual information is made available and go back to step 2, until no more qubits or frequencies are available.

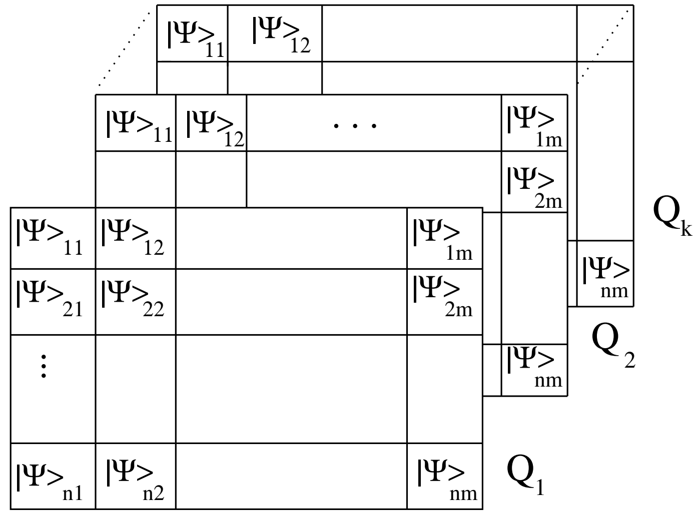

Fig. 2. Each small pigeonhole corresponds to a qubit location. The 3D grid Z is composed of n_3 qubit lattices labeled Q_k.

Note that we have assumed in algorithm 1 that visual information is made available to us in a 'serial' method. Of course, modifying this algorithm for parallel detection of monochromatic electromagnetic waves (as in a digital camera) is a straightforward exercise.

So, the spirit of algorithm 1 is: having detected a frequency, a set of n_3 qubits is initialized in a spatially-ordered fashion (as a conventional digital camera would do if it were able to initialize qubits instead of classical bits). After completion of this procedure for the whole set of qubits of a lattice, we shall have an image stored in each lattice Q_k, i.e. n_3 identically prepared qubit lattices, each containing a copy of the same image.

For instance, let us use Qubit Lattice to store the sharp photograph shown in Fig.(3) (it presents a view of Meztitla, a lovely camping site near Mexico City.) To do so, we would need to use a set of qubit lattices as those described in Eq. (4), each image copy stored in a lattice

$$Q_k = \{|q\rangle_{i,j}\}, i \in \{1, 2, \ldots, n_1\}, j \in \{1, 2, \ldots, n_2\}$$

At some point in time, we would need to retrieve Fig.(3) from Z, hence we would need to perform several measurements on each lattice Q_k. As the number of measurements, we would be retrieving visual information that would allow us to transit from blurred images like Fig(4) to the original image Fig(3).

FRQI. The image processing protocol presented in [92] proposes storing an image using the following formula:

$$|I(\theta)\rangle = \frac{1}{2^n} \sum_{i=0}^{2^{2n}-1} [\cos(\theta_i)|0\rangle + \sin(\theta_i)|1\rangle] \otimes |i\rangle \tag{5}$$

Fig. 3. Original photograph of Meztitla

Fig. 4. Blurred retrieved image of Meztitla

where $\theta \in [0, \frac{\pi}{2}]$, $i \in \{0, 1, 2, \ldots, 2^{2n} - 1\}$, and $\theta = \{\theta_0, \theta_1, \theta_2, \ldots, \theta_{2^{2n}-1}\}$ (θ is the vector of angles encoding colors). The FRQI protocol is composed of two parts:

- The expression $\cos(\theta_i) + \sin(\theta_i)$, where $\theta_i \in \{\theta_0, \theta_1, \theta_2, \ldots, \theta_{2^{2n}-1}\}$ which encodes the image color data, and
- $|i\rangle = |x\rangle_i \otimes |y\rangle_i$ is a set of qubits that contains the position coordinates of the quantum pixels $\cos(\theta_i) + \sin(\theta_i)$.

According to the results presented in [92], the transition

$$|0\rangle^{\otimes 2^{2n}-1} \rightarrow \frac{1}{2^n} \sum_{i=0}^{2^{2n}-1} [\cos(\theta_i)|0\rangle + \sin(\theta_i)|1\rangle] \otimes |i\rangle$$

which is the process of storing an image in Eq(5) with $|0\rangle^{\otimes 2^{2n}-1}$ as initial condition, can be performed in polynomial time. Furthermore, several basic image operations based on the FRQI protocol are defined in [92], including image shift and image compression.

IV. Applications of Quantum Image Processing in Marine Science

A. Phytoplankton analysis

Plankton is a key component of the food base of small fish and squids and, consequently, plays a central role on the aquatic food web. Quantum Image Processing algorithms can be used to analyze plankton in two different scenarios:

Flow cytometry, i.e. the measurement of chemical and physical properties of particles and cells in a fluid using lasers, is a technique used to analyze different plankton groups [32]. One of the most important limitations of this approach is its inability to distinguish morphologically similar cells, this limitation is particularly relevant to study nano- and micro-plankton [33].

Several approaches for enhancing the resolution of flow cytometry techniques have been developed (e.g.,[99], [100]). Among those emergent technologies, we find an interesting application of quantum dots for bioanalysis.

Quantum dots are nanostructured semiconductors of zero dimension widely used in several areas of science and engineering, ranging from solar panels [70] and ultra-high definition TV technology [71] to several applications in bioanalysis [34].

In particular, quantum dots have been used in multicolor flow citometry [72] and phytoplankton analysis [73]. In these applications, quantum dots basically play the role of quantum sensors whose information must be processed later. In this sense, a potential application of quantum image processing techniques could be to develop specific-purpose quantum hardware with quantum image processing algorithms as *quantum firmware*, for processing data delivered by quantum dots arrays.

B. Quantum Computation for Passive Sonar Tracking

The problem of tracking an underwater target using a passive sonar towed linear array may greatly benefit from the algorithmic structure of quantum computation. Let us recall that a sonar towed array is made of a linear array of omnidirectional hydrophones. These sensors are stored within a protecting covering along a cable towed by a surface or underwater vehicle.

Because the hydrophones are omnidirectional, the sensor array has the incorrectly named "left-right ambiguity". That is, a single linear array can only provide the conical angle between the linear array and the target [101]. Thus, once detected, the target is only known to be on the surface of a cone. The simplest way to get rid of such ambiguity is for the ship to steer its course to obtain a new contact cone. This way, if the target is static, its location will be given by the intersection of both cones. Further measurements using different orientations of the linear array can be used to determine, very precisely, the exact location of the target.

Search Type	Query Time	Space Resources
I	$\mathcal{O}(N)$	$\mathcal{O}(N)$
II	$\mathcal{O}(N^{1-1/d})$	$\mathcal{O}(N)$
III	$\mathcal{O}(\log^d N)$	$\mathcal{O}(N \log^{d-1} N)$
IV	$\mathcal{O}(N^{1/2})$	$\mathcal{O}(N)$

TABLE I
THEORETICAL BOUNDS FOR INTERSECTION SEARCH ALGORITHMS WITH N OBJECTS IN d DIMENSIONS.

The problem is much more complex if the target is not static. In such a case, after some interval of time, the uncertainty cone evolves into a conical shell with a width determined by the maximum possible speed of the target. And even more complicated is the case of several moving targets (imagine, for instance, a swarm of underwater autonomous vehicles). And finally, the measurement error of the hydrophones and the reflection and/or refraction of the acoustic signal at the surface of the water, the thermocline, or at the bottom of the ocean, further complicate the precise tracking of the underwater targets.

From a computational point of view, the problem of passive sonar tracking its reduced to a problem of finding the intersections of multiple conical shells. This problem has been shown to be a good candidate to be solved efficiently using quantum computation [102], [103].

The problem of finding the intersections between different d-dimensional geometric figures is solved using techniques from *computational geometry*. It is known that the problem is substantially simplified if the geometric shapes are limited to be d-dimensional coordinate-aligned boxes. In this case, for instance, a conical shell has to be approximated by a large number of small coordinate-aligned boxes. The query time to determine intersections depends on how much memory one is allowed to use. That is, one can speed-up the query time at the expense of additional space resources.

Table 1 shows a comparison between 4 four types of intersection algorithms. Type I: *classical general objects* searches for the intersection of objects of arbitrary geometry; Type II: *classical linear space coordinate-aligned boxes*, searches for the intersection of coordinate-aligned boxes using linear space resources (i.e.. computer memory); Type III: *classical non-linear space coordinate-aligned boxes* speeds up the query time at the expense of a logarithmic factor in space resources; and finally Type IV: *quantum general objects* uses a quantum algorithm for objects of arbitrary geometry.

We can make the following observations:

- The quantum solution performs better than the best linear-space classical solution for coordinate-aligned boxes in 3 or more dimensions.
- The quantum solution has a better space complexity than the best theoretical limit for non-linear space classical search using coordinate-aligned boxes.
- The quantum algorithm does not presume a specific geometry for the intersecting objects.
- The quantum algorithm is independent of the dimension of the problem.

Notice that even though Type III search is faster than the quantum algorithm, the query time is accelerated at the expense of valuable space resources. This is a problematic limitation when large databases are involved. For example, if N is one million objects, we need a memory 400 times larger.

To summarize the quantum algorithm is attractive because of its generality and linear space complexity. As such, quantum computation may substantially speed up the signal analysis of data from passive sonar towed arrays in the presence of several moving targets.

V. CONCLUSIONS

Quantum computing is an interdisciplinary field of physics and computer science whose goal is to harness the quantum-mechanical properties of Nature, in order to develop both hardware and software that overcome their classical counterparts. Quantum computing is now an established field of science & engineering and, consequently, it is constantly reaching out new application areas in order to test its state-of-the-art algorithms and hardware. Quantum Image Processing is an emerging field of quantum computing whose aims include novel quantum algorithms for storing, processing and retrieving visual information.

In this paper, we have presented a succinct introduction to the mathematical building blocks of quantum computing as well as to the foundations of quantum image processing which include two models for image storage, processing and retrieval using quantum mechanics. Finally, we have presented one application of quantum image for phytoplankton analysis as well as one application of quantum computing for passive sonar tracking.

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