Optimal Design for Consensus of Second Order Multi-agent Systems with Damping Term

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Abstract-**This paper presents a LQR-based optimal design method for consensus of second order multi-agent systems with damping term in an undireceted communication topology. Concerning about the undirected connected topology, a Lyapunov based method is carried out to guarantee the asymptotical consensus of systems. Being different from previous researches, this paper takes the damping term into consideration and forms the velocity dynamics. Besides, it is derived that the consensus value is related with its parameter. In addition, simulations are carried out with simplified heading control systems of autonomous underwater vehicles (AUVs), which show the effectiveness of the proposed optimal design method.**

Key Words-multi-agent systems; consensus; second order dynamics; optimal design

I. INTRODUCTION

Recently, cooperative control of multiple autonomous agents has drawn increasing attention of control society due to the wide application prospects of multi-agent systems. The agents may include unmanned air vehicles (UAVs), autonomous underwater vehicles (AUVs), spacecrafts and wire sensors [1-4]. And applications can cover a broad area from civil to military, such as cooperative searching of disaster area, intelligent transportation, surveillance of battlefield, formation control of unmanned autonomous vehicles and so on. Besides, comparing with single intelligent systems, multi-agent systems have advantages in robustness and efficiency.

Among all the problems brought out by multi-agent systems, consensus or synchronization, which means all agents reach an agreement on certain states of interest, is studied widely and lots of meaningful results have been obtained with applications in the field of rendezvous, formation control and flocking. As a whole, extensive researches of consensus are developed with single-integrator dynamics. For example, Olfati-Saber and Murray [5] solved the average consensus problem of multi-agent system with a balanced digraph based on disagreement function. Moreau [6] handled the consensus problem according to a set-valued Lyapunov approach. More details can be found in Ren and Atkins [7]. In fact, single-integrator dynamics can only be regarded as an abstraction of vehicle dynamics. To be more practical, second order systems are studied in order to include a broader and more general class of vehicle dynamics [8]. Tanner and Jadbabaie [9] have investigated the stability properties of multi-vehicle systems with double integrator dynamics with fixed and dynamic topology. Cao and Ren [10] studied the coordination algorithm for second order multi-agent systems under discrete time framework with sampled data.

Even though double integrator dynamics can show essential characteristics of vehicle dynamics compared with the single integral ones, this structure implies that the acceleration of the vehicle can maintain a constant value as long as the input is fixed. This is obviously unreasonable in real-world applications. So unlike the previous mentioned articles, consensus for the second order system with damping term is considered in this paper. In this paper particularly, a simplified AUV heading control system, which is a typical second order system with damping term, is adopted to testify the proposed control protocol. One widely discussed structure for multi-agent systems is the hierarchy structure which means that one agent can only receive message from its superior and send information to its inferior. Therefore, the undirected connected topology is discussed in this paper. Similar to the Lyapunov method used in Ni and Cheng [11], asymptotical consensus of the system is validated for the second order multi-agent systems.

The rest of this paper is organized as follows. In section II, the problem of consensus is formulated and optimal design method is introduced. In section III, consensus problem with connected topology is discussed and that with spanning tree is proved. Section IV contains simulation results with the AUV heading systems and a conclusion is given at last.

II. SYSTEM MODEL

Consider n agents with second order dynamics:

$$\dot{x}_i = v_i, \quad \dot{v}_i = a_i v_i + b_i u_i, \quad i = 1, \cdots, n \tag{1}$$

where $x_i \in R$ and $v_i \in R$ are system states, u_i is the control input needed to be designed and a_i, b_i are system parameters that satisfy $a_i < 0, b_i > 0$. Assume that the system have identical agents, then $a_i = a,\, b_i = b\ (i = 1, \cdots, n)$.

In previous papers, the control protocols of multi-agent systems usually are adopted as following

$$u_i = -\sum_{j \in N_i} a_{ij}((x_i - x_j) + k(v_i - v_j)), \quad k > 0$$

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In this paper, the system dynamics is firstly reformed as below

$$z_i = Az_i + Bu_i \tag{2}$$

where, $z_i = [x_i, v_i]^T$.

Define $A = \begin{bmatrix} 0 & 1 \\ 0 & a \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ b \end{bmatrix}$. Notice that matrix A here denotes the system matrix, not the adjacency matrix and z_i has no relation with Gersgorin Circle Criterion. Then the local neighborhood error of agent i can be described as $\zeta_i = \sum_{j \in N_i} \left(z_i - z_j\right)$. So the control protocol for agent i can be chosen as $u_i = -K\zeta_i$, where, $K = B^T P$ and P satisfy the Riccati equation below.

$$A^T P + PA - PBB^T P + Q = 0$$

With this definition of feedback gain K for agent i, it can be found that this protocol can minimize the performance index expressed as Eq. (3) and yield the local optimal results.

$$J_i = \frac{1}{2}\int_0^\infty x_i^T Q x_i + u_i^T u_i dt \tag{3}$$

Another advantage of optimal design method is that the choice of feedback gain only depends on the choice of matrix Q and decoupled from the graph structure. In section III, it is proved that based on this control protocol, asymptotical consensus of multi-agent systems with system dynamics depicted as Eq. (2) could be reached.

III Optimal design of consensus for multi-agent systems

In this section, the problem of asymptotical consensus shown as Eq. (2) is settled with certain requirements of Q is satisfied, where the topology is an undirected connected graph. First, a definition for system consensus is given.

Definition 1. Consensus of the system is regarded as achieved when the following equation is satisfied

$$\lim_{t\to\infty} \left\| z_i(t) - z_j(t) \right\| = 0 \quad (i, j = 1, \cdots n)$$

for arbitrary initial values of $z_i(0), i = 1, \cdots n$.

Define the states error between two distinct agents i and j as $\zeta_{ij} = z_i - z_j$. So the consensus can be achieved if $\lim_{t\to\infty} \left\| \zeta_{ij}(t) \right\| = 0$ ($i, j = 1, \cdots n$), and the system error variable can be defined as $\Delta = \begin{bmatrix} \zeta_{11}^T & \zeta_{12}^T & \cdots & \zeta_{1n}^T \end{bmatrix}^T \in R^{2n\times 1}$. Then Lemma 1 is proposed.

Lemma 1. If the system error variable satisfies that $\lim_{t\to\infty} \left\| \Delta(t) \right\| = 0$, then the consensus of system is reached.

Proof. According to the definition of $\Delta(t)$, $\lim_{t\to\infty} \left\| \Delta(t) \right\| = 0$ means $\lim_{t\to\infty} \left\| \zeta_{1j}(t) \right\| = 0$ ($i, j = 1, \cdots n$). Since $\zeta_{ij} = z_i - z_j$, it can be acquired that $\zeta_{ij} = \zeta_{1j} - \zeta_{1i}$. So if $\lim_{t\to\infty} \left\| \Delta(t) \right\| = 0$ is satisfied, it can be concluded that $\lim_{t\to\infty} \left\| \zeta_{ij}(t) \right\| = 0, \quad i, j = 1, \cdots n$. □

Therefore, based on Lemma 1, the analysis of system consensus is equivalent to the convergence of $\Delta(t)$. Next, the error system dynamics is presented. By differentiating ζ_{1i}, Eq. (4) is obtained.

$$\begin{aligned} \dot{\zeta}_{1i} &= \dot{z}_1 - \dot{z}_i = A\zeta_{1i} - BK\sum_{j=1}^n a_{1j}\zeta_{1j} + BK\sum_{j=1}^n a_{ij}(\zeta_{1j} - \zeta_{1i}) \\ &= A\zeta_{1i} - BKd_i\zeta_{1i} + BK\sum_{j=1}^n a_{ij}\zeta_{1j} - BK\sum_{j=1}^n a_{1j}\zeta_{1j} \end{aligned} \tag{4}$$

So the error system dynamics can be depicted as followed.

$$\begin{aligned} \dot{\Delta} &= (I_n \otimes A)\Delta - (L \otimes BK)\Delta - (A_{nG} \otimes BK)\Delta \\ &= (I_n \otimes A - L \otimes BK - A_{nG} \otimes BK)\Delta \end{aligned} \tag{5}$$

where, A_{nG} is defined as below. It is obvious that $rank(A_{nG}) = 1$.

$$A_{nG} = \begin{bmatrix} a_{11} & a_{11} & \cdots & a_{1n} \\ a_{11} & a_{11} & \cdots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{11} & a_{11} & \cdots & a_{1n} \end{bmatrix} \in R^{n\times n}$$

Lemma 2. For a connected graph, the matrix $M = L + L^T + A_{nG} + A_{nG}^T$ is invertible.

Proof. According to [12], strictly diagonally dominant matrix is nonsingular and invertible. Therefore, the strictly diagonally dominant property of M needs to be proved. Denote the i_{th} row of matrix M as M_i. Similarly, L_i and A_{nGi} represent the i_{th} row of matrix L, A_{nG} respectively.

For the first row of M, with $L_1 = L_1^T$ and $(A_{nG}^T)_1 = \mathbf{0} \in R^{1\times n}$, it can be found that

$$\begin{cases} L_1 + L_1^T = \begin{bmatrix} 2d_1 & -2a_{12} & \cdots & -2a_{1n} \end{bmatrix} \\ A_{nG1} + (A_{nG}^T)_1 = \begin{bmatrix} 0 & a_{12} & \cdots & a_{1n} \end{bmatrix} \end{cases} \tag{6}$$

Then $M_1 = L_1 + L_1^T + A_{nG1} + (A_{nG}^T)_1$ can be expressed as follows

$$M_1 = \begin{bmatrix} 2d_1 & -a_{12} & \cdots & -a_{1n} \end{bmatrix} \tag{7}$$

Being a strictly diagonally dominant matrix, it requires that $\left| m_{ii} \right| > \sum_{j\neq i} \left| m_{ij} \right| \quad for\ i = 1, \ldots, n$. According to the definition of $d_i = \sum_{j=1, j\neq i}^n a_{ij}$, it can be easily concluded that M_1 satisfies this condition.

For the jth row of M, with $A_{nGj} = A_{nG1}$ and $L_j + L_j^T = \begin{bmatrix} -2a_{j1} & \cdots & 2d_j & \cdots & -2a_{jn} \end{bmatrix}$, the value of A_{nGj}^T

depends on the connection between agent *1* and agent j, which is , the value of a_{1j}.

Case 1: If $a_{1j}=1$, then $(A_{nG}^T)_j=\mathbf{1}\in R^{1\times n}$.

Define $\delta=2d_j+a_{1j}+1-\sum_{n\neq j}\left|-2a_{jn}+1+a_{1n}\right|$, $\delta>0$ means that $\left|m_{jj}\right|>\sum_{k\neq j}\left|m_{jk}\right|$ is satisfied. In fact, Eq. (8) can be obtained using the form of δ.

$$\begin{aligned}\delta&=2d_j+a_{1j}+1-\sum_{n\neq j}\left|2a_{jn}-1-a_{1n}\right|\\&\geq 2d_j+a_{1j}+1-\sum_{n\neq j}(\left|2a_{jn}\right|-\left|1+a_{1n}\right|)\\&=2d_j-\sum_{n\neq j}\left|2a_{jn}\right|+a_{1j}+1+\sum_{n\neq j}\left|1+a_{1n}\right|>0\end{aligned}\tag{8}$$

Case 2: If $a_{1j}=0$, then $(A_{nG}^T)_j=\mathbf{0}\in R^{1\times n}$.

With $\delta=2d_j-\sum_{n\neq j}\left|-2a_{jn}+a_{1n}\right|$, Eq. (9) can be derived as below

$$\begin{aligned}\delta&=2d_j-\sum_{n\neq j}\left|2a_{jn}-a_{1n}\right|\\&\geq 2d_j-\sum_{n\neq j}(\left|2a_{jn}\right|-\left|a_{1n}\right|)=\sum_{n\neq j}\left|a_{1n}\right|>0\end{aligned}\tag{9}$$

Eq. (9) is derived for at least one agent i should satisfy $a_{1i}=1$, which means that the graph is connected.

As a consequence, it can be concluded that each row of matrix M satisfies $\left|m_{ii}\right|>\sum_{j\neq i}\left|m_{ij}\right|$ ($i=1,\ldots,n$) and M is strictly diagonally dominant. Lemma 2 is proved. □

Subsequently, theorem 1 of this paper is presented.

Theorem 1. For multi-agent systems, which is consisted of second order dynamics described as Eq. (2)., assuming communication topology is connected with the feedback gain $K=B^TP$ and P as the solution of Riccati equation, then with the control algorithm $u_i=-K\zeta_i$, the consensus of systems is guaranteed if the diagonal element of positive definite matrix Q in Riccati equation satisfies the following condition

$$q>\max\left(0,\frac{(1-\mu)(b+2)}{\mu b}\right)\tag{10}$$

where, q is the diagonal element of matrix $Q=\begin{bmatrix}q&0\\0&q\end{bmatrix}$, and μ is the smallest eigenvalue of matrix M which is invertible and symmetric ($\mu>0$).

Proof. Choose a Lyapunov function as

$$V(\Delta)=\Delta^T(I_n\otimes P)\Delta$$

Combing with Eq. (5), the time derivative of Lyapunov candidate is calculated as follows

$$\begin{aligned}\dot V(\Delta)&=\dot\Delta^T(I_n\otimes P)\Delta+\Delta^T(I_n\otimes P)\dot\Delta\\&=\Delta^T(I_n\otimes A-L\otimes BK-A_{nG}\otimes BK)^T(I_n\otimes P)\Delta\\&\quad+\Delta^T(I_n\otimes P)(I_n\otimes A-L\otimes BK-A_{nG}\otimes BK)\Delta\end{aligned}\tag{11}$$

For the Kronecker production has the properties as

$$(A\otimes B)(C\otimes D)=(AC)\otimes(BD)\,,\,(A\otimes B)^T=A^T\otimes B^T$$

Hence, with proper derivation, Eq. (11) can be rewritten as below

$$\begin{aligned}\dot V(\Delta)&=\Delta^T(I_n\otimes(A^TP+PA)-(L+L^T)\otimes PBB^TP\\&\quad-(A_{nG}+A_{nG}^T)\otimes PBB^TP)\Delta\\&=\Delta^T(I_n\otimes(A^TP+PA)-M\otimes PBB^TP)\Delta\end{aligned}\tag{12}$$

According to Lemma 2 and Gersgorin Circle Criterion, M is invertible and symmetric, and $\mu_1,\mu_2\cdots\mu_n$ are the positive eigenvalues of matrix. Set μ as the smallest eigenvalue of matrix M , only the negative definiteness of matrix $A^TP+PA-\mu PBB^TP$ needs to be proved to verify that $\dot V(\Delta)<0$ *for* $\Delta\neq 0$

Before further discussion about the negative definiteness of the matrix, the Riccati equation for the second order system and the feedback gain K should be studied. Recall the Riccati equation

$$A^TP+PA-PBB^TP+Q=0\tag{13}$$

Then based on Eq. (13), the following equation can be obtained

$$(AB)^TPB+B^TP(AB)+B^TQB-B^TPBB^TPB=0\tag{14}$$

Notice that for a second order system, K can be defined as $K=[k_1\quad k_2],k_1,k_2>0$. Considering that $A=\begin{bmatrix}0&1\\0&a\end{bmatrix}$ and $B=\begin{bmatrix}0\\b\end{bmatrix}$, and with no loss of generality, k_1 can be set as $k_1=1$ as long as Eq. (15) can be solved.

$$b^2k_2^2-2abk_2-2b-b^2q=0\tag{15}$$

Therefore, the solution of k_2 is $k_2=\frac{a}{b}+\sqrt{\frac{a^2}{b^2}+\frac{2}{b}+q}>0$.

Since $a<0,b>0$,

$$k_2<\frac{a}{b}+\left|\frac{a}{b}\right|+\sqrt{\frac{2}{b}+q}=\sqrt{\frac{2}{b}+q}\tag{16}$$

The negative definiteness of matrix $A^TP+PA-\mu PBB^TP$ requires that

$$q>\frac{1-\mu}{2}(1+k_2)^2\tag{17}$$

To satisfy this requirement, q can be chosen as

$$q>\frac{1-\mu}{2}\left(1+\sqrt{\frac{2}{b}+q}\right)^2\tag{18}$$

In addition, Eq. (19) is derived as follows

$$q > \frac{1-\mu}{2}\left(2+\frac{4}{b}+2q\right) \tag{19}$$

With proper transformation, Eq. (19) can be rewritten as the condition which is equal to q in Theorem 1. And it can be concluded that if $q > \max\left(0, \frac{(1-\mu)(b+2)}{\mu b}\right)$ is satisfied, the convergence of error system dynamics is guaranteed. That is, $\lim_{t\to\infty}\|\Delta(t)\| = 0$. According to Lemma 1 and Definition 1, the consensus of the multi-agent systems is reached. Theorem 1 is proved. □

In the proof of Theorem 1, it is worth to be noticed that the symmetry of Laplacian matrix L is significant for keeping the eigenvalues of $L+L^T$ nonnegative.

IV SIMULATIONS

In this section, simulations are carried out to testify the proposed theorem in section III. A second order system is adopted according to a simplified AUV heading control system as Eq. (20). The velocity of AUV is set as 2.0 m/s. And the states are heading angle and angular rate of the AUV.

$$\dot{z} = \begin{bmatrix} 0 & 1 \\ 0 & -0.6558 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0.2165 \end{bmatrix} u \tag{20}$$

Five identical agents are selected, and the communication topology in this example is described as Figure 1. Adjacency gains are set as $a_{ij} = 1$. Hence the Laplacian matrix L and the corresponding left eigenvector w_1^T for $\lambda_1 = 0$ can be decided. According to Theorem 1, choose $q = 10$.

Simulation results are shown in Figure 2 and Figure 3 for heading and angular rate values respectively. The initial values of heading are randomly selected between 0°-50°, angular rates are selected between 0-10 °/s. In this case, the initial values of heading are $\begin{bmatrix} 3.7^\circ & 26.5^\circ & 46.7^\circ & 28.4^\circ & 0.6^\circ \end{bmatrix}$, angular rate are $\begin{bmatrix} 0.54 & 7.79 & 1.30 & 4.69 & 3.37 \end{bmatrix} {}^\circ/s$, and the consensus is $\begin{bmatrix} 26.6^\circ & 0{}^\circ/s \end{bmatrix}$.

where,

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 3 & 0 & -1 & -1 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix},\ w_1^T = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

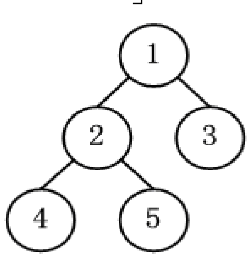

FIGURE 1: The undirected connected graph

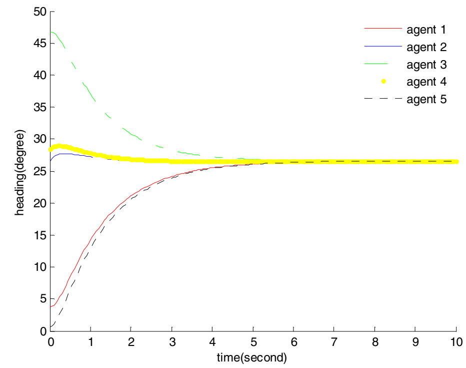

FIGURE 2: Heading angles of multi-AUV systems

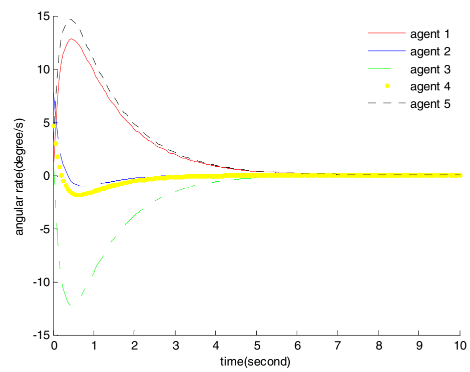

FIGURE 3: Angular rates of multi-AUV systems

The example above has concerned the consensus of heading control problem for multi-AUV with randomly selected initial values. The results have demonstrated the effectiveness of the algorithms yielded by Theorem 1.

V CONCLUSIONS

This paper studied the asymptotical consensus problem of multi-agent systems with second order dynamics. Different from previous researches about second order system, a damping term was added to the velocity dynamics. A commonly discussed communication topology was concerned, where sufficient conditions for optimal design of control protocols were derived. It was concluded that asymptotical consensus can be reached as long as the choice of matrix Q in Riccati equation was appropriate and the consensus value was calculated with related damping parameter. Simulations were carried out with AUV heading control systems which could be viewed as a typical second order system with damping term. Results had demonstrated the validity of the proposed theorems. However, one shortage of this paper is that only sufficient conditions for Q are derived, and necessary conditions will be studied in future work.

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REFERENCES

[1] Maza I., Caballero F. and Capitán, J. et al, "Experimental results in multi-UAV coordination for disaster management and civil security applications," *Journal of intelligent & robotic systems*, 2011, 61(1-4), pp. 563-585.

[2] Carlési N., Michel F. and Jouvencel B. et al, "Generic architecture for multi-AUV cooperation based on a multi-agent reactive organizational approach," IEEE International Conference on Intelligent Robots and Systems (IROS), 2011.

[3] Lugaz N., Hernandez-Charpak J.N. and Roussev I.I. et al, "Determining the azimuthal properties of coronal mass ejections from multi-spacecraft remote-sensing observations with STEREO SECCHI," The Astrophysical Journal, 2010, 715(1), pp. 493-499.

[4] S. Sitharama Iyengar and Richard R. Brooks. "Distributed sensor networks: sensor networking and applications," Chapman & Hall/CRC, 2012.

[5] Olfati-Saber R. and Murray R. M., "Consensus problems in networks of agents with switching topology and time-delays," IEEE Transactions on Automatic Control, 2004, 49(9), pp. 1520–1533.

[6] Moreau L., "Stability of multi-agent systems with time-dependent communication links," IEEE Transactions on Automatic Control, 2005, 50(2), pp. 169–182.

[7] Ren W., Beard R. W. and Atkins E. M., "A survey of consensus problems in multi-agent coordination," In Proceedings of American Control Conference, 2005.

[8] Ren W. and Beard R., "Distributed consensus in multi-vehicle cooperative control: theory and applications," Springer, 2008.

[9] Tanner H. G., Jadbabaie A., Pappas G. J. Stable flocking of mobile agents, Part I: Fixed topology. In Proceedings of Decision and Control, 2003.

[10] Cao Y. and Ren W., "Sampled-data discrete-time coordination algorithms for double-integrator dynamics under dynamic directed interaction," International Journal of Control, 2010, 83(3), pp. 506-515.

[11] Ni W. and Cheng D., "Leader-following consensus of multi-agent systems under fixed and switching topologies," Systems & Control Letters, 2010, 59(3), pp. 209-217.

[12] Poole D., "Linear algebra: A modern introduction," CengageBrain. Com, 2010.