Cooperative Localization for Multi-AUVs Based on Time-Of-Flight of Acoustic Signal Using Moving Horizon Estimation *

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***Abstract*—Research on cooperative localization of multiple autonomous underwater vehicles (AUV) is an important topic to solve the navigation problem in Middle-Depth-Zone. An algorithm based on Time-Of-Flight (TOF) of acoustic signal for multi-UUVs is presented. A Moving Horizon Estimation (MHE) estimator with EKF based arrive cost updating for multi-AUVs with the master-slaver framework is designed to fuse the proprioceptive and exteroceptive sensors. The simulation results are shown to prove the localization accuracy has been improved effectively by using the proposed cooperative localization algorithm.**

Keywords—Multi-AUVs; cooperative localization; Moving Horizon Estimation;

I. INTRODUCTION (*HEADING 1*)

With the ocean operation region expanding and underwater missions more and more complexity, multi-AUVs gradually become a mainstream trend in the field of underwater vehicle, being used for a variety of task, including oceanographic surveys, demining, and bathymetric data collection in marine and riverine environments, refer to [1]. The cooperative navigation and localization of multi-AUVs is the precondition of the cooperative system.

Cooperative localization problem is firstly proposed in mobile ground robots. An algorithm of cooperative localization and navigation for robots is presented in [2]. It is proven that a group of unmanned robots can localize themselves by sharing pose estimates with relative measurements. In [3, 4], an analysis of the propagation of the estimate error is studied. The conception of cooperative localization and is introduced in underwater navigation field. Time- synchronized acoustic data and relative range are used to calculate the relative range and update the position of the group vehicles in [4]. In [5] the author develops and testes an algorithm for multiple UUVs with both range measurement and communication. A geometric method based on a tetrahedral configuration to obtain a deterministic fix for position of multi-AUVs is proposed in [6]. The work in [7] focuses on cooperative bathymetry-based localization and performs by decentralized particle filtering. The experimental implementations of cooperative localization were presented in [8, 9].

It is well known that multi-sensor data fusion is the key to cooperative localization problem. Extended Kalman Filter (EKF) is widely used because of its easy to use and low computational complexity. In [10, 11], an EKF based algorithm is used to update the position. But EKF may cause big linearization errors owing to the linearization of nonlinear system, which is analyzed in [12]. In [13], Maximum a Posteriori Estimation (MAP) methods overcomes the disadvantage of Maximum Likelihood Estimation (MLE) , it considers the prior probability and is also suitable for cooperative localization problem, which can deal with the nonlinear system models. However, general MAP method cannot run in real-time or long-range terms because its computational complexity grow more and more over time since all the previous states and measurements are considered in every step.

Although these methods mentioned above have been used many years, none of them considered the constraints in the states and noise of the system. There are kinds of constrains and burst-interference in the system. In[14], the common kinds of constrains are given, including physical constrain and process constrain. Physical constrain is physical characteristics quantification. Process constrain is the information in system operating, including the bound of states, the noise`s form, size and statistical characteristics. When there are constrains in the real system, unconstrained state estimator will cause low precision estimation, even be beyond the physical limits. Meanwhile, the environment information obtained by sensors grows more over the time, the calculation will be more complicated and time-consuming, which cannot meet the requirements of the real-time estimation. So the environment information must be optimized.

A Moving Horizon Estimation method for nonlinear constrained systems has been proposed by the present paper in [15]. In [16-18], nonlinear moving horizon state estimation for multi-robot relative localization has been given. And MHE is designed for multi-robots using single beacon in [19]. In this paper, we proposed an MHE based cooperative localization algorithm for multi-AUVs with the master-slaver framework in which structure a master AUV, the leader vehicle, equips high precision INS and slave AUVs equip low cost INS for dead-reckoning. Both AUVs

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equip acoustic modem to measure relative location. The MHE based algorithm takes full account of constrains of the real system, which are difficult to be incorporated into EKF. Arrival cost, which is approximated by EKF covariance updating, restricts the size of the calculation window, avoids the problem of so-called “data explosion” and transforms the state estimation of the multi-AUVs` cooperative localization into the limited quadratic optimization problem with constrains. The proposed algorithm can improve the positioning precision, which balances estimate accuracy and computational complexity.

The rest of this paper is organized as follows. Section 2 describes the research problem. The MHE based cooperative localization algorithm is proposed in section 3. And the simulation results is shown and analyzed in section 4. At last, the conclusion is given in section 5.

II. Cooperative Localization Problem in Master-Slaver

A. System Architecture

There are two forms of cooperative navigation. One is parallel form: all of the AUVs have equipped the same sensors. They are equal and can get the information of any other one. The other is Leader-Fellow form: few master AUVs equip with high level sensors and many slaver AUVs equip with low-cost sensors. Slaver gets the information from the master, and then updates its position. This form reduces the cost and reaches the request. So it has been the most important approach to solve the underwater navigation of multiple AUVs. We consider the Master-Slaver form in the paper. Both the master and slaver are equipped with acoustic modem, time-synchrony-clock, and dead-reckoning sensors. Fig.1 shows the scheme. Firstly the master sends the acoustic pings at the fixed time which is known by all the slavers in advance. Then, after an interval the master broadcasts its true position by acoustic modem. At the end the slavers use the information from master to calculate its position.

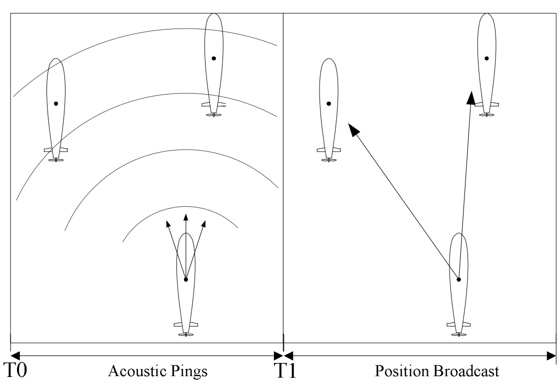

Fig.1. Scheme of Cooperative Localization

B. System Model

Consider the AUV as a point. $[x_k, y_k, z_k]$ is the trajectory of AUV at time k. Where x_k is the longitude, y_k is the latitude and z_k is the depth. z_k can be measured by the depth sensor directly, so we do not take into account in the model. The model can be described as follows:

$$\begin{cases} x_{k+1} = x_k + \Delta t \cdot v_k \cos\phi_k. \\ y_{k+1} = y_k + \Delta t \cdot v_k \sin\phi_k. \\ \phi_{k+1} = \phi_k. \end{cases} \tag{1}$$

Where Δt is the sampling time interval, v_k is the velocity measured by DVL. ϕ_k is the heading angle measured by compass. The process model of AUV also can be represented as the following nonlinear discrete time system:

$$\begin{aligned} \mathbf{X}_{k+1} &= f(\mathbf{X}_k, \mathbf{u}_k, \omega_k) = \mathbf{X}_k + \Gamma(\mathbf{u}_k + \omega_k) \\ &= \mathbf{X}_k + \Delta t F_x \mathbf{u}_k + \omega_k \end{aligned} \tag{2}$$

Where $\mathbf{X}_k = (x_k, y_k, \phi_k)^T$ represents AUVs position at time k. $\mathbf{u}_k = (v_k, \phi_k)^T$, $\Gamma(\mathbf{u}_k + \omega_k)$ is a nonlinear item. F_x is the Jacobian of f evaluated at X_k. ω_k is white Gaussian noise process with zero mean and known covariance, described as follow:

$$\mathbf{Q}_k = E\{\omega_k \omega_k^T\} = \begin{bmatrix} \sigma_{v_k}^2 & 0 \\ 0 & \sigma_{\phi_k}^2 \end{bmatrix}. \tag{3}$$

C. Measurement Model

AUVs calculate the relative range between them by using Time-Of-Flight of acoustic signal from the acoustic modem. Slaver AUV will get the information from the master to bound its localization error. Fig.2 shows how the cooperative localization algorithm works.

At time k, slaver gets the acoustic pings and position of master, then it can calculate the relative distance, additional with the depth sensor we can reduce the algorithm from 3D to 2D. Then all the AUVs travel a period. At time k+1, we can also get the distance in horizontal space. Meanwhile, the slaver can get the distance $(DX_{k,k+1}, DY_{k,k+1})$ traveled from t_k to t_{k+1}. As shown in Fig.2, system of equations is obtained from the intersections of two circles in horizontal space. It has the position of the slaver at t_{k+1}. So the measurement model is given by the system of equations as follow:

$$\begin{cases} r_k^2 = (x_{k+1}^S - DX_{k,k+1} - x_k^M)^2 + (y_{k+1}^S - DY_{k,k+1} - y_k^M)^2 \\ r_{k+1}^2 = (x_{k+1}^S - x_{k+1}^M)^2 + (y_{k+1}^S - y_{k+1}^M)^2 \end{cases} \tag{4}$$

Where the superscript M or S represents master AUV or slaver AUV. r_k is the distance between them in horizontal space.

Then the measurement model is given as follow:

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$$
\begin{aligned}
Z_{k+1} &= H(X_{k+1}^S, X_k^M, X_{k+1}^M, D_{k,k+1}) + \upsilon_z \\
&= \begin{bmatrix} r_k^2 \\ r_{k+1}^2 \end{bmatrix} + \upsilon_z \\
&= \begin{bmatrix} (x_{k+1}^S - DX_{k,k+1} - x_k^M)^2 + (y_{k+1}^S - DY_{k,k+1} - y_k^M)^2 \\ (x_{k+1}^S - x_{k+1}^M)^2 + (y_{k+1}^S - y_{k+1}^M)^2 \end{bmatrix} + \upsilon_z
\end{aligned} \tag{5}
$$

Where $\upsilon_z \sim \mathrm{N}(0, \mathbf{R})$ is the Gaussian noise on the measurement. $\mathbf{R}$ is given as follow:

$$
R = E\{V_z V_z^{\mathrm{T}}\} = \begin{bmatrix} \sigma_{r_k^2}^2 & 0 \\ 0 & \sigma_{r_{k+1}^2}^2 \end{bmatrix} \tag{6}
$$

We give the Jacobian matrix of H_{k+1} as follow:

$$
\begin{aligned}
H_{k+1} &= \left.\frac{\partial h}{\partial X_{k+1}}\right|_{\hat{X}_{k+1,k}} \\
&= \begin{bmatrix} 2(x_{k+1}^S - DX_{k,k+1} - x_k^M) & 2(y_{k+1}^S - DY_{k,k+1} - y_k^M) & 0 \\ 2(x_{k+1}^S - x_{k+1}^M) & 2(y_{k+1}^S - y_{k+1}^M) & 0 \end{bmatrix}
\end{aligned} \tag{7}
$$

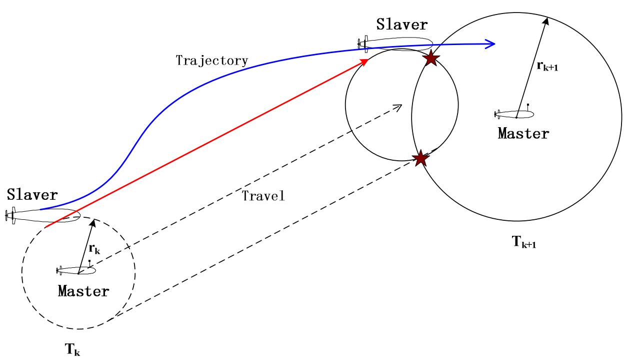

Fig.2. Scheme of measurement model

D. State Constrains

The priori knowledge of the system and environment, including the size, form or statistical property of states and noise, can be introduced into state estimation as constrains in order to improve the precision and reasonability. In the cooperative system of multi-AUVs, the size of mission operating and the range of sensors can be constrains in the estimation. The constrain can be described as follow:

$$
g_i(\mathbf{X}_i, \mathbf{u}_i, \omega_i) \le \mathbf{d}_i, i = 0, \ldots, k. \tag{8}
$$

III. Cooperative Localization Algorithm Using MHE

A. EKF based cooperative localization algorithm

Equation (2) is a nonlinear system, the Kalman Filter (KF) is a recursive linear estimator which successively calculates a minimum variance estimate for a state, that evolves over time, on the basis of periodic observations that are linearly related to this state. The Extended Kalman Filter is developed to solve the problem of estimation for the nonlinear systems.

1) State Prediction: State prediction $\hat{X}_{k+1,k}$ is the state at the next time k based only on the information available up to time k-1. The equation is given as follow:

$$
\hat{X}_{k+1,k} = f(\hat{X}_k, u_k, 0) = \hat{X}_k + \Gamma(u_k) \tag{9}
$$

The prediction covariance of the previous estimate is given as follow:

$$
P_{k+1,k} = F_x \bullet P \bullet F_x^T + F_u \bullet Q_k \bullet F_u^T \tag{10}
$$

Where F_x is the Jacobian matrix of f evaluated at X_k, and F_u is the Jacobian matrix of f evaluated at u_k. And F_x, F_u are defined as follow:

$$
F_x = \frac{\partial f}{\partial X_k} = \begin{bmatrix} 1 & 0 & \text{-T}\bullet\text{V}_k\bullet\sin(\phi_k) \\ 0 & 1 & \text{T}\bullet\text{V}_k\bullet\cos(\phi_k) \\ 0 & 0 & 1 \end{bmatrix} \tag{11}
$$

$$
F_u = \frac{\partial f}{\partial u_k} = \begin{bmatrix} T\bullet\cos(\phi_k) & 0 \\ T\bullet\sin(\phi_k) & 0 \\ 0 & 1 \end{bmatrix} \tag{12}
$$

2) Update Equation: Equipped with the prediction and innovation equations, a linearized estimator can be proposed.

$$
P_{k+1} = [I - K_{k+1} H_{k+1}] P_{k+1,k} \tag{13}
$$

$$
P_{k+1,k} = F_x \bullet P \bullet F_x^T + F_u \bullet Q_k \bullet F_u^T \tag{14}
$$

$$
S_{k+1} = H_{k+1} P_{k+1,k} H_{k+1}^T + R \tag{15}
$$

The gain is given as follow:

$$
K_{k+1} = P_{k+1,k} H_{k+1}^T S_{k+1}^{-1} \tag{16}
$$

The estimation is:

$$
\hat{X}_{k+1} = \hat{X}_{k+1,k} + K_k [Z_{k+1} - h(\hat{X}_{k+1,k})] \tag{17}
$$

Where the estimation $\hat{X}_{k+1}$ is the current AUV pose and P_{k+1} is the current covariance.

B. MHE based cooperative localization algorithm

Specific to system model (2) and measurement equation (5), EKF method given in the previous section obtains the estimated information from the measurement in real-time recursively, which is used to correct error. Different from EKF, MHE transforms the cooperative localization problem into the limited quadratic optimization problem with constrains and optimizes the estimation error with both the previous and current pose information.

The first step of our proposed algorithm combining EKF and MHE is cooperative localization algorithm in Master-Slaver with EKF updating. The second step is to optimize the estimation error by using MHE to fuse the measurement with constrains. The environment information obtained by sensors grows more over the time, the full information estimation problem (18) will also meet the "data explosion" problem.

$$\begin{aligned}
&\min_{\mathbf{X}_0,\mathbf{X}_1,\ldots,\mathbf{X}_k} \Phi_k(\mathbf{X}_{0:k}) \\
&= \min_{\mathbf{X}_0,\mathbf{X}_1,\ldots,\mathbf{X}_k} \left\| \mathbf{X}_0 - \hat{\mathbf{X}}_0 \right\|_{P_0}^2 \\
&+ \sum_{i=0}^{k} \left\| \mathbf{X}_{i+1} - f(\mathbf{X}_i, \mathbf{u}_i) \right\|_{\mathbf{Q}_i}^2 + \sum_{i=0}^{k-1} \left\| \mathbf{Z}_i - H_i(\mathbf{X}_i) \right\|_R^2
\end{aligned} \tag{18}$$

In order to avoid this problem, MHE treats the whole time into two parts, $\{t: 0 \le t \le k-N\}$ and $\{t: k-N+1 \le t \le k\}$. In the first part, we use EKF to update states and define its estimation of P as an arrival cost. The second part the size of sliding estimation window N is introduced to MHE and arrival cost is approximated by EKF results to evaluate the influence from the measurements of the period (0, k-N), which is shown in Fig.3. Then the problem (18) can be described as:

$$\begin{aligned}
&\arg\min_{\mathbf{X}_0,\mathbf{X}_1,\ldots,\mathbf{X}_k} \Phi_k(\mathbf{X}_{0:k}) \\
&= \left\| \mathbf{X}_0 - \hat{\mathbf{X}}_0 \right\|_{P_0}^2 \\
&+ \sum_{i=0}^{k-N} \left\| \mathbf{X}_{i+1} - f(\mathbf{X}_i, \mathbf{u}_i) \right\|_{\mathbf{Q}_i}^2 + \sum_{i=0}^{k-N} \left\| \mathbf{Z}_i - H_i(\mathbf{X}_i) \right\|_R^2 \\
&+ \sum_{i=k-N+1}^{k-1} \left\| \mathbf{X}_{i+1} - f(\mathbf{X}_i, \mathbf{u}_i) \right\|_{\mathbf{Q}_i}^2 + \sum_{i=k-N+1}^{k} \left\| \mathbf{Z}_i - H_i(\mathbf{X}_i) \right\|_R^2
\end{aligned} \tag{19}$$

Where N is the size of the moving horizon window. The first part of the object function can be approached by arrival cost function. Then the MHE based algorithm with constrains (8) is given as follow:

$$\begin{aligned}
&\min_{\mathbf{x}_{T_N}, \{\omega_i\}_{T_N}^{k-1}} \sum_{i=T_N}^{k-1} \left\| \mathbf{X}_{i+1} - f(\mathbf{X}_i, \mathbf{u}_i) \right\|_{\mathbf{Q}_i}^2 + \sum_{i=T_N}^{k} \left\| \mathbf{Z}_i - H_i(\mathbf{X}_i) \right\|_R^2 + \Theta_{T_N}(\mathbf{X}_{T_N}) \\
&s.t. f_i(\mathbf{X}_i, \mathbf{u}_i, \omega_i) \le \mathbf{d}_i, i = T_N \ldots\ldots k
\end{aligned} \tag{20}$$

Where the arrival cost is given as follow:

$$\Theta_{T_N}(\mathbf{X}_{T_N}) = \left\| \mathbf{X}_{T_N} - \hat{\mathbf{X}}_{T_N} \right\|_{P_{T_N}}^2 \tag{21}$$

Where the covariance P_{T_N} is updated by (13):

$$P_{T_N} = P_{k-N+1} = [I - K_{k-N+1} H_{k-N+1}] P_{k-N+1,\mathrm{k-N}} \tag{22}$$

When the proposed algorithm works, the current time step will be compared with the size of window N. When $0 \le t \le k-N$, problem (18) is solved to estimate states. Otherwise the solution of the problem (20) works in

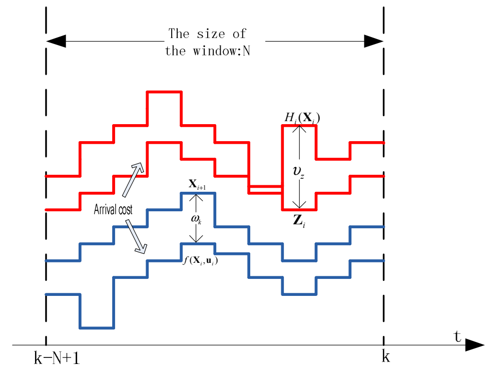

Fig.3. Scheme of MHE

$t: k-N+1 \le t \le k$, then update P and predict $\hat{X}_{k+1,k}$ by the system model and EKF updating. At next step, the latest measurement $\mathbf{Z}_T$ will be used to form the new measurement data set.

IV. Simulations Results

To prove the efficiency of the algorithm, a simulation based on Matlab was studied. In the simulation, the Master AUV and Slave AUVs were programmed to execute the curved path in the mission field. Basic parameter includes the noise covariance $\sigma_V^2 = (0.5kn)^2$, $\sigma_\phi^2 = (0.3\deg)^2$ and $\sigma_{R_k^2}^2 = \sigma_{R_{k+1}^2}^2 = (10m)^2$. The size of moving horizon window is set to be 6. The simulation was studied for 100 hundred times, and then we derived the error by Monte Carlo method.

First of all, AUVs get their initial position and clock synchronization by GPS. Then the AUVs move in a curve path. The localization results of the system are shown in Fig.4. The trajectory of DR deviates from the true path, while the results of the EKF works well. The EKF method bounded the errors by the use of the relative distance measurement. However, it cannot follow the whole path accurately. The trajectory of MHE performs better because the proposed algorithm can handle the nonlinearity more accurately and the constrains have been introduced into the system.

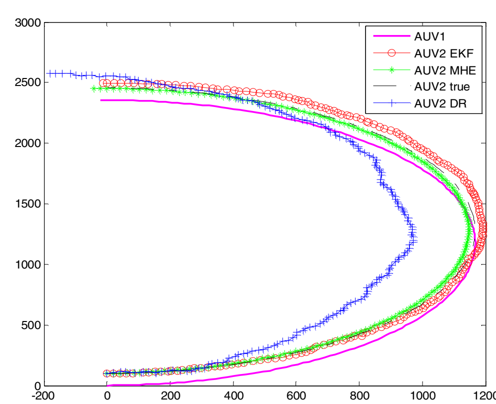

Fig.4. The trajectories of AUVs

The localization error of each algorithm is shown in Fig.5. The localization error of DR is unbounded growing during the mission. And it can be seen that MHE gives a higher accurate and smoother estimation than EKF during the entire path..

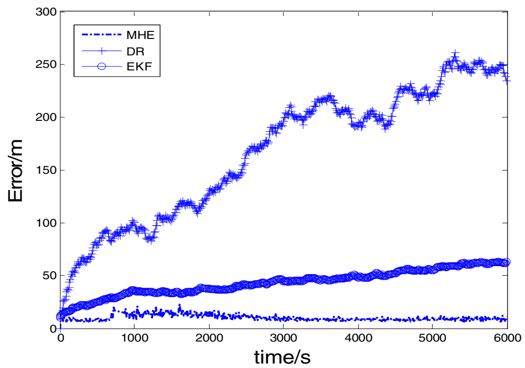

Fig.5. The localization errors of each algorithm

V. CONCLUSIONS

In this paper, an MHE based cooperative localization algorithm is presented. With the EKF based arrival cost updating, the MHE estimator was introduced that optimally combines information from master and slaver AUVs. Simulations show that the algorithm can perform more effectively than traditional method. It is believed that the algorithm proposed may present a significant in cooperative navigation and localization of multi-AUVs.

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