# A Novel Calibration Method for Acoustic Vector Sensor when the Phase is Inconsistent

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ABSTRACT

The Acoustic Vector Sensor can measure both pressure and particle velocity information of acoustic field at a point in space. By using the acoustic pressure and particle velocity information, a single AVS can get the unbiased estimation of target's bearing. But when the phase of each channel is inconsistent, the estimation precision will be badly affected. In order to solve this problem, this paper uses the instantaneous correlation integral as a preprocessing algorithm. After the instantaneous correlation integral, the phase of each channel becomes the same, and the signal-to-noise (SNR) ratio of the input signal is improved, which can guarantee the effectiveness of target bearing estimation. Theoretical analysis and simulation results verify the effectiveness of this method.

KEY WORDS: Acoustic Vector Sensor; Phase; Instantaneous Correlation Integral; SNR;

## 1. INTRODUCTION

Vector acoustic field is a very important part of sound wave, but the vector information of acoustic wave has been unwitnessed for a long time. Acoustic Vector sensor (AVS), also called combined sensor, is combined by traditional and omni-directional pressure hydrophone and natural dipole independent on frequency, which can co-locating and simultaneously measures pressure(scalar field) and particle velocity(vector field) of acoustic field. Combined processing based on pressure and particle velocity can get better detection effect and better estimation precision than processing only on pressure. So the technology associated with vector hydrophone becomes a very important investigative aspect in underwater acoustic field [1-7]. Signal processing based on vector hydrophone is the main content of the thesis, which has academic and engineering significance.

In practical application of single AVS and array, due to the production process, the amount of amplification and other reasons, inconsistency of amplitude and phase in the acoustic pressure and particle velocity channel may appear, finally affect the accuracy of its position estimation. So it is necessary to calibrate the AVS in different frequency before usage, to compensate the inconsistency [8].

In this paper, we propose a pre-processing algorithm for the phase inconsistency of AVS. The instantaneous correlation integral method is used to keep the phase of each channel the same, so the azimuth of the received signal can be gotten precisely, and the signal-to-noise ratio is also improved effectively. Theoretical analysis and simulation experiments verified the effectiveness of the proposed algorithm.

This paper is organized as follows. In Section 2, the measure model of the AVS is showed; some basic methods that AVS use to estimate the DOA are presented too. The reason to calibrate the AVS is also given there. The instantaneous correlation integral algorithm is presented in Section 3. Include the Formulation of this method and the processing gain. In Section 4, simulation results are presented. Finally conclusions are given in Section 5.

## 2. THE DOA ESTIMATION METHOD FOR AVS

### 2.1 The measurement model of AVS

The measurement model is shown in Figure 1, $\theta$ is the angle of azimuth, while $\varphi$ is the angle of pitch.

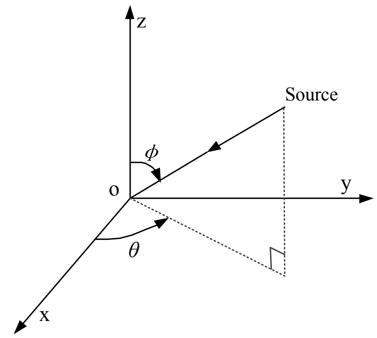


Fig1 the measurement model

The sound pressure and the particle velocity signal is expressed as $p(r,t)$ and $v(r,t)$ respectively, the $v(r,t)$ contains three orthogonal directions, which is $v_x(r,t)$, $v_y(r,t)$ and $v_z(r,t)$, the output of each channel is:

$$\begin{cases} p(t)=x(t) \\ v_x(t)=\sin\phi\cos\theta x(t) \\ v_y(t)=\sin\phi\sin\theta x(t) \\ v_z(t)=\cos\phi x(t) \end{cases} \tag{1}$$

### 2.2 Several azimuth estimation algorithms

According to the relationship expressed in the above formula, the following methods can be got.

(1) The particle velocity method

Its calculation formula is:

$$\begin{cases} \hat{\theta}=\arctan\left[\dfrac{\overline{v_y(t)}}{\overline{v_x(t)}}\right] \\ \hat{\phi}=\arctan\left[\dfrac{\sqrt{\overline{v_x(t)}^2+\overline{v_y(t)}^2}}{\overline{v_z(t)}}\right] \end{cases} \tag{2}$$

In the above formula, $\overline{()}$ expressed averaged.

(2) The average acoustic intensity method

The expression of average acoustic intensity is:

$$\overline{I(t)}=\overline{p(t)v(t)} \tag{3}$$

Then the average acoustic intensity of three orthogonal particle velocity components can be obtained:

$$\begin{cases} \overline{I_x(t)}=\overline{I(t)}\sin\phi\cos\theta \\ \overline{I_y(t)}=\overline{I(t)}\sin\phi\sin\theta \\ \overline{I_z(t)}=\overline{I(t)}\cos\phi \end{cases} \tag{4}$$

Thus the estimated value of the azimuth is:

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$$\begin{cases} \hat{\theta} = \arctan\left[\dfrac{\overline{I_y(t)}}{\overline{I_x(t)}}\right] \\ \hat{\phi} = \arctan\left[\dfrac{\sqrt{\overline{I_x(t)}^2 + \overline{I_y(t)}^2}}{\overline{I_z(t)}}\right] \end{cases} \tag{5}$$

Studies show that the average acoustic intensity method performs well in resisting isotropic interference. In the condition of noise jamming, the average sound intensity method is superior to the particle velocity method.

(3) The cross-spectrum acoustic intensity method

The definition of complex acoustic intensity is:

$$I(\omega) = P(\omega) \cdot V^*(\omega) \tag{6}$$

In the above formula, $P(\omega)$ and $V(\omega)$ is the Fourier transform of $p(t)$ and $v(t)$ respectively, $*$ express the complex conjugate operation. The real part of the complex acoustic intensity involved in signal propagation. The azimuth angle $\theta$ and pitch angle $\phi$ of the acoustic source is given by:

$$\begin{cases} \hat{\theta} = \arctan\left\{\dfrac{\mathrm{Re}\left[I_y(\omega)\right]}{\mathrm{Re}\left[I_x(\omega)\right]}\right\} \\ \hat{\phi} = \arctan\left[\dfrac{\sqrt{(\mathrm{Re}[I_x(\omega)])^2 + (\mathrm{Re}[I_y(\omega)])^2}}{\mathrm{Re}[I_z(\omega)]}\right] \end{cases} \tag{7}$$

The Re means to take the real operation. This method is called cross-spectrum acoustic intensity method. Because signals in different frequency can be distinguished in frequency domain, multi target can be resolved.

All the method above assumed that the amplitude and phase of each channel is consistent, but in practical application, it is inevitable that will be inconsistencies in the amplitude and phase, which will influence the target's azimuth estimation greatly.

### 2.3 The influence of phase inconsistency

According to reference [8], when the amplitude and phase of each channel is inconsistent, the result by using cross-spectrum acoustic intensity method is:

$$\begin{cases} \hat{\theta}(\omega) = \arctan\left\{\Delta A_{yx}(\omega)\dfrac{\cos[\Delta\varphi_{py}(\omega)]}{\cos[\Delta\varphi_{px}(\omega)]} \cdot \tan\theta\right\} \\ \hat{\phi}(\omega) = \arctan\left\{\sqrt{\dfrac{1 + [\Delta A_{yx}(\omega)\dfrac{\cos[\Delta\varphi_{py}(\omega)]}{\cos[\Delta\varphi_{px}(\omega)]}]^2 \cdot \tan\theta}{[\Delta A_{zx}(\omega)\dfrac{\cos[\Delta\varphi_{pz}(\omega)]}{\cos[\Delta\varphi_{px}(\omega)]}]^2 \cdot [1+\tan^2\theta]}} \cdot \tan\phi\right\} \end{cases} \tag{8}$$

$\Delta A_{yx}(\omega) = A_y(\omega)/A_x(\omega)$ , $\Delta A_{zx}(\omega) = A_z(\omega)/A_x(\omega)$ represent the atio of the amplitude. The phase difference of each channel is: $\Delta\varphi_{py}(\omega) = \varphi_p(\omega) - \varphi_y(\omega)$ , $\Delta\varphi_{px}(\omega) = \varphi_p(\omega) - \varphi_x(\omega)$ , $\Delta\varphi_{pz}(\omega) = \varphi_p(\omega) - \varphi_z(\omega)$ .

From the above formula, it can be seen that the estimation of the azimuth and pitch angle is related to the signal's DOA $\theta$ , the inconsistencies of amplitude $\Delta A$ and phase $\Delta\varphi$ of each channel.

If at a deterministic frequency, the amplitude gain of each channel be the same, then the formula (8) become:

$$\begin{cases} \hat{\theta}(\omega) = \arctan\left\{\dfrac{\cos[\Delta\varphi_{py}(\omega)]}{\cos[\Delta\varphi_{px}(\omega)]} \cdot \tan\theta\right\} \\ \hat{\phi}(\omega) = \arctan\left\{\sqrt{\dfrac{\cos^2[\Delta\varphi_{px}(\omega)] + \cos^2[\Delta\varphi_{py}(\omega)] \cdot \tan^2\theta}{\cos^2[\Delta\varphi_{pz}(\omega)] \cdot [1+\tan^2\theta]}} \cdot \tan\phi\right\} \end{cases} \tag{9}$$

In this case, the estimated result of the azimuth and pitch angle is related to $\theta$ and $\Delta\varphi$ . Assuming that $\Delta\varphi_{px} = 0$ , as the phase difference of particle velocity channel X and Y $\Delta\varphi_{xy}$ change from 10 degree to 30 degree, the estimation error curves is shown in figure (2):

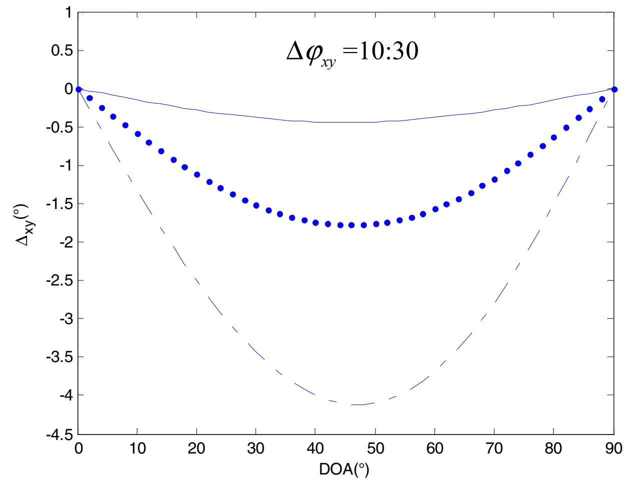



Fig2 the estimate error versus $\Delta\varphi_{xy}$

It can be seen from the diagram (2), the angle estimation error increases with the increase of $\Delta\varphi_{xy}$ , and the azimuth angle of the target is closer to 45, the greater the error is.

## 3. INSTANTANEOUS CORRELATION INGEGRAL (ICI) ALGORITHMS

### 3.1 The instantaneous correlation integral algorithms

The instantaneous correlation function (or bilinear transformation) of the signal is defined as:

$$r_s(t,\tau) = s(t+\frac{\tau}{2}) \cdot s^*(t-\frac{\tau}{2}) \tag{10}$$

If the signal is $s(t) = A\cos(\omega t + \varphi)$ , then its instantaneous correlation function is:

$$r_x(t,\tau) = A^2\cos[\omega(t+\frac{\tau}{2}) + \varphi] \cdot \cos[\omega(t-\frac{\tau}{2}) + \varphi] \tag{11}$$

Integrated it in the time domain:

$$\begin{aligned} X(\tau) &= \frac{1}{T}\int_0^t x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})\,\mathrm{d}t \\ &= \frac{A^2}{2T}\int_0^t [\cos(2\omega t + 2\varphi) + \cos(\omega\tau)]dt \end{aligned} \tag{12}$$

According to the orthogonality of the sine function, if the integral time is long enough, the integral $\int_0^t \cos(2\omega t + 2\varphi)dt$ tend to be 0, Then formula (10) can be simplified as:

$$X(\tau) \approx \frac{A^2}{2}\cos(\omega\tau) \tag{13}$$

As can be seen from the formula (13), through the integral of the instantaneous correlation function, there is no change in the frequency components of the signal. The amplitude is changed to its square, and the phase is changed to 0. Using this method to each channel, the phase of the AVS can be kept the same.

### 3.2 The processing gain of ICI

The received signal with noise is:

$$s(t) = x(t) + n(t) \tag{14}$$

Take $s(t)$ into formula (10), we can get:

$$\begin{aligned} r_s(t,\tau) &= s(t+\frac{\tau}{2})s^*(t-\frac{\tau}{2}) \\ &= \left[x(t+\frac{\tau}{2}) + n(t+\frac{\tau}{2})\right]\left[x(t-\frac{\tau}{2}) + n(t-\frac{\tau}{2})\right] \\ &= X(t,\tau) + N(t,\tau) \end{aligned} \tag{15}$$

$$X(t,\tau) = x(t+\frac{\tau}{2})x(t-\frac{\tau}{2}) \tag{16}$$

$$N(t,\tau) = x(t+\frac{\tau}{2})n(t-\frac{\tau}{2}) + x(t-\frac{\tau}{2})n(t+\frac{\tau}{2}) + n(t+\frac{\tau}{2})n(t-\frac{\tau}{2}) \tag{17}$$

$X(t,\tau)$ is the signal component, and the $N(t,\tau)$ is the noise component. The noise is in Gaussian distribution, uncorrelated with signal. Integrated the $N(t,\tau)$ in the time domain, the result can be expressed as:

$$\begin{aligned} &\frac{1}{T}\int_0^T N(t,\tau)dt \\ &= \frac{1}{T}\int_0^T \left[x(t+\frac{\tau}{2})n(t-\frac{\tau}{2}) + x(t-\frac{\tau}{2})n(t+\frac{\tau}{2}) + n(t+\frac{\tau}{2})n(t-\frac{\tau}{2})\right]\mathrm{d}t \\ &= \sqrt{\frac{dt_1}{T}}x(\tau)n(\tau) + \sqrt{\frac{dt_2}{T}}n^2(\tau) \end{aligned} \tag{18}$$

In the above formula, $dt_1$ and $dt_2$ is the time correlation radius, which are related to the bandwidth of the signal and the noise. When the signal's bandwidth is $B$ , the time correlation radius can be expressed as: $dt = 1/B$ . If the bandwidth of the signal is $B_0$ , the bandwidth of the noise is $B_n$ , then the bandwidth of the first term in formula (18) is $B_1 = B_0 + B_n$, causing the signal being line spectrum signal. So $B_1 \approx B_n$ , and the second term's bandwidth is $B_2 \approx 2B_n$ . Thus formula (16) can be represented as:

$$\frac{1}{T}\int_0^T h(t,\tau)dt = \sqrt{\frac{1}{B_nT}}x(\tau)n(\tau) + \sqrt{\frac{1}{2B_nT}}n^2(\tau) \tag{19}$$

It is obviously that the result is related to the bandwidth of the noise and the integrate time. When time is longer, and the bandwidth of noise is wider, and the integrate result is smaller.

The signal to noise ratio (SNR) of the input signal is defined as:

$$SNR_i = 10\log\frac{P_i}{\sigma^2} \tag{20}$$

$\sigma^2$ is the average power of noise, $P_i = \frac{A^2}{2}$ is the average power of signal. After ICI, the SNR is:

$$SNR_o = 10\log\frac{P_o}{P_n} \tag{21}$$

Based on formula (13) and (19), the

$$P_o = \frac{A_i^4}{8},\ P_n \le \frac{P_i\sigma^2 + \sigma^4}{2B_nT},$$

So the output SNR is:

$$SNR_o \ge 10\left(\log\frac{A^4}{8} - \log\frac{P_i\sigma^2 + \sigma^4}{2B_nT}\right) \tag{22}$$

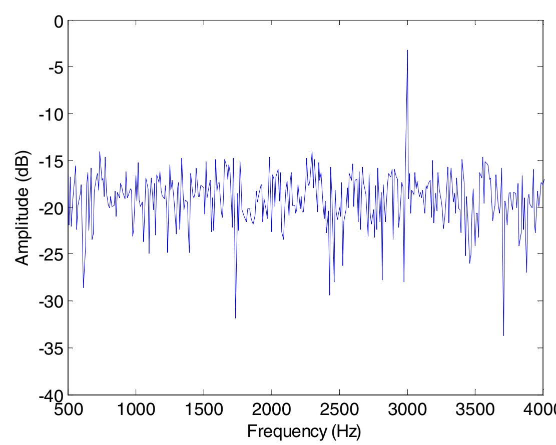
Then the gain of the ICI is:

$$\begin{aligned} G = SNR_o - SNR_i &\ge 10\log\frac{P_o}{P_n} - 10\log\frac{P_i}{\sigma^2} \\ &= 10\left[\log A^2 - \log 2(\frac{A^2}{2} + \sigma^2) + \log B_nt\right] \end{aligned} \tag{23}$$

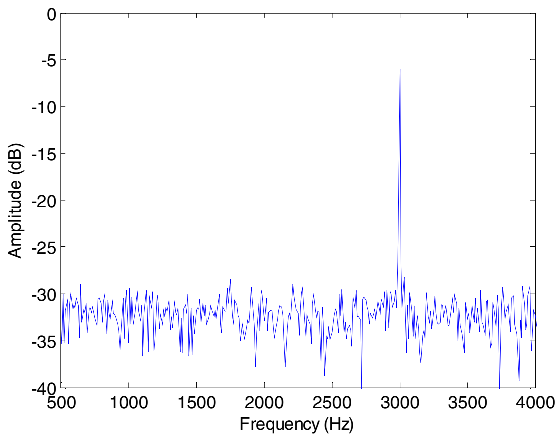
Formula (23) shows that, the processing gain of the ICI algorithm is related to the average power of the signal, the average power of the noise, the time length of the integral, the bandwidth of the signal and noise. The gain achieved by the integral is constant $10\log B_nt$ . When the signal power and the integration time is constant, the total processing gain decreases with the increase of the noise power, which means that the processing gain decreases with the increase of the input SNR.

## 4. SIMULATION ANALYSIS

Simulation experiment of the algorithm proposed in this paper is carried out. Simulation conditions is: The frequency of the CW signal is 3 kHz, the duration is T=0.2s, and the sampling frequency of the system is fs=36 kHz. The input signal to noise ratio is 0dB, related integration time is 0.1 s, the output signal length is 0.1s, and the spectrum of the signal before and after the ICI is shown in figure (3).




(a) Signal's spectrum before ICI




(b) Signal's spectrum after ICI

Fig 3 Signal's spectrum before and after ICI

From the graph (3), it is obvious that the noise is effectively suppressed and the signal to noise ratio has been improved after the ICI.

An analysis is made on this algorithm only consider the horizontal direction. Despite the loss of generality, only to consider the horizontal direction. Assuming that the magnitude of the both particle velocity channels $V_x$ and $V_y$ is 1, target angle θ distributed in the (0, 45) ° range, the phase difference Δφ distribution within the scope of the (0, 60) °, and the rest of the conditions are the same as before experiment. Cross spectrum method are used to get the azimuth estimation. The error of estimation before and after ICI is e1 and e2. Take Monte Carlo simulation 400 times, and the result is shown in table 1:

Table 1 The estimate error versus phase

| $\Delta\varphi_{xy}$ \ θ | 10° | 20° | 30° | 45° |
|---|---|---|---|---|
| 10 | $e_1$=1.35<br>$e_2$=0.51 | $e_1$=1.21<br>$e_2$=0.50 | $e_1$=1.31<br>$e_2$=0.53 | $e_1$=1.22<br>$e_2$=0.50 |
| 20 | $e_1$=1.49<br>$e_2$=0.51 | $e_1$=1.43<br>$e_2$=0.50 | $e_1$=1.89<br>$e_2$=0.53 | $e_1$=1.96<br>$e_2$=0.50 |
| 30 | $e_1$=1.83<br>$e_2$=0.51 | $e_1$=2.35<br>$e_2$=0.51 | $e_1$=3.59<br>$e_2$=0.54 | $e_1$=4.13<br>$e_2$=0.51 |
| 40 | $e_1$=2.56<br>$e_2$=0.50 | $e_1$=4.21<br>$e_2$=0.51 | $e_1$=6.32<br>$e_2$=0.55 | $e_1$=7.58<br>$e_2$=0.51 |
| 60 | $e_1$=5.11<br>$e_2$=0.48 | $e_1$=9.50<br>$e_2$=0.52 | $e_1$=9.84<br>$e_2$=0.56 | $e_1$=18.51<br>$e_2$=0.51 |

From table 1, it can be seen that the error without ICI increases with the increase of the phase difference. In the same condition, the target orientation close more to 45 °, and the estimation error is bigger. After the ICI, the estimation error is kept on a small level. Regardless of the phase difference and the target azimuth, the main factor that affects the accuracy is SNR.

## 5. CONCLUSIONS

In view of the phase in each channel of the AVS being inconsistent, this paper puts forward a method to deal with this problem. With the usage of the ICI algorithm, the phase of each channel is consistent and the signal noise ratio is improved. The method is verified with simulation analysis.

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