Effect of Array Tilt on Estimates of Vertical Ambient Noise Spectrum

Mehdi Farrokhrooz
Electrical and Computer Engineering Department
George Mason University
Email: farrokhrooz@ieee.org

Kathleen E. Wage
Electrical and Computer Engineering Department
George Mason University
Email: k.e.wage@ieee.org

***Abstract*—The vertical spectrum of ambient noise in the SOFAR channel carries useful information about noise sources and trapped modes in the channel. Measuring the directionality of low frequency ambient noise requires very long hydrophone arrays. Array motion can obscure the measurements of vertical spectra. This paper is motivated by the need to characterize the ambient noise spectrum using moored arrays such as those deployed in the North Pacific [1] and the Philippine Sea [2]. Using a straight-line model for array motion, this paper derives an approximate expression for the vertical spectrum measured by a tilted array. Analysis show that array tilt broadens the measured spectrum and reduces the noise notch at broadside. Numerical simulations using transport theory confirm the results.**

I. Introduction

Ambient noise is an important topic in underwater acoustics. Wenz [3] and Urick [4] summarized the results of early experiments. Shipping is the dominant source of noise in the 20-100 Hz band. Carey and Evans describe measurements of the vertical spectrum of noise in this band using vertical hydrophone arrays [5]. The vertical spectrum of ambient low-frequency noise has most of its energy concentrated around angles near broadside. Broadside energy is due to distant sources whose energy is coupled into the SOFAR channel where it becomes trapped and propagates long distances. At short ranges from its source, the trapped energy is concentrated along ray paths that turn near the surface. These rays typically arrive at the array with angles between 10 and 20 degrees with respect to broadside. [1] For these shorter range source, the vertical spectrum has peaks at these angles and a notch at broadside. As range increases, scattering occurs and the notch is filled in. The shape of the vertical ambient noise spectrum provides valuable information about the SOFAR channel and the nature of scattering.

This paper is motivated by the need to characterize the ambient noise spectrum using moored arrays such as those deployed in the North Pacific [1] and the Philippine Sea [2]. At low frequency a long array of hydrophones is required to achieve a desired resolution in spatial spectrum estimation [6]. A floating vertical long array bends in water and this array deformation causes an observation error to the estimated vertical spectrum. While there are some studies on the effect of array tilt on the horizontal spectrum of ambient noise by Hodgkiss and Fisher [7] and Felisberto and Jesus [8], there hasn't been a similar analysis for vertical arrays.

This paper analyzes the effect of array tilt on vertical spectrum. First, we mathematically model the effect of array tilt on vertical spectrum. For simplicity, this model just considers the linear tilted array not an array with arbitrary hydrophone locations. The approximated model helps us to understand the effect array tilt on vertical spectrum. Then, different numerical simulations indicate the effect of array tilt on the vertical spectrum. The normal mode model will be used in Monte Carlo simulations. In simulations, internal waves [10], [11], [12] couples the energy from distant surface source to SOFAR channel. Transport theory [13], [14], [15] calculates the mode powers at different ranges from source. The results of simulations for deterministic and random source distributions well fit the analytical model.

This paper is organized as follows. Section II describes the characteristics of the vertical spectrum of ambient noise in deep ocean. Section III analyzes the distortion of the vertical noise spectrum due to measurement with a tilted array. Section IV provides the results of numerical simulations for several different scenarios. Section V concludes the paper.

II. Vertical directionality of ambient noise in the deep ocean

The vertical spectra of noise trapped in the SOFAR channel can be interpreted as the normal mode model. The lower modes and higher modes are associated with broadside and steeper angles of spectrum, respectively. The sea surface sources highly stimulate the higher modes which causes a very deep notch at broadside angle in short range. Dashen and Munk [9] considered three mechanisms that could transfer shipping noise into the low modes which are downslope conversion on the continental slope, volume scattering due to internal waves, and direct excitation of the low modes at high latitudes where the sound channel intersects the surface. They ruled out the scattering of internal waves because it requires impractically large propagation ranges. Kuryanov and Klyachin [16], [17] disagree with Dashen and Munk [9] and think that scattering of internal waves could be one of the effective causes for transferring the surface noise into SOFAR channel. We have showed that the limitation of Dashen and Munk's [9] internal waves analysis is that it does not consider the effect of bottom loss [18]. For the purposes of the simulations, this paper will focus on the internal waves scattering that can be modeled using Transport theory.

Garrett and Munk [10], [11], [12] have presented a random model for internal waves spectrum. Dozier and Tappert [13], [14] provided the master equations and statistics of of normal mode amplitudes in presence of random perturbations as

[1] Angle depends on sound speed profile.

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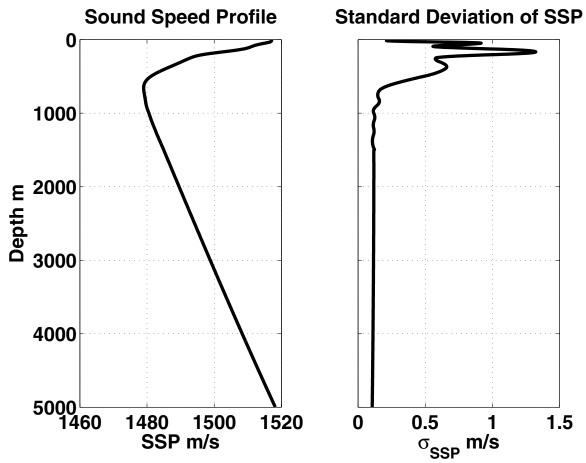

Fig. 1. The mean ($c_0(z)$) of sound speed profile (left) and the standard deviation ($\sigma_{\delta c(r,z)}$) of sound speed profile (right).

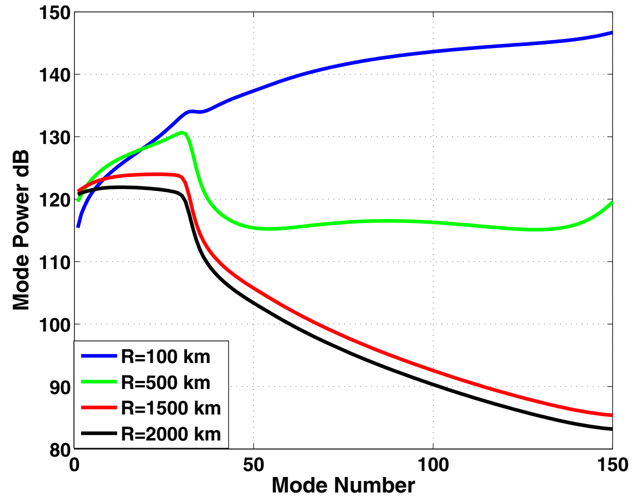

Fig. 2. The mode power associated with the fluctuating environment of Fig.1 at four different ranges from a 190 dB source located in a depth of 10 m.

Transport Theory. Colosi and Morozov [15] simplified this theory for long range propagation in deep water. Transport theory predicts the normal mode amplitudes in the form of a differential equation described in Equation 1:

$$\frac{d\psi_n}{dr} = -i \sum_{m=1}^{N_{mode}} \rho_{mn}(r) e^{il_{mn}r} \psi_m(r) \tag{1}$$

Where, $\psi_m(r) = a_m(r)e^{-il_m r}$, $l_m = k_m + i\alpha_m$ is the complex wavenumber and $l_{mn} = l_m - l_n$. ρ_{mn} is the coupling between m^{th} and n^{th} modes and defined as:

$$\rho_{mn} = \frac{k_0^2}{\sqrt{k_m k_n}} \int_0^D \frac{\phi_n(z)\phi_m(z)}{\rho_0(z)} \frac{\delta c(r,z)}{c_0} \tag{2}$$

In Equation 2, D is the water column and $\delta c(r,z)$ is a random variable of sound speed fluctuation as a function of depth and range and depends on deterministic sound speed profile (c), gravity acceleration (g), buoyancy frequency (N), random variable (μ) and displacement ($\zeta(z,r,t)$) as shown in Equation 3:

$$\delta c(r,z,t) = c_0(z)(\frac{\mu}{g})N^2(z)\zeta(r,z,t) \tag{3}$$

$\zeta(r,z,t)$ is described by Garrett-Munk spectrum. In the simulation results of Section IV, we use the mean of sound speed profile ($c_0(z)$) and standard deviation of sound speed fluctuation $\sigma_{\delta c(r,z)}$ for one Garrett-Munk spectrum as shown in Fig. 1.

Applying transport theory to a fluctuating environment shown in Fig. 1, the mode powers at range of 100, 500, 1500 and 2000 km from a 33 Hz source with source level of 190 dB located in depth of 10 m are as shown in Fig. 2. The higher modes interact with the sea floor and attenuate very fast while low modes conserve their powers. Also, scattering of internal waves caused mode coupling and transfers energy to the lower modes.

III. Vertical Directionality of Tilted Array

A bottom-anchored array bends and rotates around its anchor. This paper simplifies the bent array as a linear array tilted from the vertical as shown in Fig. 3. Although the vertical tilt angle θ_a is very small, it can affect the vertical spectrum estimate. In this section the vertical spectrum of tilted array shown in Fig. 3 will be derived.

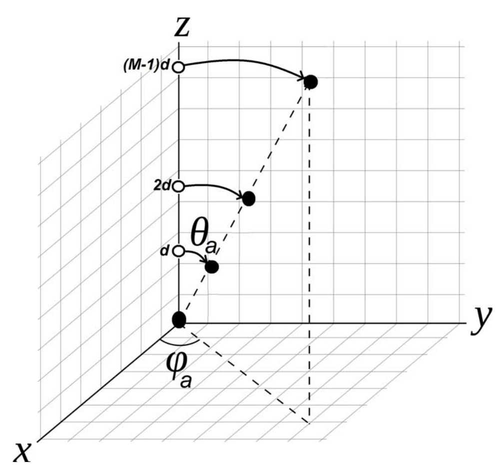

Fig. 3. Tilted array of M hydrophones.

θ_a is the angle from the presumed vertical position. Since θ_a is very small $sin\theta_a \simeq \theta_a$ and $cos\theta_a \simeq 1$, and the projections of the m^{th} hydrophone on $x-y$ plane and z-axis are $(m-1)\theta_a d$ and $(m-1)d$, respectively.

In general, source and array are not co-planar. If we assume a source located at range of R_s from z-axis on a plane P described by $ycos\phi_s = xsin\phi_s$, the projection of displacement of m^{th} hydrophone on cross-section of P and xy is: $(m-1)\theta_a dcos(\phi_s - \phi_a)$.

The pressure received by l^{th} hydrophone of straight array can be interpreted as summation of normal modes as:

$$p_{straight}(l) = \sum_{n=1}^{N_{mode}} \frac{a_n \phi_n(z_l)}{\sqrt{k_n R_s}} \tag{4}$$

Where $l = 0, \cdots, M-1$ and $l = 0$ refers to the shallowest hydrophone. ϕ_n, a_n and, k_n are the modeshape, amplitude, and wavenumber for the n^{th} mode. R is the radial range of source from array and $R \gg (M-1)d$. The displacement of hydrophones causes a phase shift of the received signal. The pressure generated by single source and received by l^{th} hydrophone of the tilted linear array is as Equation 5.

$$p_s[l] = \sum_{n=1}^{N_{mode}} e^{ik_n(M-l-1)\theta_a d cos(\phi_s-\phi_a)} \frac{a_n\phi_n(z_l)}{\sqrt{k_n R_s}} \quad (5)$$

The total pressure generated by N_s sources at l^{th} hydrophone is the superposition of pressure generated by each source:

$$p_p[l] = \sum_{s=1}^{N_s} p_s[l] = \\ = \sum_{s=1}^{N_s}\sum_{n=1}^{N_{mode}} e^{ik_n(M-l-1)\theta_a d cos(\phi_s-\phi_a)} \frac{a_{n,s}\phi_n(z_l)}{\sqrt{k_n R_s}} \quad (6)$$

Where $a_{n,s}$ is the amplitude of n^{th} mode excited by s^{th} source. the mode amplitudes and sources are uncorrelated i.e.,

$$E\{a_{n,s}a^*_{m,q}\} = \begin{cases} 0 & \text{if } n \neq m \text{ or } s \neq q \\ E\{|a_{n,s}|^2\} & \text{if } n = m \text{ and } s = q \end{cases} \quad (7)$$

Maranda [20] suggested an efficient FFT-based beamforming algorithm which we use to estimate the vertical spectrum estimation:

$$S[k] = E\{|FFT\{w[l]p_p[l]\}|^2\} \\ l = 0, \cdots M-1 \ , \ k = 0, \cdots NFFT-1 \quad (8)$$

where $w[l]$ is the spatial window which we have applied to the array receptions to control spatial resolution and sidelobe level [6]. Considering the normality of modeshapes and uncorrelation of mode amplitudes and independence of sources (Equation 7), the vertical spectrum is:

$$S(k) = \sum_{s=1}^{N_s}\sum_{n=1}^{N_{mode}} E\{|a_{n,s}|^2\} \times \\ \frac{|W[k] * \Psi_n[k + k_n\theta_a cos(\gamma_s)]|^2}{k_n R_s} \\ k = 0, \cdots NFFT-1 \quad (9)$$

where $W[k]$ and $\Psi_n[k]$ are Fourier Transform of $w[l]$ and $\phi_n(z_l)$, respectively and also $\gamma_s = \phi_s - \phi_a$. As Equation 9 shows, Fourier transform of modes are shifted over wavenumber axis for tilted array. The shift value depends on the wavenumber (k_n), tilt angle (θ_a) and azimuthal angle difference (γ_s). In addition to these factors, the shifted spectrum of each mode shape is weighted by the mode power ($E\{|a_{n,s}|^2\}$).

Equation 9 is an approximate formula of the vertical spectrum that provides insight about how array tilt affects the vertical noise spectrum. The following section uses Monte Carlo simulation to investigate tilt effects for several different scenarios.

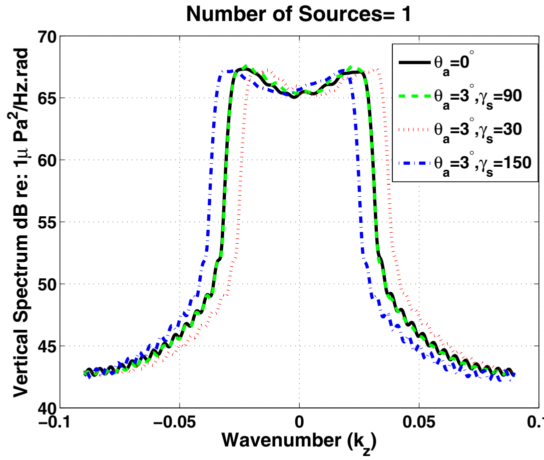

Fig. 4. Comparison of vertical spectrum of straight array and 3° tilted array for three source angles: $\gamma_s = 0°$, $\gamma_s = 30°$, $\gamma_s = 150°$.

IV. Numerical results

An approximate closed form of vertical spectrum of a linear tilted array was derived in Section III. This section presents some numerical results to show the effect of array tilt on vertical spectrum. Simulation of the acoustical pressure in the ocean needs the mode shapes and wavenumers. Complex KRAKEN (KRAKENC) [19] is an accurate, robust and efficient model to calculate mode shapes and wavenumbers. As mentioned in Section II, scattering of internal waves couples the surface sources into SOFAR channel. In the simulations, SOFAR channel (300-1665 m) has been spanned by an array which is similar to the array designed for NPAL [1] and consists of 40 hydrophones with 35 m spacing.

A. *Three scenarios with known source locations*

This subsection shows the results of Monte Carlo simulations for three deterministic source scenarios. Deterministic scenarios presume that source(s) work(s) in 33 Hz with source level of 190 dB and located at depth of 10 m in a range of 1500 km from the array. The estimated mode powers for each source at range of 1500 km are shown in Fig. 2. The Monte Carlo vertical spectrum of tilted and straight arrays are averaged on 1000 independent realizations.

The first scenario compares the vertical spectrum of a straight array with tilted array ($\theta_a = 3°$) at three different azimuth angles ($\gamma_s = 0°, 30°$ and $150°$). As shown in Fig.4, the angle of source respect to the tilted array affects on the vertical spectrum by shifting it on wavenumber axis. If the source is located perpendicular on the array's plane, i.e. $\gamma_s = 90°$, the effect of the tilted array is vanished.

The second scenario investigates the effect of tilt angle (θ_a) for a single source. Fig.5 shows four vertical spectrum of different tilt angles using Monte Carlo simulation when $\gamma_s = 0^{circ}$. As we expect from Equation 9, the higher tilt angle causes more shift of vertical spectrum on wavenumber axis. In multiple source scenario, 360 identical sources at 33 Hz with source level of 190 dB and depth of 10 m located on a circle around array in radius of 1500 km are assumed. The azimuth angle between each two adjacent sources is one degree. As shown in Fig.6, the effect of array tilt on the width

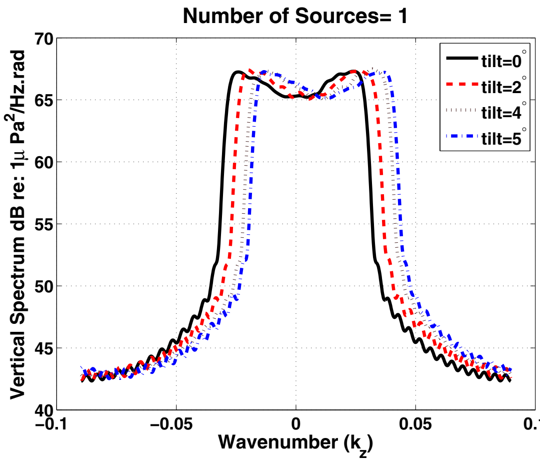

Fig. 5. Comparison of vertical spectra of straight array and three tilted arrays ($2°$, $4°$ and $5°$) for a 33 Hz source located 1500 km from the array.

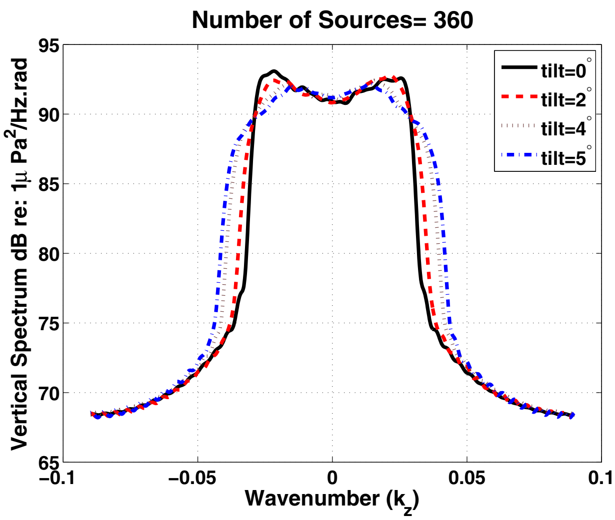

Fig. 6. Comparison of vertical spectra of straight array and three tilted arrays ($2°$, $4°$ and $5°$) for 360 sources located on a circle with radius of 1500 km around the array.

of vertical spectrum is very clear when we compare the vertical spectrum of different array tilts. In this scenario, increasing the array tilt causes broadening of vertical spectrum and also decreases the depth of broadside notch.

B. Randomly distributed source scenario

According to examples of subsection IV-A, the array tilt causes a kind of observation error to the vertical spectrum estimation. The goal of this subsection is finding the minimum angle in which the array tilt is observable. This value must be close to the real experiments. In real cases, there are many surface sources randomly distributed in the ocean which we do not have enough information about them. The random source scenario which is more realistic assumes the same array as previous examples but randomly distributed sources. All 360 random sources are working at 33 Hz and located in random ranged from array at random azimuth angles and random source levels. The simulation program has been run

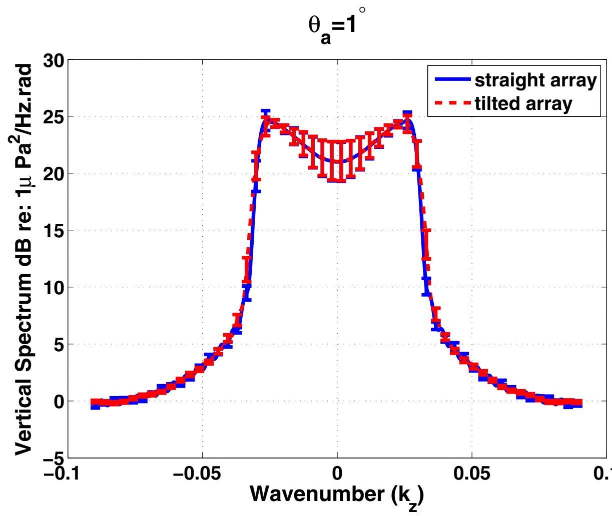

Fig. 7. The mean value and error bars of tilted and straight array lies on each other at $\theta_a = 1°$.

100 times for each tilt angle and the location and source level of sources have been randomly changed at each run. The range, azimuth angle and source level are uniformly randomly picked in the range of 400-2000 km, $0°$-$360°$ and 150-190 dB, respectively. Since the range of sources are different in this scenario, the mode powers corresponding to these sources must be calculated by transport theory. At each specific tilt angle, the acoustical pressure of each random source is generated by adding up 1000 Monte Carlo realizations and for both tilted and straight arrays and then estimated the vertical spectrum.

find the minimum observable tilt angle needs the mean value and standard deviation of vertical spectrum for each tilt angle. Since the power and range of sources vary at eat run, their vertical spectrum have different offset values. These offset values have been removed by normalization respect to the base level. Then mean value and standard deviation of the normalized spectrum for each tilt angle estimated to find the minimum detectable tilt angle. If the mean spectra for a tilted array lies within the standard deviation bars of straight array spectrum then the tilt is not detectable. Fig. 7 shows that for $\theta_a = 1\circ$ the vertical spectrum of tilted and straight arrays are such close to each other that tilt is not recognizable. As Fig. 8 by increasing the tilt angle to $\theta_a = 2°$ the broadening of spectrum is clear but tilt effect at broadside is still unrecognizable.

At $\theta_a = 3°$ the vertical spectrum of tilted array broadens and also the peak locations have been suppressed then the array tilt is obvious from broadening the spectrum and suppressing the peaks. When the tilted angle increases to $\theta_a = 5°$ Fig. 10, the mean value of tilted spectra no longer lies on the mean value of straight array's spectra but the error bars are still overlapping. It means that in some cases the noise notch variation of tilted array is still undetectable.

The effect of array tilt for the tilt angles less than $\theta_a = 1°$ is not recognizable by increasing the array tilt the discrimination between spectrum of tilted and straight arrays shows up by broadening the width of spectra for tilted array's spectra, and then follows by suppressing the peaks of spectra in tilted array. In the extreme case ($\theta_a = 5°$), the mean

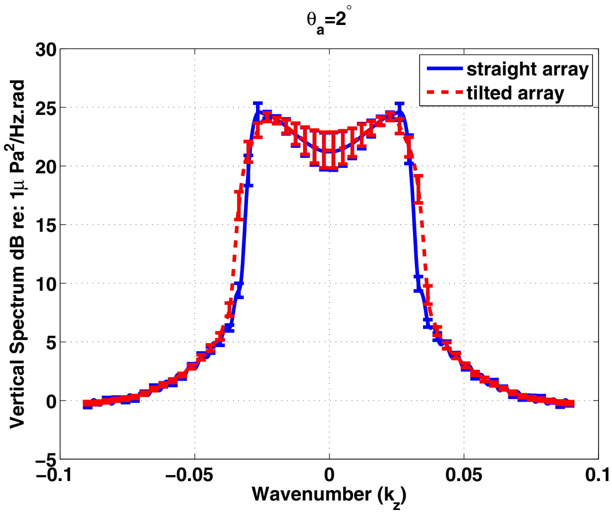

Fig. 8. The vertical spectrum of tilted array starts broadening at $\theta_a = 2°$ but at broadside direction tilted array spectrum lies within the error bars of straight array.

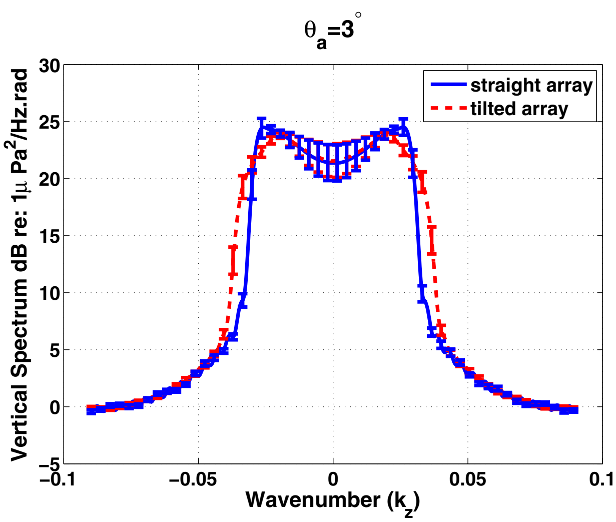

Fig. 9. The vertical spectrum of tilted array broadens at $\theta_a = 3°$ and peaks are suppressed enough to be recognizable, but at broadside direction tilted array spectrum still lies within the error bars of straight array.

value of tilted spectra is recognizable from the straight array

Examples in subsection IV-A and IV-B show that the array tilt affects on the measured vertical spectrum of ambient noise by broadening of spectrum and decreasing of noise notch. Both of these features of vertical spectrum carry important information about the modes trapped in the SOFAR channel and sources of ambient noise.

V. Conclusion

This paper investigated the effect of array tilt on the vertical spectrum of ambient noise at low frequencies. The approximate analytical expression and numerical simulations show that array tilt broadens the vertical spectrum and reduces the notch depth at broadside. The width of the vertical spectrum and the depth of the noise notch carry important information about the modes trapped in the SOFAR channel and the origin of

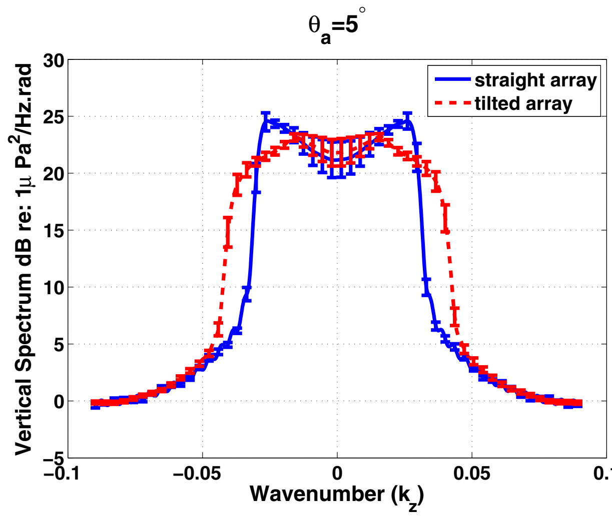

Fig. 10. The vertical spectrum of tilted array broadens at $\theta_a = 5°$ and peaks are suppressed enough to be recognizable, at broadside direction the mean of tilted array's spectrum no longer lies on the mean value of straight array.

sources.

Work on this topic is ongoing. Specifically, we are expanding the analysis to use a general model for array shape instead of simple straight-line tilt model used in this paper. Results will be applied in the analysis of measurements made during the SPICEX and PhilSea experiments. This work will continue by simulating the effect of real array shape on the vertical spectrum estimates. And also, we will explore NPAL [1] and the PhilSea [2] databases to find evidences which support the idea of vertical spectrum broadening and noise notch depth reduction caused by array deformation.

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