An Improved Whitening Transformation for Snapshot-Deficient Scenarios

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***Abstract*—Many adaptive signal processing algorithms, such as the MUltiple SIgnal Classification (MUSIC) spectral estimator, require knowledge of the number of sources. Source enumeration algorithms typically use the eigenvalues of the sample covariance matrix to infer the rank of the signal subspace. These algorithms often assume a white noise background. In colored noise environments, the standard solution is to whiten the data prior to processing. Using results from random matrix theory, Nadakuditi and Silverstein predict the fundamental signal detection limit as a function of the number of snapshots used to define the whitening filter and the number of snapshots used for the detector. They also describe a whitening-based detection algorithm whose performance approaches their asymptotic predictions. Motivated by prior work on the Dominant Mode Rejection beamformer (DMR), this paper proposes a modified whitening transform that uses a structured estimate of the noise-only covariance matrix. Simulations demonstrate that the new whitening transform performs substantially better than Nadakuditi and Silverstein's algorithm when the number of snapshots available to estimate the whitening filter is small.**

***Keywords*—detection, colored noise, whitening, random matrix theory, source enumeration**

I. Introduction

Many adaptive signal processing algorithms, such as the MUltiple SIgnal Classification (MUSIC) spectral estimator [1], require knowledge of the number of sources. Most source enumeration algorithms use the eigenvalues of the sample covariance matrix (SCM) to determine the number of signals, e.g., the classic Akaike Information Criterion (AIC) and Minimum Description Length (MDL) estimators described by Wax and Kailath [2]. The common assumption is that the sources are embedded in a white noise background, however in many applications the background noise is colored. When the environment has colored noise, these algorithms can overestimate the number of signals. Colored noise is a common problem in deep underwater arrays where filtering effects of the ocean propagation channel define the ambient noise field at low frequencies. The dominant source of low frequency noise in the ocean is due to shipping traffic [3], [4], [5].

Various researchers have investigated the problem of source enumeration when the background noise is colored [6], [7], [8], [9], [10], [11], [12]. The standard approach is to apply a whitening transformation to the data before applying the signal enumeration algorithm. The whitening transform is typically designed using the inverse square root of the SCM estimated using a set of noise-only snapshots. Estimating a full-rank SCM requires the number of snapshots to be greater than or equal to the array size. For large arrays it is difficult to meet this snapshot requirement when the environment is non-stationary.

Classical methods for source enumeration such as the one proposed by Wax and Kailath [2] are derived in the limit as the number of snapshots approaches infinity while the number of sensors remains fixed. The limitation of these methods is that the number of snapshots is not much larger than the number of sensors in practice. Recently, Nadakuditi and Edelman [13] used random matrix theory (RMT) to design a source enumeration algorithm that requires less data than the classical methods. RMT results are derived in the limit as both the number of sensors and the number of snapshots approach infinity, while the ratio of sensors to snapshots approaches a constant c. This type of asymptotic analysis facilitates performance predictions for snapshot-deficient scenarios, i.e., where the ratio is $c > 1$.

Using RMT results Nadakuditi and Silverstein [11] derive the fundamental eigenvalue-based detection limit for the whitened eigenvalues in scenarios with arbitrary noise. This limit is the minimum required SNR, as a function of the number of sensors and snapshots, for the signals to be detectable. The authors also propose an RMT-based source enumeration method for scenarios with arbitrary noise when the whitening transform is designed with a noise-only SCM. Their method improves the SNR limits for signal detection over previous methods that consider colored noise. It uses the probability density function of the largest whitened noise eigenvalue to define a threshold for signal detection. The key requirement to implement this algorithm is that the number of snapshots has to be larger the number of sensors in order for the noise-only SCM to be invertible. This paper presents a novel whitening method that improves signal detection over Nadakuditi and Silverstein's method in snapshot-deficient scenarios, when the number of snapshots is smaller than the number of sensors.

Motivated by prior work on the Dominant Mode Rejection (DMR) beamformer [14], this paper proposes a modified whitening transformation that uses a structured estimate of the noise-only SCM. The structured estimate improves performance of the whitening filter when the number of snapshots available to estimate it is on the same order as the array size. It also facilitates the implementation of the whitening filter when the number of snapshots is less than the number of sensors N. The first part of the method utilizes Nadakuditi and Edelman's source enumeration algorithm [13] for sources in white noise to find the rank R of the colored noise in the noise-only data set. The structured covariance includes the R dominant terms of the colored noise subspace as estimated from data (largest estimated eigenvalues and eigenvectors). The

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smallest $N-R$ eigenvalues are averaged. Then using a method similar to Abraham and Owsley in [14] for DMR, this average replaces the smallest eigenvalues of the noise-only SCM to define a new whitening transform. The eigenvalues whitened with this structured whitening transform are evaluated to determine the number of sources. Monte Carlo simulations show an improvement in the probability of correct determination of the proposed algorithm over Nadakuditi and Silverstein's algorithm [11], specially at low sample support.

The paper is organized as follows. Section II presents the problem formulation and Section III discusses the RMT-based source enumeration method proposed by Nadakuditi and Silverstein [11]. Section IV describes the proposed whitening transform, Section V presents the simulations, and Section VI provides the summary. Throughout the paper uppercase bold letters represent matrices, lowercase bold letters represent vectors, and non-bold variables represent scalars. The letter H indicates the Hermitian of a matrix.

II. Problem Formulation

The goal of source enumeration is to determine the number of sources from a set of noisy observations. This paper focuses on a narrowband signal model consisting of a set of D planewaves and noise. The noise $\mathbf{n}$ has two components: a white noise component $\mathbf{n}_w$ and a colored noise component $\mathbf{n}_c$. $\mathbf{n}_w$ and $\mathbf{n}_c$ are independent complex Gaussian random vectors with zero mean and covariance matrices $\sigma_w^2\mathbf{I}$ and $\mathbf{S}_c$, respectively. The variance of the white sensor noise is σ_w^2. The model for the received signal is

$$\mathbf{p} = \sum_{i=1}^{D} a_i\mathbf{v}_i + \mathbf{n}_w + \mathbf{n}_c. \tag{1}$$

The planewave amplitudes (a_i's) are complex Gaussian random variables with zero mean and variance σ_i^2. The replica vector $\mathbf{v}_i$ contains directional information for each signal: $\mathbf{v}_i = \left[1\, e^{-jk_z\Delta z} \dots e^{-j(N-1)k_z\Delta z}\right]^H$. The wavenumber k_z is a function of the arrival angle, the narrowband frequency, and the propagation speed. The separation between sensors is Δz.

In the absence of signals, the ensemble covariance matrix is

$$\mathbf{\Sigma_n} = \varepsilon\left\{\mathbf{n}\mathbf{n}^H\right\} = \sigma_w^2\mathbf{I} + \mathbf{S}_c. \tag{2}$$

When there are signals present, the ensemble covariance matrix is

$$\mathbf{\Sigma_p} = \varepsilon\left\{\mathbf{p}\mathbf{p}^H\right\} = \sum_{i=1}^{D}\sigma_i^2\mathbf{v}_i\mathbf{v}_i^H + \sigma_w^2\mathbf{I} + \mathbf{S}_c. \tag{3}$$

The ensemble eigenvalues and eigenvectors of $\mathbf{\Sigma_p}$ are stored in matrices $\mathbf{\Lambda}$ and $\mathbf{\Phi}$, respectively. For a single signal in white noise, the ensemble covariance matrix is $\mathbf{\Sigma_p} = \sigma_1^2\mathbf{v}_1\mathbf{v}_1^H + \sigma_w^2\mathbf{I}$. In this case the maximum eigenvalue of $\mathbf{\Sigma_p}$ is $\lambda_1 = N\sigma_1^2 + \sigma_w^2$, and the remaining $N-1$ eigenvalues are equal to σ_w^2.

A detector designed for signals in white noise will overestimate the number of signals when the white noise assumption is violated. A common solution to this problem is to whiten the data prior to applying the source enumeration algorithm. A whitening transform rotates and scales $\mathbf{\Sigma_p}$, removing the colored noise. The ideal whitening transform uses the inverse square root of $\mathbf{\Sigma_n}$, i.e.,

$$\mathbf{W}_{\text{ens}} = \mathbf{\Sigma_n}^{-1/2} = \mathbf{\Gamma}^{-1/2}\mathbf{\Xi}^H, \tag{4}$$

where $\mathbf{\Gamma}$ and $\mathbf{\Xi}$ are the eigenvalues and eigenvectors of $\mathbf{\Sigma_n}$, respectively. Assuming the ensemble whitening filter is available, the whitened signal-plus-noise covariance is

$$\hat{\mathbf{\Sigma}}_\mathbf{p} = \mathbf{W}_{\text{ens}}\mathbf{\Sigma_p}\mathbf{W}_{\text{ens}}^H = \hat{\mathbf{\Phi}}\hat{\mathbf{\Lambda}}\hat{\mathbf{\Phi}}^H, \tag{5}$$

where $\hat{\mathbf{\Lambda}}$ and $\hat{\mathbf{\Phi}}$ contain the eigenvalues and eigenvectors of the whitened covariance matrix, respectively. The resulting whitened covariance matrix provides a better input for source enumeration algorithms since the largest eigenvalues in $\hat{\mathbf{\Lambda}}$ can now be associated with signals.

The ensemble covariance matrix is not available in practice. Instead it has to be estimated using data received by the sensors. To perform whitening, two data sets are needed: one that contains noise-only snapshots, and another that contains signal-plus-noise snapshots. The noise-only SCM is estimated by averaging the outer products of L_n snapshots, i.e.,

$$\mathbf{S_n} = \frac{1}{L_n}\sum_{i=1}^{L_n}\mathbf{n}_i\mathbf{n}_i^H. \tag{6}$$

The sample eigenvalues and eigenvectors of $\mathbf{S_n}$ are contained in the matrices $\mathbf{G}$ and $\mathbf{E}$. The inverse square root of the estimated noise-only SCM is used to design the whitening transform, i.e.,

$$\mathbf{W} = \mathbf{S_n}^{-1/2} = \mathbf{G}^{-1/2}\mathbf{E}^H. \tag{7}$$

The signal-plus-noise SCM is estimated from L_p snapshots:

$$\mathbf{S_p} = \frac{1}{L_p}\sum_{i=1}^{L_p}\mathbf{p}_i\mathbf{p}_i^H. \tag{8}$$

The whitened covariance is obtained as follows

$$\mathbf{Y} = \mathbf{W}\mathbf{S_p}\mathbf{W}^H, \tag{9}$$

with the resulting matrix eigendecomposition

$$\mathbf{Y} = \mathbf{U}\mathbf{B}\mathbf{U}^H = \sum_{k=1}^{N} b_k\mathbf{u}_k\mathbf{u}_k^H. \tag{10}$$

The eigenvalues and eigenvectors of the whitened SCM ($\mathbf{Y}$) are given by $\mathbf{B}$ and $\mathbf{U}$, respectively, where b_k and $\mathbf{u}_k$ are the individual eigenvalues and eigenvectors.

Figure 1 shows a block-diagram that describes the whitening transformation process preceding the source enumeration algorithm. The eigenvalues of $\mathbf{Y}$ are used by the enumeration algorithm to determine the number of signals.

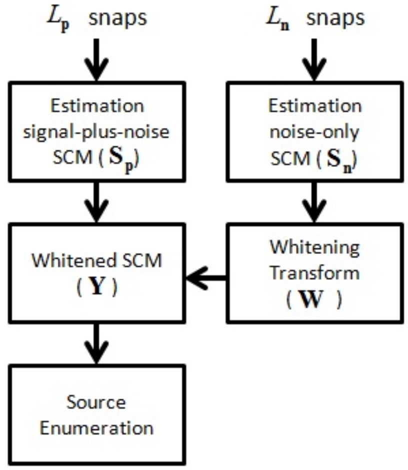

Figure 1. Block-diagram illustrating the whitening transformation process prior to source enumeration.

III. RMT-BASED SOURCE ENUMERATION

Using tools from random matrix theory, Nadakuditi and Silverstein derive the fundamental limit for eigenvalue-based signal detection [11] using a whitening transform. The detection limit depends on the number of snapshots used to estimate the noise-only covariance $\mathbf{S_n}$ and the signal-plus-noise covariance $\mathbf{S_p}$. The RMT derivation assumes that the array size and the number of snapshots approach infinity while the ratios of sensors to snapshots remain constant. The ratios are defined as follows: $c_n = N/L_n$ is the ratio of sensors to noise-only snapshots and $c_p = N/L_p$ is the ratio of sensors to signal-plus-noise snapshots. Nadakuditi and Silverstein show that a signal is detectable if its associated eigenvalue is above the threshold T defined below. T is a function of the ratios c_n and c_p:

$$T\left(c_p, c_n\right) = \frac{1 + \tau - \tau c_n + \sqrt{\left(1 + \tau - \tau c_n\right)^2 - 4\tau}}{2}, \quad (11)$$

where τ and μ are defined as follows:

$$\tau = \frac{\left(1 + c_n\right)\mu + \sqrt{\mu}\left(2c_n + c_p\left(1 - c_n\right)\right)}{\left(1 - c_n\right)^2 \mu}, \quad (12)$$

and

$$\mu = c_n + c_p - c_p c_n. \quad (13)$$

Figure 2 shows the required input signal SNR for reliable detection assuming one source. Using the description for the largest eigenvalue in the single signal case in Section II, the ensemble input signal SNR is $\frac{\sigma_1^2}{\sigma_w^2} = \left(\hat{\lambda}_1 - 1\right)/N$. The eigenvalues b in (10) are detectable if their amplitude is greater than $T\left(c_p, c_n\right)$. Using the description for the input signal SNR along with $T\left(c_p, c_n\right)$, the minimum input signal SNR for reliable detection can be expressed as $\left(T\left(c_p, c_n\right) - 1\right)/N$. In Figure 2, $\left(T\left(c_p, c_n\right) - 1\right)/N$ is plotted for a fixed array size N and a variable L_n (corresponding to a range of c_n values).

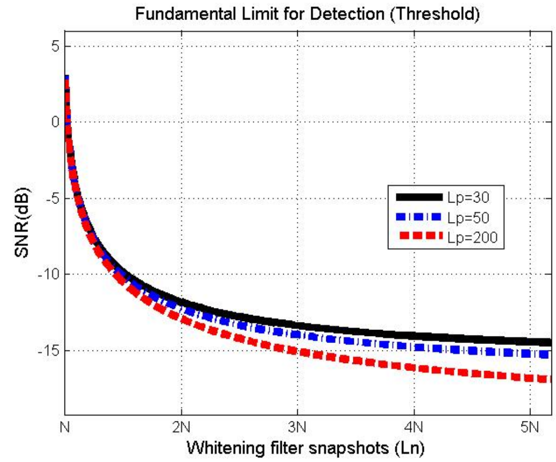

Figure 2. Plot of the asymptotic eigenvalue-based detection limit using the input signal SNR $\left(T\left(c_p, c_n\right) - 1\right)/N$ in function of the number of noise-only snapshots L_n for different values of L_p. These lines represent the minimum signal power required for detection using the eigenvalues of $\mathbf{Y}$. These results assume a 50-element array $(N = 50)$.

Three values of L_p are considered corresponding to c_p= 1.67, 1, and 0.25. Note that the required SNR for detection decreases as the number of noise-only snapshots L_n increases. In general these plots show that eigenvalue-based detection is better when both c_n, $c_p \to 0$.

Nadakuditi and Silverstein also propose a source enumeration algorithm for signals in colored noise that is based on RMT. The algorithm makes use of Johnstone's result for the distribution of the eigenvalues of a whitened SCM. Johnstone shows that in the signal-absent case, the maximum eigenvalue of the whitened SCM can be transformed to yield a Tracy-Widom-distributed random variable [15][1]. Given a known distribution, it is straightforward to set a detection threshold to provide a desired false alarm rate. Nadakuditi and Silverstein's algorithm cycles through the eigenvalues of the whitened SCM, comparing each to a threshold to determine the number of signals in the whitened data.

IV. PROPOSED APPROACH

This section presents an improved whitening transform that requires less data support than the standard whitening filter and improves detection below the fundamental limit described in the previous section. The proposed approach is motivated by the DMR beamformer [14], which uses a structured covariance matrix constructed from the eigendecomposition of the SCM. DMR assumes that the large eigenvalues are associated with loud signals and the small eigenvalues correspond to weak signals and noise. It further assumes that the large eigenvalues and eigenvectors accurately define the dominant signal subspace. Recognizing that the small eigenvalues are often estimated poorly, particularly with low sample support, DMR replaces these eigenvalues with their average. This improves the conditioning of the structured covariance matrix, meaning

[1]The transformation consists of taking the natural log of the eigenvalue and shifting and scaling. The shift and scale parameters are a function of the array size N and the number of snapshots for whitening L_n and for detection L_p. See Johnstone's paper [15] for more details.

that it can be inverted without difficulty to obtain the beamformer weights. The DMR method also enables estimation of a structured covariance using fewer snapshots than sensors.

Assuming that the colored noise is relatively low rank compared to the array size, a structured covariance can be used to implement the whitening filter. The main components of the colored noise form the dominant subspace defined by the large eigenvalues and eigenvectors. The remaining components are assumed to correspond to white sensor noise. The first task in estimating the structured covariance for DMR-based whitening is to estimate the rank of the dominant colored noise subspace. This paper uses the source enumeration algorithm proposed by Nadakuditi and Edelman [13] to determine the rank. The algorithm, designed to estimate the number of signals in white noise, is based on random matrix theory results and performs better than classical approaches when snapshots are limited. If a colored noise component is detected then it will be included in the signal part of the structured covariance. The noise part of the structured covariance will include the mean of the small eigenvalues not detected by the algorithm, which are assumed to be white sensor noise.

Nadakuditi and Edelman's algorithm uses the number of sensors N, the ratio of the number of sensors to noise-only snapshots c_n, and the eigenvalues of the noise-only SCM $(\mathbf{S_n})$ to calculate the rank of the colored noise using the following equations

$$R = \arg\min_{q} \left\{ \frac{\psi^2}{2c_n^2} + 2\left(q+1\right) \right\}, \tag{14}$$

and

$$\psi = N\left[\left(N-q\right) \frac{\sum_{i=q+1}^{N} g_i^2}{\left(\sum_{i=q+1}^{N} g_i\right)^2} - \left(1+c_n\right) \right]. \tag{15}$$

The number of sources q is varied iteratively from 1 to N. The rank of the colored noise R is the value of q that minimizes the expression in (14).

Using this estimated number of large eigenvalues R, the number of white noise eigenvalues is the difference between the number of sensors and this estimated number of sources, $N-R$. Once the number of large eigenvalues is known, the whitening transform can be divided in two parts using the eigendecomposition of the noise-only $\mathbf{S_n}$: one part that belongs to colored noise and another that corresponds to white noise, i.e.,

$$\mathbf{S_n} = \overbrace{\sum_{r=1}^{R} g_r \mathbf{e}_r \mathbf{e}_r^H}^{colored\ noise} + \overbrace{\sum_{n=R+1}^{N} g_n \mathbf{e}_n \mathbf{e}_n^H}^{white\ noise}. \tag{16}$$

The average of the small eigenvalues is given by[2]

$$\hat{\sigma}_w^2 = \left(\frac{L_n}{L_n-1}\right)\left(\frac{1}{N-R}\right)\sum_{n=R+1}^{N} g_n. \tag{17}$$

Since we know how many eigenvalues are part of the white noise from (14), the small eigenvalues of the noise-only SCM are replaced by this mean as shown in the following equation.

$$\mathbf{S}_{\text{DMR}} = \overbrace{\sum_{r=1}^{R} g_r \mathbf{e}_r \mathbf{e}_r^H}^{colored\ noise} + \overbrace{\sum_{n=R+1}^{N} \hat{\sigma}_w^2 \mathbf{e}_n \mathbf{e}_n^H}^{white\ noise} \tag{18}$$

The new whitening transform is the inverse square root of this modified SCM

$$\mathbf{W}_{\text{DMR}} = \left(\mathbf{S}_{\text{DMR}}\right)^{-1/2}. \tag{19}$$

The resulting whitened SCM using this modified transform is given by

$$\hat{\mathbf{Y}} = \mathbf{W}_{\text{DMR}} \mathbf{S_p} {\mathbf{W}_{\text{DMR}}}^H, \tag{20}$$

whose eigendecomposition is given by

$$\hat{\mathbf{Y}} = \hat{\mathbf{U}}\hat{\mathbf{B}}\hat{\mathbf{U}}^H. \tag{21}$$

To determine the number of signals we consider the eigenvalues $\hat{\mathbf{B}}$ of the whitened covariance matrix $\hat{\mathbf{Y}}$. The number of sources is determined by counting the number of individual eigenvalues $\hat{b}$ that exceed an empirically-determined threshold. While it is known that the standard whitening transform produces transformed eigenvalues that obey a Tracy-Widom distribution, the distribution of the eigenvalues resulting from DMR whitening is not yet known. For the results presented in the following section, the threshold required for a given false alarm rate was obtained through simulations. The threshold was determined from 10,000 noise-only Monte Carlo trials.

V. Simulations

This section presents numerical simulations to illustrate the performance of the proposed algorithm. The simulations consider the case of a single signal in colored noise. The colored noise is concentrated around broadside and the signal is close to endfire. The colored noise covariance matrix $\mathbf{S}_c$ corresponds to the noise spatial correlation function in example 5.3.3 of Van Trees [1]:

$$S_c\left(f, \Delta z\right) = \sigma_c^2 e^{-\frac{k_o^2 \Delta z^2 \sigma_u^2}{2}}. \tag{22}$$

This example represents noise whose spatial power spectrum has a Gaussian shape in the direction perpendicular to the array. The variable σ_u is selected to be much smaller than one in order to represent low rank colored noise, with the wavenumber equal to $k_o = |k_z|$. f is the temporal frequency.

The simulations described below use (22) to generate colored noise with a rank of 7. The power of the colored noise is set to $\sigma_c^2 = 4000$ and $\sigma_u = 0.021$. The 7 largest eigenvalues of this simulated noise range from 2.0209 to $118,830$. The

[2]The term $\frac{L_n}{L_n-1}$ is included to make $\hat{\sigma}_w^2$ an unbiased estimate.

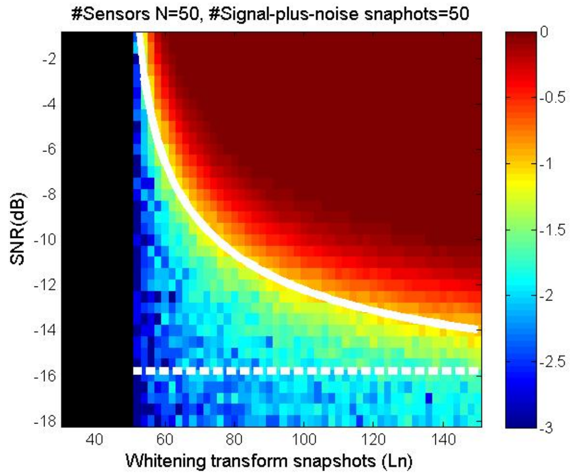

Figure 3. Heat map of the log10 probability of correct determination using Nadakuditi and Silverstein's algorithm. Results are shown for 1000 Monte Carlo simulations. The solid white line represents the fundamental limit described in [11] divided by the number of sensors N minus one. The dashed white line represents the fundamental limit if the ensemble whitening filter is known [11]. Probability of false alarm is 0.01. The black region represents the area where the algorithm cannot be implemented because the number of noise-only snapshots is below or equal to N. (0 = always detected, and -3 = rarely detected)

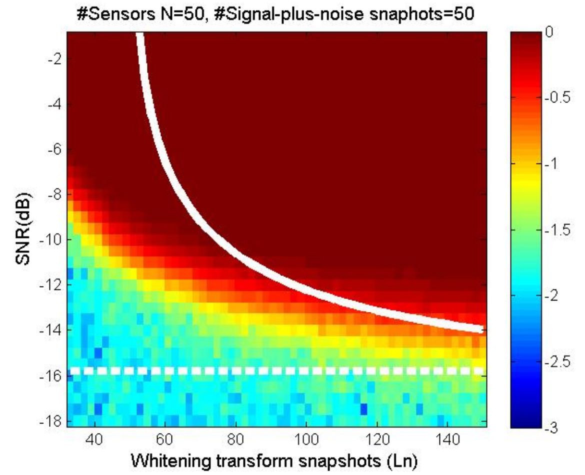

Figure 4. Heat map of the log10 probability of correct determination using the new whitening transform. Results are shown for 1000 Monte Carlo simulations. The solid white line represents the fundamental limit described in [11] divided by the number of sensors N minus one. The dashed white line represents the fundamental limit if the ensemble whitening filter is known [11]. Probability of false alarm is 0.01. (0 = always detected, and -3 = rarely detected)

rank of the colored noise was estimated correctly using (14) for all the whitening filters in the simulation.

Consider a comparison of the proposed method with the source enumeration algorithm of Nadakuditi and Silverstein [11]. The false alarm rate for both algorithms is set to 0.01. Figures 3 and 4 show the performance of both algorithms for the case of a single source in colored noise received by a 50-element array $(N = 50)$ .There are $L_p = 50$ signal-plus-noise snapshots for the detection, i.e., $c_p = 1$. The plots show the logarithm (base 10) of the probability of correct detection as the number of whitening filter snapshots (L_n) is varied from 30 to 150 ($c_n = 0.33$ to $c_n = 1.67$). Note that when the number of snapshots to estimate the whitening filter increases, the detection improves for both algorithms. When the number of snapshots to estimate the whitening filter is about the same order of the number of sensors, the proposed algorithm has a better performance than Nadakuditi and Silverstein's approach. At low sample support the proposed algorithm is able to determine the number of signals when the number of snapshots is below the number of sensors. Nadakuditi and Silverstein's approach cannot be implemented in that setting because it requires more than N snapshots for the whitening filter.

Figure 5 shows how the proposed whitening method improves separation between the signal eigenvalue and the largest noise eigenvalue. The DMR-based whitening method mitigates the degradation introduced by poor estimation of the small eigenvalues in the noise-only SCM at low sample support. As a result, the signal eigenvalue shows increased separation from the maximum noise eigenvalue following DMR whitening, as compared to standard whitening.

VI. Summary

This paper presents an improved whitening transform that requires less data to implement than the standard whitening filter. The proposed whitening transform is based on the DMR beamformer's idea of using a structured covariance matrix. Simulations show that DMR-based whitening has better performance than standard whitening for low numbers of snapshots ($L_n > N$ but still small). Unlike other approaches, DMR-based whitening can be implemented even when $L_n \leq N$, assuming that the noise is low-rank. Section V shows that a detector based on the new whitening filter has better performance than Nadakuditi and Silverstein's algorithm [11], especially for low sample support. Additional work on DMR-based whitening is in progress. In particular the investigation focuses on characterizing the statistics of the noise eigenvalues after whitening using a structured covariance matrix. This will facilitate analytical, rather than empirical, determination of the detection threshold required for a given false alarm rate.

Acknowledgments

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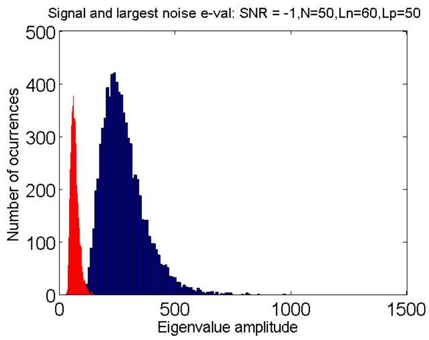
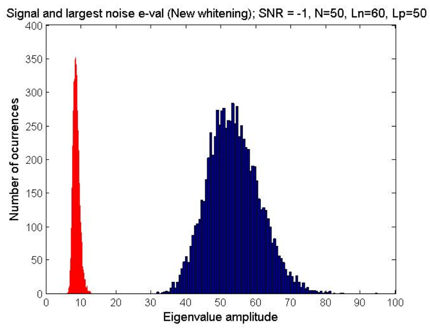


(a)



(b)

Figure 5. Whitened largest noise eigenvalue (red histogram) and whitened signal eigenvalue (blue histogram) for the case when the input signal SNR=-1 dB, the number of sensors is 50, the number of signal-plus-noise snapshots is 50, and the number of noise-only snapshots is 60. Fig. 5a shows the eigenvalues resulting from application of the standard whitening filter $\mathbf{W}$ designed using the noise-only SCM. Fig. 5b shows eigenvalues resulting from application of the DMR-based whitening filter $\mathbf{W}_{\mathrm{DMR}}$. The plot shows how DMR-based whitening improves the separation between the whitened signal and the noise eigenvalues.

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