Preliminary Feasibility Study of Cooperative Navigation of Underwater Vehicles with Range and Range-Rate Observations

Zachary J. Harris and Louis L. Whitcomb

Abstract— **This paper reports a preliminary feasibility study of a method for combined underwater communication and navigation of underwater vehicles (cooperative navigation) in which synchronous-clock modems are employed to measure both range and range rate on every acoustic data transmission. Few previously reported studies have utilized acoustic range rate in addition to acoustic range for the general navigation of underwater vehicles. This paper reports an observation model for range-rate measurements and preliminary anecdotal simulation studies for cooperative navigation of one underwater vehicle client with one surface vehicle server employing both range and range-rate observations to estimate the vehicle states with a delayed-state extended Kalman filter. In the simulation studies presented, the underwater vehicle is equipped with Doppler sonar navigation, attitude, and depth sensors, and the surface vehicle is equipped with a GPS. The preliminary studies suggest that the addition of range-rate measurements may not significantly improve the filter's estimate of the client's state over the case of range-only observations. The present study is very preliminary, however, and directions for future study are suggested.**

I. Introduction

Navigation methods for underwater vehicles utilizing velocity measurements (e.g., Doppler sonar navigation) or acceleration measurements (e.g., inertial navigation) will accumulate errors due to sensor bias, sensor noise, and a variety of other calibration issues, and thus require regular external corrections. The corrections are independent observations of absolute position or velocity from a separate sensor or system that are used to correct the drift accumulated in the navigation estimate [11].

For land and air vehicle navigation, the global positioning system (GPS) provides an ideal independent source of position corrections for acceleration and velocity-based navigation systems [11], but GPS is unavailable to submerged underwater vehicles. Bottom-lock Doppler sonar velocity measurements are an excellent correction source for inertial navigation velocity estimates. Pressure depth sensor measurements are an excellent correction source for the Z dimension (only) of Doppler sonar and inertial position estimates.

For underwater vehicles, few methods currently exist for absolute position corrections for Doppler sonar and inertial navigation systems. The most common XY-correction methods for Doppler sonar and inertial underwater navigation systems are time-of-flight acoustic navigation systems, such as long-baseline (LBL) and ultra-short baseline (USBL) acoustic navigation [26], [30], [43].

Range-only one-way travel-time (OWTT) cooperative underwater navigation uses ranges estimated from the acoustic time-of-flight between subsea nodes, e.g., between two vehicles or between a client vehicle and a server reference beacon of known (fixed or moving) location. When all vehicles and beacons (nodes) are equipped with precision clocks, each node's acoustic data transmission can be received by multiple receiving nodes—enabling all nodes within acoustic range to simultaneously (a) measure range to the transmitting node from the measured time-of-flight and (b) decode the data encoded in the acoustic data packet. This method provides both bounded-error position estimates and long range capabilities with reduced need for multiple costly fixed beacons, as is the case with most LBL systems. OWTT navigation also provides scalability by allowing all vehicles within acoustic range to simultaneously use the same acoustic data packet broadcast independent of the number of vehicles.

In this paper, we examine the feasibility of employing cooperative navigation with underwater acoustic networks to provide both position and velocity corrections for low-cost underwater vehicles.

II. Literature Review

Numerous studies by the authors and others report the development and extensive at-sea experimental evaluation of OWTT systems (including hardware and software) for the navigation of underwater vehicles using maximum-likelihood estimation [15]–[17], the extended Kalman filter (EKF) [54]–[56], and the extended Information filter (EIF) [57]–[59].

The range-only OWTT navigation and communication system has been experimentally evaluated at sea on several different AUVs worldwide: on the *SeaBed* AUV in shallow water deployments in the Atlantic Ocean [15] and in the Mediterranean Sea [16], [17]; on the *Jaguar* and *Puma* AUVs in a deep water ($\sim 5,000$ m) deployment at 4°S on the Southern Mid-Atlantic Ridge [54]–[59]; and on the *Nereus* AUV [10], [14] to $10,903$ m depth in the Mariana Trench [47] and to $5,000$ m depths in the Cayman Rise in the Caribbean [23].

Several other groups worldwide also reported results in range-only OWTT navigation. Several authors have reported least-squares methods for single-beacon range-only navigation [25], [33], [42], [46]. Range-only target tracking has been addressed using EKFs and maximum-likelihood estimators (MLE) [1], [44], [48]. The use of EKFs for homing and single-beacon navigation is reported in [2], [3], [34], [35], [37], [52].

We gratefully acknowledge the support of the National Science Foundation under Award 1319667 and the National Defense Science and Engineering Graduate Fellowship.

The authors are with the Department of Mechanical Engineering, Johns Hopkins University, Baltimore, MD 21218, USA, zharris7,llw@jhu.edu

978-0-933957-43-5 ©2015 MTS

Several different studies addressed the observability of single-beacon range-only navigation with linear EKF estimation methods [19]–[22], [40] and with nonlinear methods [27], [45]. Several authors have addressed OWTT navigation of surface and underwater vehicles in a simultaneous localization and mapping (SLAM) framework using distributed estimators [4]–[7], [18]. Multi-beacon, range-only navigation for terrestrial vehicles in a SLAM framework is addressed in [12], [13], [28], [29], [31], [32] using radio-frequency beacons for range measurement, in [41] using audible sound, and in [39], [50] using wireless sensor networks.

To the best of our knowledge, the earliest reported comprehensive study of underwater-vehicle navigation using acoustic ranging was reported by Spindell et al. in which they reported full-scale experimental evaluation of an acoustic approach to underwater-vehicle navigation in which a single underwater vehicle could detect range from a set of fixed acoustic navigation transponders whose location was known *a priori*—a method that has since been widely practiced and is now commonly known as long-baseline (LBL) acoustic navigation [26].

To the best of our knowledge, the earliest study of underwater-vehicle navigation employing acoustic detection of *both range and range-rate* was the 1978 study by Spindell et al., [49], which extends the approach reported in [26] by reporting the full-scale experimental evaluation of an approach to underwater-vehicle navigation in which a single underwater vehicle could detect both range and range rate from a set of fixed navigation transponders whose location was known *a priori*.

In [36], [38], the author, apparently unaware of [26], [49], reported the notion of employing acoustic range rate in addition to acoustic range for LBL navigation but did not report specific navigation algorithms for employing range-rate measurements nor any experimental evaluation. In [24], the author reported the notion of an underwater acoustic modem estimating and compensating for the Doppler shift of a received acoustic data packet transmission but did not address how a Doppler estimate might be used for navigation.

In [9], the author reported the experimental evaluation of algorithms for acoustically determining the *relative position* of two marine vehicles by employing measurement of acoustic range and acoustic range rate with specific focus on estimating relative positioning conditions, such as the closest-point-of-approach (CPA) of two vessels for the purpose of collision-avoidance.

III. Extended Kalman Filter

This section briefly describes the extended Kalman filter and associated process model and observation models used.

A. EKF Formulation and Implementation

The extended Kalman filter is a non-optimal extension of the Kalman filter to nonlinear plants and observations by linearizing about the time-varying estimated state. For a full derivation and formulation of the EKF, see [8], [51].

A discrete-time EKF was implemented using a discrete process model to fuse depth, gyrocompass, and Doppler velocity log (DVL) measurements from the vehicle, GPS measurements from the ship, and OWTT and range-rate measurements between the vehicle and the ship.

This paper utilizes the centralized extended Kalman filter (CEKF), which assumes real-time access to vehicle and ship sensor data simultaneously. As such, it is suitable for post-processing simulated and experimental data. Recent results have shown that this approach can be extended exactly to a decentralized extended information filter (DEIF) form [57].

B. State Description

As is typical for the CEKF cooperative navigation algorithm [53]–[56], we define the state vector, $\mathbf{x}$, as the composite of the current vehicle state, current ship state, and delayed ship states. Delayed states are required for causal processing because the range measurement occurs between the ship at time-of-launch (TOL) and the vehicle at time-of-arrival (TOA).

$$\mathbf{x} = \begin{bmatrix} \boldsymbol{x}_v^T & \boldsymbol{x}_s^T & \boldsymbol{x}_{v-1}^T & \boldsymbol{x}_{s-1}^T & \dots & \boldsymbol{x}_{v-n}^T & \boldsymbol{x}_{s-n}^T \end{bmatrix}^T \tag{1}$$

where the current ship state is a 6-degree-of-freedom (DOF) vector containing the XY-position and heading and respective velocities

$$\boldsymbol{x}_s = \begin{bmatrix} x & y & \psi & \dot{x} & \dot{y} & \dot{\psi} \end{bmatrix}^T, \tag{2}$$

and the current vehicle state is a 12-DOF vector containing the local-level pose and body-frame velocities

$$\boldsymbol{x}_v = \begin{bmatrix} \boldsymbol{s}^T & \boldsymbol{\varphi}^T & \boldsymbol{v}^T & \boldsymbol{\omega}^T \end{bmatrix}^T \tag{3}$$

$$\boldsymbol{s} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \boldsymbol{\varphi} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \quad \boldsymbol{\nu} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad \boldsymbol{\omega} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{4}$$

where $\boldsymbol{s}$ is the local-level position, $\boldsymbol{\varphi}$ is the local-level attitude, $\boldsymbol{v}$ is the body-frame linear velocity, and $\boldsymbol{\omega}$ is the body-frame angular velocity.

C. Process Model

We assume a kinematic, nonlinear process model for both the vehicle and the ship, identical to the one reported in [55].

$$\dot{\boldsymbol{x}}_v = \begin{bmatrix} 0 & 0 & \boldsymbol{R}(\boldsymbol{\varphi}) & 0 \\ 0 & 0 & 0 & \boldsymbol{J}(\boldsymbol{\varphi}) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{x}_v + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \mathbb{I} & 0 \\ 0 & \mathbb{I} \end{bmatrix} \boldsymbol{w}_v \tag{5}$$

$$\dot{\boldsymbol{x}}_s = \begin{bmatrix} 0 & \mathbb{I} \\ 0 & 0 \end{bmatrix} \boldsymbol{x}_s + \begin{bmatrix} 0 \\ \mathbb{I} \end{bmatrix} \boldsymbol{w}_s \tag{6}$$

where $\boldsymbol{R}(\boldsymbol{\varphi})$ is the transformation between inertial and body-frame linear velocities, $\boldsymbol{J}(\boldsymbol{\varphi})$ is the transformation between inertial and body-frame angular velocities, and $\boldsymbol{w}_v \sim \mathcal{N}(0, \boldsymbol{Q}_v)$ and $\boldsymbol{w}_s \sim \mathcal{N}(0, \boldsymbol{Q}_s)$ are zero-mean Gaussian process noise terms. The process model for the vehicle and ship is linearized and discretized using standard methods [8]. The reader is referred to [55] for a full description and derivation, including the linearized discrete-time process model and the subtleties of the modified process prediction, which occurs at the top of the second when the state augmentation is performed in concert with the process-prediction step.

D. Observation Models

The range and range-rate observation models are nonlinear functions of the vehicle state at TOA and the ship state at TOL. Observation models of the additional sensors, including the DVL, GPS, depth sensor, and gyrocompass, are detailed in [53].

a) Range Observation Model: As reported in [53], [55], the OWTT range observation model can be written in matrix notation as

$$z_{rng} = \left(\mathbf{x}^T \boldsymbol{M}^T \boldsymbol{M} \mathbf{x}\right)^{\frac{1}{2}} + v_{rng} \tag{7}$$

where $v_{rng} \sim \mathcal{N}(0, \boldsymbol{R}_{rng})$ and $\boldsymbol{M} = \left[\begin{array}{cccc} \boldsymbol{J}_v & \mathbf{0}\ldots\mathbf{0} & \boldsymbol{J}_s & \mathbf{0}\ldots\mathbf{0} \end{array}\right]$, with $\boldsymbol{J}_v$ and $\boldsymbol{J}_s$ defined such that

$$\boldsymbol{J}_v\, \boldsymbol{x}_v = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } \boldsymbol{J}_s\, \boldsymbol{x}_s = \begin{bmatrix} x_s \\ y_s \\ 0 \end{bmatrix}.$$

The measurement covariance, $\boldsymbol{R}_{rng}$, represents the noise of the range measurement. The Jacobian of the range measurement with respect to the full state, $\mathbf{x}$, is

$$\begin{aligned} \boldsymbol{H}_k = & \frac{\partial\, z_{rng}(\mathbf{x})}{\partial\, \mathbf{x}} \bigg|_{\mathbf{x}=\boldsymbol{\mu}_{k|k-1}} \\ = & \left(\boldsymbol{\mu}_{k|k-1}^T \boldsymbol{M}^T \boldsymbol{M} \boldsymbol{\mu}_{k|k-1}\right)^{-\frac{1}{2}} \boldsymbol{\mu}_{k|k-1}^T \boldsymbol{M}^T \boldsymbol{M} \end{aligned} \tag{8}$$

b) Range-Rate Observation Model: The range-rate observation model is the time derivative of (7). Formally,

$$\begin{aligned} z_{rr} = & \dot{z}_{rng} \\ = & \left(\mathbf{x}^T \boldsymbol{M}^T \boldsymbol{M} \mathbf{x}\right)^{-\frac{1}{2}} \mathbf{x}^T \boldsymbol{M}^T \boldsymbol{M} \dot{\mathbf{x}} + v_{rr} \end{aligned} \tag{9}$$

where $v_{rr} \sim \mathcal{N}(0, \boldsymbol{R}_{rr})$ represents the noise covariance of the range-rate measurement. The Jacobian of the range-rate measurement with respect to the full state is

$$\begin{aligned} \boldsymbol{H}_k = & \frac{\partial\, z_{rr}(\mathbf{x})}{\partial\, \mathbf{x}} \bigg|_{\mathbf{x}=\boldsymbol{\mu}_{k|k-1}} \\ = & -\left(\boldsymbol{\mu}_{k|k-1}^T \boldsymbol{M}^T \boldsymbol{M} \boldsymbol{\mu}_{k|k-1}\right)^{-\frac{3}{2}} \\ & \left(\boldsymbol{\mu}_{k|k-1}^T \boldsymbol{M}^T \boldsymbol{M} \boldsymbol{\mu}_{k|k-1}^T \boldsymbol{M}^T \boldsymbol{M} \dot{\boldsymbol{\mu}}_{k|k-1}\right) + \\ & \left(\boldsymbol{\mu}_{k|k-1}^T \boldsymbol{M}^T \boldsymbol{M} \boldsymbol{\mu}_{k|k-1}\right)^{-\frac{1}{2}} \left(\boldsymbol{M}^T \boldsymbol{M} \dot{\boldsymbol{\mu}}_{k|k-1}\right)^T \end{aligned} \tag{10}$$

where $\boldsymbol{\mu}_{k|k-1}$ is the estimated mean of the world-frame position and $\dot{\boldsymbol{\mu}}_{k|k-1}$ is the estimated mean of the world-frame velocity.

IV. Simulation Results

In this preliminary feasibility study, we utilized a simulation to investigate the effect of the range-rate observation on the performance of the CEKF cooperative navigation algorithm described in Section III. A full analysis of the effect the range-rate observation by varying the ship and vehicle trajectories and measurement covariances is beyond the scope of this paper; thus, anecdotal results are reported herein. We computed a true simulated vehicle path and simulated data for each of the navigation sensors by generating simulated measurements with the measurement-noise characteristics outlined in Table I. In the simulation presented here, the vehicle conducts a simulated survey mission of ten 1 km track lines spaced 100 m apart at a velocity of 2 knots and a depth of 3 m. The ship circles continuously on a 300 m radius at a velocity of 1.5 knots broadcasting acoustic packets every minute. The speed of sound was assumed constant at 1500 m/s throughout the simulation.

TABLE I

Vehicle state measurement sources, resolutions, and accuracies

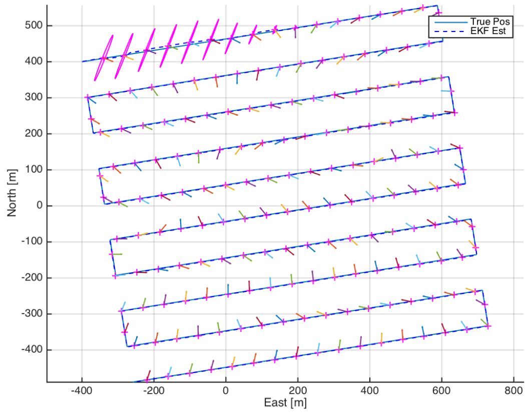
State	Source	Update Rate	Measurement Std Dev
XY Trans	modem	60 s	1m (range) 0.1m/s (range rate)
Z Trans	Paroscientific	7 Hz	6 cm
Heading	OCTANS	3 Hz	0.10°
Pitch, Roll	OCTANS	3 Hz	0.05°
Trans Velocity	300 kHz RDI DVL	5 Hz	0.01 m/s
Ang Vel	OCTANS	3 Hz	0.4-0.6°/s

Figure 1 shows the true and estimated two-dimensional vehicle position with the filter's covariance plotted at 60 s intervals. The arrows point from the vehicle to the ship along acoustic path with the length scaled by the angle from vertical.

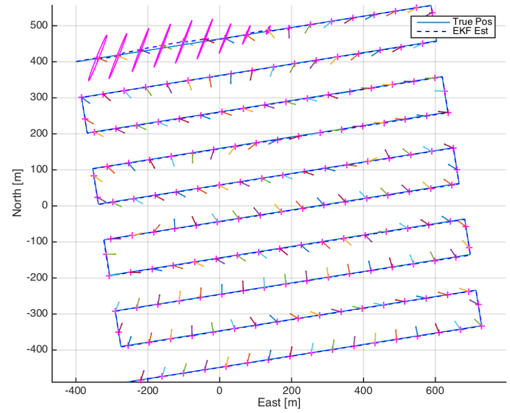
Figure 2 shows histograms of the error in the X- and Y-directions. These histograms indicate the noise statistics may not be uni-modal in X-direction and may not be zero-mean in the Y-direction. However, the variability of the simulation noise in disparate realizations prevents us from drawing broad conclusions based on these results alone.

Figure 3 shows the XY error magnitude, i.e., the magnitude of the difference between the EKF-estimated and true XY position, for both range-only and combined range and range-rate observations. It should be noted that all other simulated measurements were held constant during this comparison; the only change was the addition of range-rate observations.

For the kinematic process model described in Section III-C and the observation models described in Section III-D, the addition of range-rate observations does not appear to offer a significant improvement in CEKF performance on this simulated survey mission when utilizing a high-accuracy velocity and acceleration navigation sensors and reasonable range and range-rate noise statistics. However, we note that

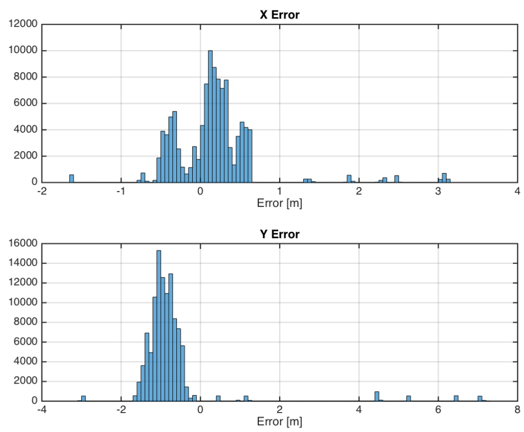


(a) Range-only observation model.

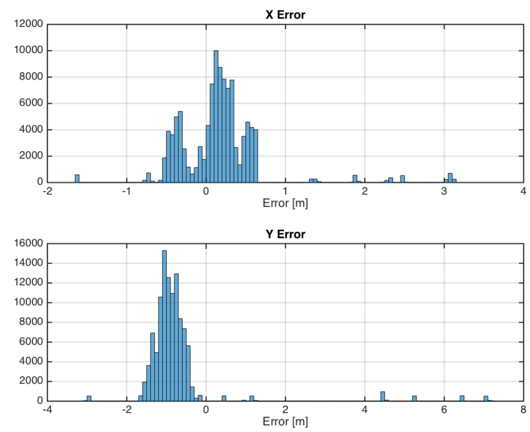


(b) Range and range-rate observation model.

Fig. 1. Vehicle XY Position



(a) Range-only observation model.



(b) Range and range-rate observation model.

Fig. 2. Error Histogram in the X- and Y-directions

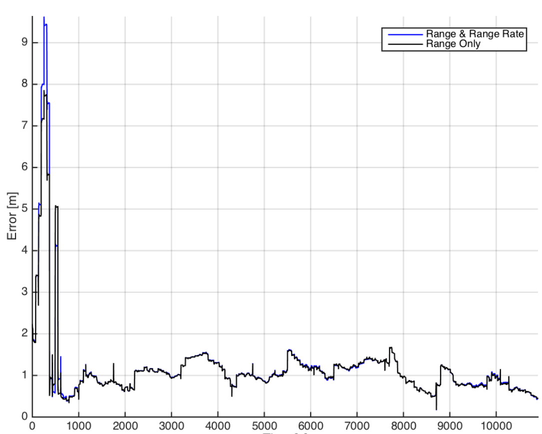


Fig. 3. XY Error magnitude versus mission time

these anecdotal results are preliminary, and more studies are needed before any broad conclusions can be made.

We also briefly investigated in simulation the performance of this cooperative navigation approach for the case in which the underwater vehicle is not equipped with a Doppler sonar but observed instability of the resulting CEKF. We conjecture that improvements in the process model may be required to obtain stable performance in the absence of DVL measurements.

V. Conclusion

This paper reports preliminary anecdotal simulation studies for cooperative navigation of one underwater vehicle client (equipped with Doppler sonar navigation, attitude, and depth sensors) with one surface vehicle server (equipped with GPS), employing both range and range-rate observations to estimate the vehicle states with a delayed-state extended Kalman filter. This preliminary study suggests that the addition of range-rate measurements may not significantly improve the performance of the CEKF cooperative navigation algorithm over the case of range-only observations. The present study is very preliminary, however. In future studies, we intend to evaluate a broader variety of simulated missions, both shallow-water and deep-water, and to more precisely experimentally characterize actual range and range-rate covariances with our acoustic modem systems for various modem configurations, and to investigate the effect of different process models on system performance with differing vehicle sensor configurations.

VI. Acknowledgements

We very gratefully acknowledge using and extending, with permission, a CEKF implementation originally developed by our colleague Dr. Ryan Eustice at the University of Michigan and subsequently extended by Dr. Sarah Webster who is presently with the University of Washington Applied Physics Laboratory.

References

[1] J. Alleyne, "Position estimation from range only measurements," Master's thesis, Naval Postgraduate School, Monterey, CA, Sept. 2000.

[2] P. Baccou and B. Jouvencel, "Homing and navigation using one transponder for AUV, post-processing comparisons results with long base-line navigation," in *Proceedings of the IEEE International Conference on Robotics and Automation*, vol. 4, Washington, DC, May 2002, pp. 4004–4009.

[3] ——, "Simulation results, post-processing experimentations and comparisons results for navigation, homing and multiple vehicles operations with a new positioning method using on transponder," in *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, vol. 1, Oct. 2003, pp. 811–817.

[4] A. Bahr and J. Leonard, "Cooperative localization for autonomous underwater vehicles," in *Proc. 10th Intl. Symp. on Experimental Robotics (ISER)*, Rio de Janeiro, Brasil, July 2006, pp. 387–395.

[5] A. Bahr, "Cooperative localization for autonomous underwater vehicles," Ph.D. dissertation, Massachusetts Institute of Technology / Woods Hole Oceanographic Institution Joint Program, Cambridge, MA, USA, Feb. 2009.

[6] A. Bahr, J. J. Leonard, and M. F. Fallon, "Cooperative localization for autonomous underwater vehicles," *International Journal of Robotics Research*, vol. 28, no. 6, pp. 714–728, June 2009.

[7] T. Bailey, M. Bryson, H. Mu, J. Vial, L. McCalman, and H. Durrant-Whyte, "Decentralised cooperative localisation for heterogeneous teams of mobile robots," in *Proceedings of IEEE International Conference on Robotics and Automation*, May 2011, pp. 2859–2865.

[8] Y. Bar-Shalom, X. Rong Li, and T. Kirubarajan, *Estimation with applications to tracking and navigation*. New York: John Wiley & Sons, Inc., 2001.

[9] B. S. Bourgeois, "Using range and range rate for relative navigation," Naval Research Laboratory, Mapping, Charting, Geodesy Branch, Marine Geosciences Division, Tech. Rep., September 2007.

[10] A. D. Bowen, L. L. Whitcomb, and D. R. Yoegrer, "Nereus project homepage," 2012, http://www.whoi.edu/page.do?pid=10076.

[11] A. Chatfield, *Fundamentals of high accuracy inertial navigation*. AIAA (American Institute of Aeronautics & Astronautics), 1997, vol. 174.

[12] D. Djugash, S. Singh, and P. Corke, *Field and Service Robotics*, ser. Springer Tracts in Advanced Robotics. Berlin, Heidelberg: Springer Berlin / Heidelberg, 2006, vol. 25, ch. Further Results with Localization and Mapping Using Range from Radio, pp. 231–242.

[13] J. Djugash, S. Singh, G. Kantor, and W. Zhang, "Range-only SLAM for robots operating cooperatively with sensor networks," in *Proc. IEEE Intl. Conf. Robot. Auto. (ICRA)*, May 2006, pp. 2078–2084.

[14] C. Dybas and S. Murphy, "The abyss: Deepest part of the oceans no longer hidden," National Science Foundation Press Release, 2009, http://www.nsf.gov/news/news_summ.jsp?cntn_id=114913.

[15] R. M. Eustice, L. L. Whitcomb, H. Singh, and M. Grund, "Recent advances in synchronous-clock one-way-travel-time acoustic navigation," in *Proceedings of the IEEE/MTS OCEANS Conference and Exhibition*, Boston, MA, USA, Sept. 2006, pp. 1–6.

[16] R. M. Eustice, H. Singh, and L. L. Whitcomb, "Synchronous-clock one-way-travel-time acoustic navigation for underwater vehicles," *Journal of Field Robotics, Special Issue on State of the Art in Maritime Autonomous Surface and Underwater Vehicles*, vol. 28, no. 1, pp. 121–136, Jan./Feb. 2011.

[17] R. M. Eustice, L. L. Whitcomb, H. Singh, and M. Grund, "Experimental results in synchronous-clock one-way-travel-time acoustic navigation for autonomous underwater vehicles," in *Proceedings of the IEEE International Conference on Robotics and Automation*, Rome, Italy, Apr. 2007, pp. 4257–4264.

[18] M. F. Fallon, G. Papadopoulos, J. J. Leonard, and N. M. Patrikalakis, "Cooperative auv navigation using a single maneuvering surface craft," *International Journal of Robotics Research*, vol. 29, no. 12, pp. 1461–1474, October 2010.

[19] A. Gadre, "Observability analysis in navigation systems with an underwater vehicle application," Ph.D. dissertation, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, Jan. 2007.

[20] A. Gadre and D. Stilwell, "Toward underwater navigation based on range measurements from a single location," in *Proceedings of the IEEE International Conference on Robotics and Automation*, vol. 5, New Orleans, Apr. 2004, pp. 4472–4477.

[21] ——, "A complete solution to underwater navigation in the presence of unknown currents based on range measurements from a single location," in *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, Edmonton, Alberta, Canada, Aug. 2005, pp. 1420–1425.

[22] ——, "Underwater navigation in the presence of unknown currents based on range measurements from a single location," in *Proc. Am. Control Conf.*, vol. 1, June 2005, pp. 656–661.

[23] C. R. German, A. Bowen, M. L. Coleman, D. L. Honig, J. A. Huber, M. V. Jakuba, J. C. Kinsey, M. D. Kurz, S. Leroy, J. M. McDermott, B. M. de Lépinay, K. Nakamura, J. S. Seewald, J. L. Smith, S. P. Sylva, C. L. V. Dover, L. L. Whitcomb, and D. R. Yoerger, "Diverse styles of submarine venting on the ultraslow spreading Mid-Cayman Rise," *Proceedings of the National Academy of Sciences*, vol. 107, no. 32, p. 14020, 2010.

[24] M. Green and K. Scussel, "Underwater data communication and instrument release management system," Patent, March, 2007, uS Patent 7,187,623. [Online]. Available: https://www.google.com/patents/US7187623

[25] J. C. Hartsfield, "Single transponder range only navigation geometry (STRONG) applied to REMUS autonomous under water vehicles," Master's thesis, Massachusetts Institute of Technology and Woods Hole Oceanographic Institution, Aug. 2005.

[26] M. Hunt, W. Marquet, D. Moller, K. Peal, W. Smith, and R. Spindel, "An acoustic navigation system, Tech. Rep. WHOI-74-6, Dec. 1974.

[27] J. Jouffroy and J. Reger, "An algebraic perspective to single-transponder underwater navigation," in *Proceedings IEEE 2006 CCA/CACSD/ISIC*, Munich, Germany, 2006, pp. 1789–1794.

[28] G. Kantor and S. Singh, "Preliminary results in range-only localization and mapping," in *Proc. IEEE Intl. Conf. Robot. Auto. (ICRA)*, 2002, pp. 1818–1823.

[29] A. Kehagias, J. Djugash, and S. Singh, "Range-only SLAM with interpolated range data," Robotics Institute, Pittsburgh, PA, Tech. Rep. CMU-RI-TR-06-26, May 2006.

[30] J. C. Kinsey, R. M. Eustice, and L. L. Whitcomb, "A survey of underwater vehicle navigation: Recent advances and new challenges," in *IFAC Conference of Manoeuvring and Control of Marine Craft*, Lisbon, Portugal, 2006, invited paper.

[31] D. Kurth, G. Kantor, and S. Singh, "Experimental results in range-only localization with radio," in *Proc. IEEE/RSJ Intl. Conf. Intell. Robots Systems (IROS)*, vol. 1, Oct. 2003, pp. 974–979.

[32] D. Kurth, "Range-only robot localization and SLAM with radio," Master's thesis, Carnegie Mellon University, Pittsburgh, PA, May 2004.

[33] C. E. LaPointe, "Virtual long baseline (VLBL) autonomous underwater vehicle navigation using a single transponder," Master's thesis, Massachusetts Institute of Technology and Woods Hole Oceanographic Institution, June 2006.

[34] M. Larsen, "High performance autonomous underwater navigation: Experimental results," *Hydro Intl.*, vol. 6, no. 1, pp. 6–9, Jan./Feb. 2002.

[35] ——, "Synthetic long baseline navigation of underwater vehicles," in *Proceedings of the IEEE/MTS OCEANS Conference and Exhibition*, vol. 3, Providence, RI, Sept. 2000, pp. 2043–2050.

[36] ——, “Synthetic long baseline navigation of underwater vehicles,” in *OCEANS 2000 MTS/IEEE Conference and Exhibition*, vol. 3, 2000, pp. 2043–2050 vol.3.

[37] ——, “Autonomous navigation of underwater vehicles,” Ph.D. dissertation, Technical University of Denmark, Ørsteds Plads, Building 348, DK-2800 Kgs. Lyngby, Denmark, 2001.

[38] ——, “Methods and systems for navigating under water,” Patent, Nov, 2006, uS Patent 7,139,647. [Online]. Available: https://www.google.com/patents/US7139647

[39] E. Larsson, “Cramer-Rao bound analysis of distributed positioning in sensor networks,” *Signal Processing Letters, IEEE*, vol. 11, no. 3, pp. 334–337, March 2004.

[40] P.-M. Lee, B.-H. Jun, and Y.-K. Lim, “Review on underwater navigation system based on range measurements from one reference,” in *OCEANS 2008 - MTS/IEEE Kobe Techno-Ocean*, April 2008, pp. 1–5.

[41] E. Martinson and F. Dellaert, “Marco Polo localization,” in *Proc. IEEE Intl. Conf. Robot. Auto. (ICRA)*, vol. 2, Sept. 2003, pp. 1960–1965 vol.2.

[42] S. McPhail and M. Pebody, “Range-only positioning of a deep-diving autonomous underwater vehicle from a surface ship,” *IEEE J. Oceanic Eng.*, vol. 34, no. 4, pp. 669–677, Oct. 2009.

[43] P. H. Milne, *Underwater Acoustic Positioning Systems*. Houston: Gulf Publishing, 1983.

[44] B. Ristic, S. Arulampalam, and J. McCarthy, “Target motion analysis using range-only measurements: algorithms, performance and application to ISAR data,” *Signal Processing*, vol. 82, no. 2, pp. 273–296, Feb. 2002.

[45] A. Ross and J. Jouffroy, “Remarks on the observability of single beacon underwater navigation,” in *Proceedings of the International Symposium on Unmanned Untethered Submersible Technology*, Durham, New Hampshire, Aug. 2005.

[46] A. Scherbatyuk, “The AUV positioning using ranges from one transponder LBL,” in *Proceedings of the IEEE/MTS OCEANS Conference and Exhibition*, vol. 3, San Diego, CA, Oct. 1995, pp. 1620–1623.

[47] S. Singh, S. E. Webster, L. Freitag, L. L. Whitcomb, K. Ball, J. Bailey, and C. Taylor, “Acoustic communication performance of the WHOI micro-modem in sea trials of the *Nereus* vehicle to 11,000 m depth,” in *Proc. IEEE/MTS OCEANS Conf. Exhib.*, Biloxi, MS, Oct 2009, pp. 1–6.

[48] T. Song, “Observability of target tracking with range-only measurements,” *IEEE Journal of Oceanic Engineering*, vol. 24, no. 24, pp. 383–387, July 1999.

[49] R. Spindel, R. Porter, W. Marquet, and J. Durham, “A high-resolution pulse-doppler underwater acoustic navigation system,” *Oceanic Engineering, IEEE Journal of*, vol. 1, no. 1, pp. 6–13, September 1976.

[50] J. R. Spletzer, “A new approach to range-only SLAM for wireless sensor networks,” Lehigh University, Bethlehem, PA, Tech. Rep., 2003.

[51] S. Thrun, W. Burgard, and D. Fox, *Probabilistic Robotics*, 2005.

[52] J. Vaganay, P. Baccou, and B. Jouvencel, “Homing by acoustic ranging to a single beacon,” in *Proceedings of the IEEE/MTS OCEANS Conference and Exhibition*, vol. 2, Providence, RI, Sept. 2000, pp. 1457–1462.

[53] S. E. Webster, “Decentralized single-beacon acoustic navigation: Combined communication and navigation for underwater vehicles,” Ph.D. dissertation, Johns Hopkins University, Baltimore, MD, June 2010.

[54] S. E. Webster, R. M. Eustice, C. Murphy, H. Singh, and L. L. Whitcomb, “Toward a platform-independent acoustic communications and navigation system for underwater vehicles,” in *Proceedings of the IEEE/MTS OCEANS Conference and Exhibition*, Biloxi, MS, Oct. 2009, pp. 1–7.

[55] S. E. Webster, R. M. Eustice, H. Singh, and L. L. Whitcomb, “Advances in single-beacon one-way-travel-time acoustic navigation for underwater vehicles,” *The International Journal of Robotics Research*, vol. 31, no. 8, pp. 935–950, 2012, http://ijr.sagepub.com/content/31/8/935.

[56] ——, “Preliminary deep water results in single-beacon one-way-travel-time acoustic navigation for underwater vehicles,” in *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, St. Louis, MO, Oct. 2009, pp. 2053–2060.

[57] S. E. Webster, J. M. Walls, L. L. Whitcomb, and R. M. Eustice, “Decentralized extended information filter for single-beacon cooperative acoustic navigation: Theory and experiments,” *Robotics, IEEE Transactions on*, vol. 29, no. 4, pp. 957–974, Aug 2013.

[58] S. E. Webster, L. L. Whitcomb, and R. M. Eustice, “Advances in decentralized single-beacon acoustic navigation for underwater vehicles: Theory and simulation,” in *Proceedings of the IEEE/OES Autonomous Underwater Vehicles Conference*, Monterey, CA, Sept. 2010.

[59] ——, “Preliminary results in decentralized estimation for single-beacon acoustic underwater navigation,” in *Proceedings of the Robotics: Science & Systems Conference*, Zaragoza, Spain, June 2010.