Probability Density Function Analysis for Optimization of Underwater Optical Communications Systems

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Abstract— **Underwater optical systems have real potential in the domain of wireless underwater communications. However, beam propagation through water is not well understood. While success has been achieved modeling the underwater channel using Monte Carlo methods, underwater beam propagation has, to our knowledge, yet to be fit to a true probabilistic model. This paper investigates the utility of the Lognormal, Weibull, and exponentiated Weibull probability density functions in modeling the underwater channel, using parameters developed for the free space domain. Beam propagation is tested under various salinity, temperature and turbidity conditions, and compared with brackish water from the Severn River.**

Index Terms—**Probability Density Function, underwater, optical communications, beam propagation**

I. Introduction

In the realm of underwater wireless communication, transmission options are extremely limited. Due to heavy absorption by water, electromagnetic communications in the radio frequency range are not viable, and acoustic communication methods, while effective, are limited in bandwidth to the kbps to low Mbps range [1]. Current demand for high bandwidth underwater communication systems is growing, and acoustic methods of transmitting data do not appear to be suited to shoulder the load.

The utility and effectiveness of laser based underwater optical wireless communication systems is well established. Numerous tests conducted in the past several years have proven the potential for high bandwidth, low bit error rate optical communication in the underwater domain. It has been shown that laser communication using 400-600 nm wavelength beams minimizes absorption by water [2-6]. Using beams in the specified range, it is possible to establish a communication link underwater with extremely high fidelity and data rate (rates into the Gbps range have been realized) [3-4].

However, laser propagation through water is generally not well understood. The underwater channel is inherently more unpredictable than the free space channel, in that factors such as turbidity, salinity and temperature can significantly affect the propagation of the beam [7-8]. Additionally, while beam propagation has been successfully modeled using Monte Carlo simulation (see [9] for example), it has not, as to our knowledge yet been fit to a probabilistic model.

Knowledge of the probability density function that describes beam propagation in water would provide valuable insight into the performance of an underwater wireless optical link. For a free space link, the probability density function is an important tool for measuring bit error rate and performance. In this project, we attempt to fit various probability density function models to an underwater link, and investigate the effect that water characteristics such as salinity have on these models.

We constructed a lab emulator for a short-range underwater optical wireless link, simply consisting of a laser transmitter, CMOS camera, and 1 meter long plexiglass water tank. Specific knowledge of the volume of water present allows for precise control of factors such as salinity and turbidity, and a warm water drip allows for introduction of a temperature gradient. Using the emulator, we can with reasonable accuracy model the water in a desired location around the world given characteristics such as salinity, turbidity and average temperature, and gain valuable insight into some of the scaled effects of the actual environment while in a controlled laboratory setting.

We discuss the accuracy of several probability density functions in the underwater environment; namely, the Lognormal, Weibull, and exponentiated Weibull distributions.

To prove the utility of these measurements in the real world, we conducted a series of tests on brackish water gathered at the United States Naval Academy, from the Severn River. We first took measurements of the various characteristics of the collected Severn water, specifically salinity and temperature gradient. Using the emulator, we then recreated these conditions in the laboratory, and compared them to the PDF models.

In our analysis, we made use of several parameters normally applied to probabilistic analysis of optical beam propagation in the free space domain. One of the main goals of this project was to determine the applicability of these parameters to the underwater regime. In the future, we plan to attempt to alter these parameters to better fit the underwater propagation model.

Knowledge of the probabilistic model for underwater optical beam propagation would provide significant insight into the performance of an underwater optical link. The ability to adjust the model to fit various salinities and turbidities from water around the globe would facilitate the efficient use of underwater optical communications more universally.

Additionally, the United States Navy is interested in this area of research. It envisions underwater optical communications systems being used by combat and salvage divers to communicate wirelessly and covertly while conducting operations underneath the surface. The currently feasible range for such devices is likely <100m, and this range is perfectly applicable to diver-to-diver communication. In the future, the Navy also hopes to develop optical communications for Unmanned Underwater Vehicle (UUV) control and wireless communication between submarines and surface ships. Knowledge of the probability density function associated with the underwater channel would greatly aid in the development of these technologies.

In the next section, we introduce the probabilistic models we investigate, Lognormal, Weibull, and exponentiated Weibull. These models were chosen due largely to their relevance to free space optical communication. In the following section, the testbed setup and measurement methods are detailed, and results for each probabilistic model under various laboratory conditions are shown and discussed. In the final section, conclusions are given, and a way forward is proposed.

II. Theoretical Background

One of the goals of this paper is to assess the utility of probability models used to assess Free Space Optics (FSO) links in assessing similar optical links underwater. Therefore, several definitions and beam propagation characteristics applicable to FSO are used, under the assumptions that they could apply to laser propagation underwater. One of the most important of these is scintillation index. In FSO, scintillation index is a mathematical description of the normalized atmospheric variations that affect the propagation of the beam, resulting from optical turbulence [10]. Such variations in free space are primarily the result of temperature variations in the atmosphere which cause small fluctuations in the index of refraction and subsequently cause optical turbulence. Mathematically, scintillation index is defined by

$$\sigma_I^2 = \frac{<I^2> - <I>^2}{<I>^2} = \frac{<I^2>}{<I>^2} - 1 \qquad (1)$$

Where I denotes irradiance of the beam and < > denotes an ensemble average. How we measure irradiance over time will be discussed in depth in section 3.3.

Research has proven that certain wavelengths of light propagate much more effectively through water than others. Specifically, lasers in the blue-green portion of the spectrum (470 – 570 nm) experience the smallest amount of scattering and absorption in water. For this paper, a 632 nm collimated beam generated by a 1.5 mW HeNe laser was used.

A. Probability Density Functions

The overall goal of this project was to investigate the utility of probability density function analysis in characterizing the underwater channel. Three separate distributions that are used for the FSO channel were analyzed; one model commonly used for weak fluctuation regime (Lognormal), and two recent models introduced to FSO analysis by Barrios et al [12] (Weibull and exponentiated Weibull).

1) Lognormal (LN)

The Lognormal distribution is a commonly used and accepted probabilistic model for characterization of the free space channel. Under first-order Rytov approximation, the logarithm of the irradiance is Gaussian distributed, so the overall distribution is said to be *Lognormal* [10]. The probability density function for this distribution is defined by

$$p_I(I) = \frac{1}{I\sigma_{\ln I}\sqrt{2\pi}} \exp\left(-\frac{(\ln I - \mu_{\ln I})^2}{2\sigma_{\ln I}}\right), \ I > 0 \qquad (2)$$

where I is irradiance. The σ term is the variance of the log-irradiance, and the μ term is the mean of the log-irradiance [11].

2) Weibull

Until fairly recently, the Weibull probability distribution was not widely used in description of the FSO channel. A 2013 paper first introduced it as a viable alternative to the generally accepted Lognormal and gamma gamma distribution [12]. It showed that the Weibull and exponentiated Weibull distributions offer an excellent fit for data under all turbulence conditions in the free space domain. The exponentiated Weibull was proven to show a near-perfect fit in the tail for irradiance data [12]. This paper aims to investigate whether these distibutions apply to the underwater regime. The Weibull probability density function is defined by

$$p_I(I) = \frac{\beta}{\eta}\left(\frac{I}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{I}{\eta}\right)\right]^{\beta} \quad (3)$$

where β and η are constants related to scintillation index, defined by

$$\sigma_I^2 \approx \beta^{-6/11} \quad (4)$$

$$\eta = \frac{1}{\Gamma(1+\frac{1}{\beta})} \quad (5)$$

where Γ represents the gamma function. β and η are both shape parameters used to describe beam propagation in the free space domain [12].

3) Exponentiated Weibull

The exponentiated Weibull distribution, also introduced by Barrios and Dios, may prove to be a unique and interesting model for the underwater channel. While complex, it may be useful in the inherently unpredictable underwater environment. The probability density function is defined by

$$p_I(I) = \frac{\beta}{\eta}\left(\frac{I}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{I}{\eta}\right)\right]^{\beta} \left\{1-\exp\left[-\left(\frac{I}{\eta}\right)\right]^{\beta}\right\}^{\alpha-1} \quad (6)$$

α, β and η are shape parameters [12] calculated using a fitting procedure to simulate data and relate α, β and η to the scintillation index. These parameters are defined by

$$\alpha \cong \frac{7.220\sigma_I^{2/3}}{\Gamma(2.487\sigma_I^{2/6} - 0.104)} \quad (7)$$

$$\beta = 1.012(\alpha\sigma_I^2)^{-13/25} + 0.142 \quad (8)$$

$$\eta = \frac{1}{\alpha\Gamma(1+\frac{1}{\beta})g_1(\alpha,\beta)} \quad (9)$$

In the calculation of η, a separate function, *g*, is used, which is defined by

$$g_n(\alpha,\beta) = \sum_{k=0}^{\infty} \frac{(-1)^k\Gamma(\alpha)}{k!\,(k+1)^{1+\frac{n}{\beta}}\Gamma(\alpha-1)} \quad (10)$$

which, converges very quickly. For data processing purposes, we use 10 terms to approximate g. The added shape parameter α controls lower tail steepness [12]. As α approaches 1, the exponentiated Weibull distribution becomes the standard Weibull distribution.

B. Least Squared Error for Assessing Fit

In assessing the various probabilistic models introduced above, we use least squared error (LSE) analysis to determine deviation of the data from the expected curve. In this report, we use both overall LSE of the curve and LSE in the tail. For communication systems, LSE in the tail of the curve is the determining factor in the accuracy of a probability model in predicting bit-error rate.

LSE for this project was calculated by using the value at the histogram bin center to calculate the probability of the PDF at the specified intensity value, and calculating the difference between the data value and the value of the PDF model at that point. This difference was then squared and summed across the number of bins to give the total LSE. For the tail LSE calculation, the LSE for the first 10 bins on the left side of the PDF function was calculated to show the quality of fit in the tail as compared to the quality of overall fit.

In all figures shown in Section IV, red dots indicate the values for the data histogram bin centers, and the curves indicates the calculated Probability Density Functions.

III. Testbed and Measurement Methods

In order to gather irradiance data in a coherent and reproducible way, a simple lab bench testbed was constructed wherein various characteristics of water could be measured and controlled. A 1 meter long plexiglass tank was used, so that the volume of water used could be precisely measured. The laser, a 1.5 mW HeNe along with a collimator, was placed at one end of the tank, and a CMOS camera (1280x1024 pixels) was placed at the other. Additionally, a red bandpass filter was affixed to the camera in order to minimize the effects of ambient light on the measurements, and a neutral density filter was also used to protect the camera from pixel saturation.

Another of the major goals of this project was to investigate the effect of temperature gradients on beam propagation using probabilistic models. To achieve such gradients, several droppers were affixed to the top of the tank, and warm water was added during data collection. This ensured even distribution of warm water vertically along the beam, to simulate minute temperature changes in water, proportional to depth. Thermocouples were distributed at varying depths of approximately 2, 4 and 6 cm at 4 locations, 20, 40, 60 and 80 cm along the length of the tank to measure the gradient. Figures 1 and 2 show the testbed setup.

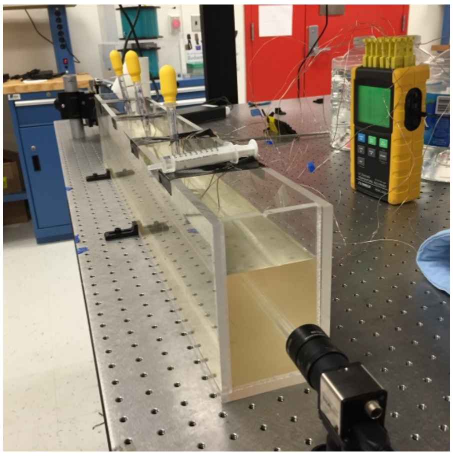

Figure 1: Testbed Setup

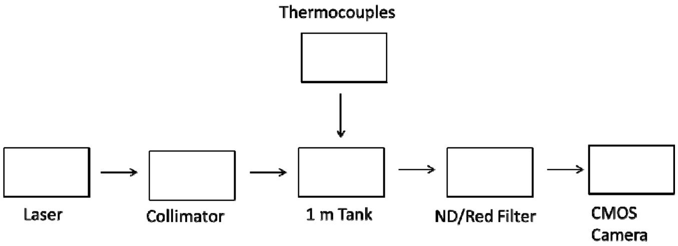

Figure 2: Block Diagram for Testbed

We measured the irradiance of the propagating beam by compiling average pixel brightness at the center of the beam. To ensure consistency, we use a 28x28 pixel box. For each condition, two separate 20-second runs were conducted, and the pixel brightness values averaged. The frame rate for the tests was 180 frames per second.

IV. Results and Discussion

A. Results for Distilled Water

To establish a baseline for standard water, we take a series of samples with simple distilled water. We also subtract ambient light from each trial, obtained by taking the average of the 28x28 pixel box with no laser illumination. The following figure is a time series of irradiance over 20 seconds for distilled water. Scintillation index here is almost negligible, on the order of 1e-5. The time series for this trial is shown in figure 3.

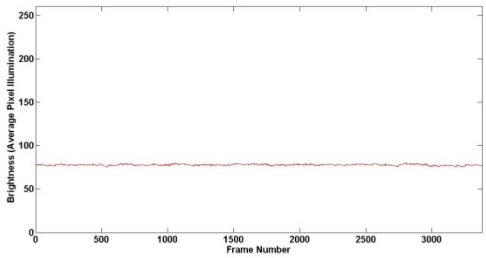

Figure 3: Time Series of Distilled Water Data

Probability analysis shows the Lognormal distribution to be most accurate for such a low turbulence value. The PDF-histogram plot is shown in Figure 4.

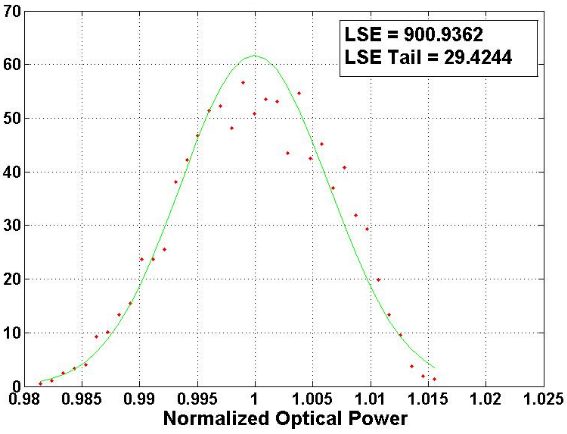

Figure 4: Lognormal Distribution for Distilled Water Irradiance

However, the distilled water trials become more interesting when a temperature gradient is introduced. Using the warm water drip, a gradient of 0.2 °C was created from top to bottom starting 1.5 cm above the beam and ending 1.5 cm below it. Scintillation index increased by a factor of 100, to 0.0011. Figures 5-7 show the PDF-Histogram plots for the distilled water condition.

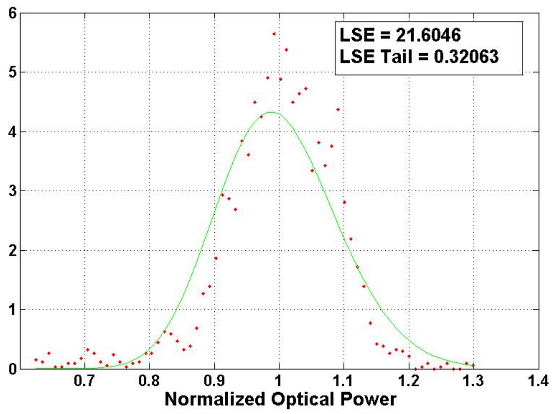

Figure 5: Lognormal distribution for Distilled Water with Temperature Gradient

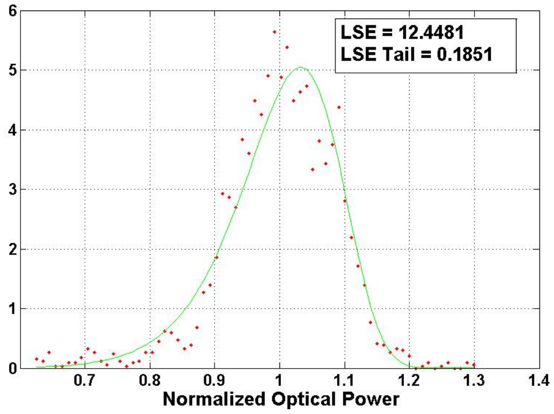

Figure 6: Weibull distribution for Distilled Water with Temperature Gradient

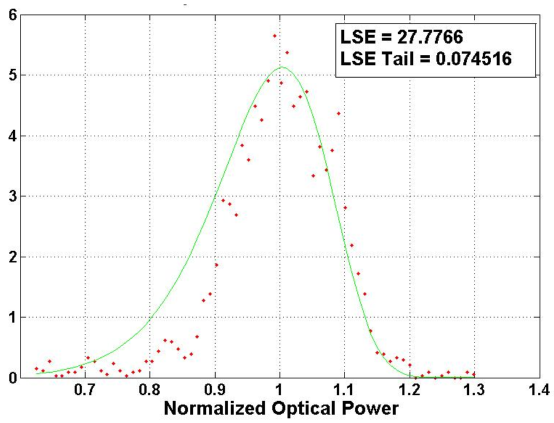

Figure 7: Exponentiated Weibull distribution for Distilled Water with Temperature Gradient

As shown in figures 5-7, in the case of distilled water with a slight temperature gradient, the Weibull and exponentiated Weibull distributions outperform the Lognormal distribution in the tail. The Weibull distribution showed the best fit, and the exponentiated Weibull showed similarly good performance in the tail, but had a higher overall LSE than the Lognormal and the Weibull distributions.

B. Results for Severn Water

Analysis of brackish water from the Severn River at the United States Naval Academy in Annapolis, MD showed far more interesting results than simple distilled water. Without a temperature gradient, for an average over two trials, the scintillation index was 0.00215. Figures 8-10 show the PDF-histogram plots for Severn water without an introduced temperature gradient. The unit used to measure salinity is the Practical Salinity Unit (PSU), which is equivalent to 1 gram of salt per kilogram of water. The turbidity unit used is the Nephelometric Turbidity Unit (NTU), which is a measure of the amount of light scattered by an incident beam by particles suspended in water. The salinity of the water at the time the samples were taken from the river was 12 PSU, and the turbidity 1.2 NTU.

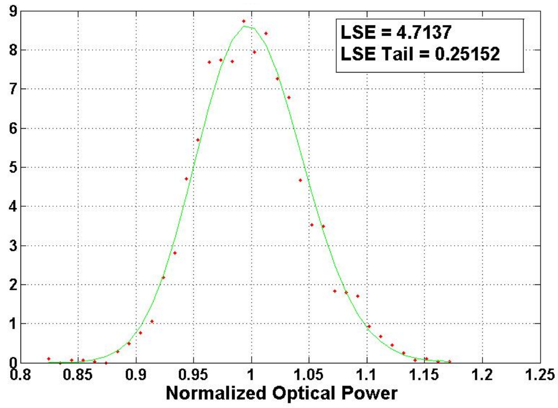

Figure 8: Lognormal distribution for Severn Water

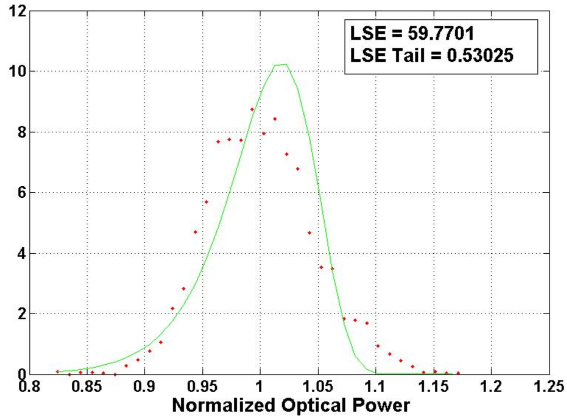

Figure 9: Weibull distribution for Severn Water

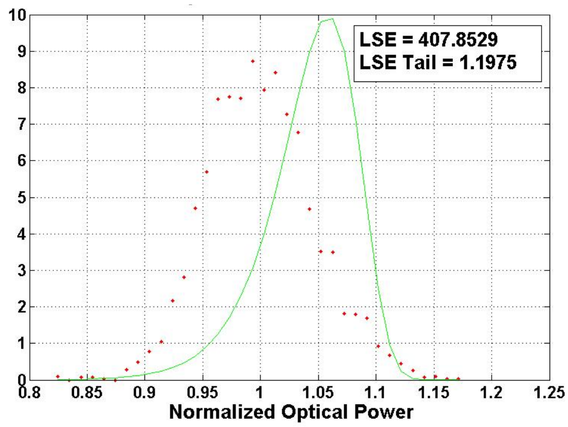

Figure 10: Exponentiated Weibull distribution for Severn Water

As shown in Figures 8-10, in Severn River water, the Lognormal distribution clearly outperformed the Weibull and exponentiated Weibull distributions. While all three were acceptable accurate in the tail, the overall fit for the LN distribution is better than the Weibull distribution by a factor of 10, and better than the exponentiated Weibull by a factor of 100.

With an added temperature gradient, the scintillation index increased to 0.0131. This condition is illustrated in Figures 11-13.

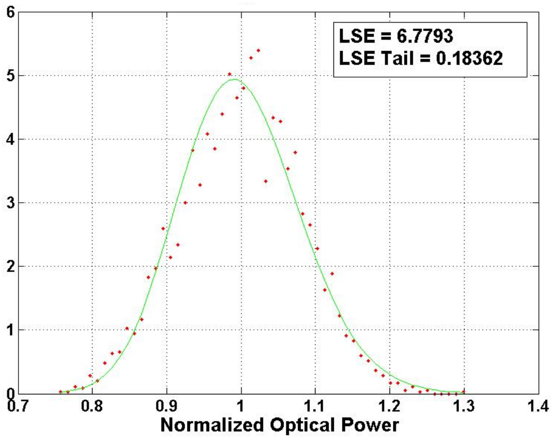

Figure 11: Lognormal distribution for Severn Water with Temperature Gradient

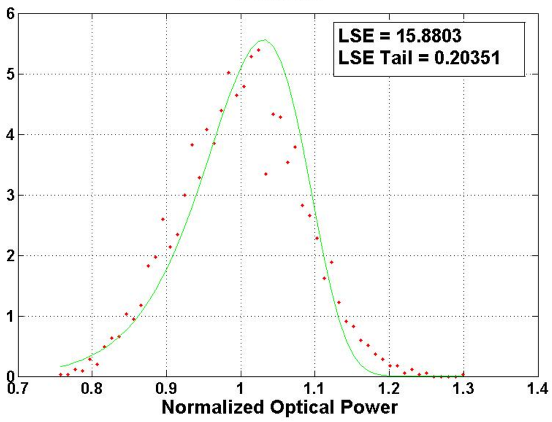

Figure 12: Weibull distribution for Severn Water with Temperature Gradient

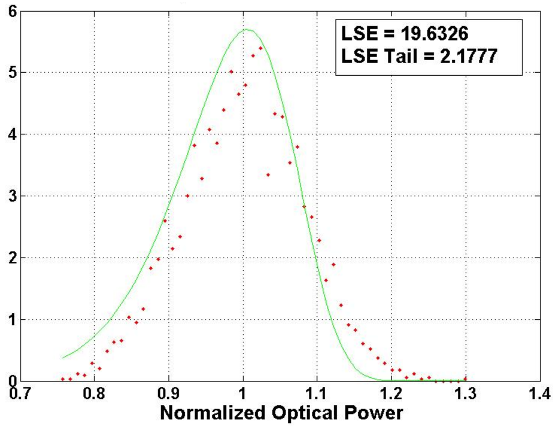

Figure 13: Exponentiated Weibull distribution for Severn Water with Temperature Gradient

In the case of an added temperature gradient, all distributions show promise. Lognormal was again narrowly the best performer, but both Weibull and exponentiated Weibull showed a high quality fit for the data. Figure 14 shows the each PDF for Severn water with a temperature gradient overlayed on the same graph.

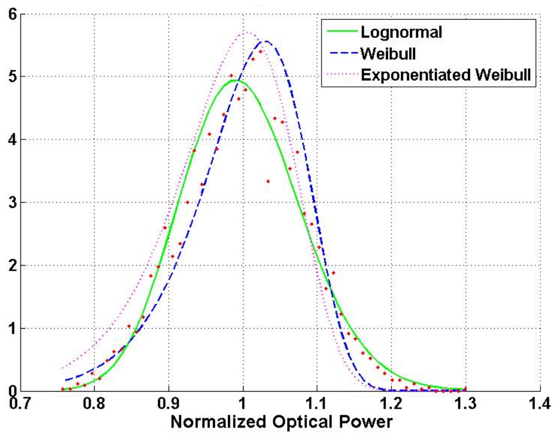

Figure 14: Overlay of 3 PDFs for Severn Water with Temperature Gradient

From the figure, it is clear that while all PDFs show promise, especially in their tails, the Lognormal PDF is the best overall fit for the dataset.

C. Results for Simulated Severn Water

The final major goal of this project was to create an emulator where water from around the globe could be simulated under laboratory conditions. In order to simulate the salinity of Severn water, a proportionate amount of salt was added to distilled water to achieve a nominal salinity of 12 PSU, to match that of the actual Severn on the day the sample was taken.

From the analysis, it appears that salinity in and of itself is not a major factor in determining laser propagation through water. The Lognormal distribution, again the most accurate, was nearly identical to the one seen with distilled water, and scintillation index was similar to that seen with distilled water, at 9.0275e-05. Figure 15 shows the PDF-histogram plot for this dataset.

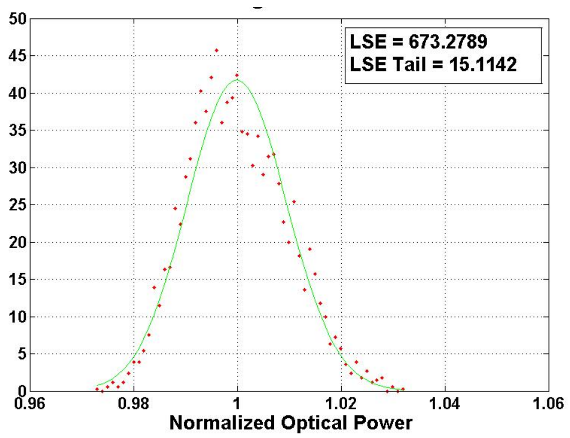

Figure 15: Lognormal distribution for Simulated Severn Water (12 PSU)

With an added temperature gradient, however, scintillation index again increased significantly, to 0.0101. Figures 16-18 show the PDF-histogram plots for this condition.

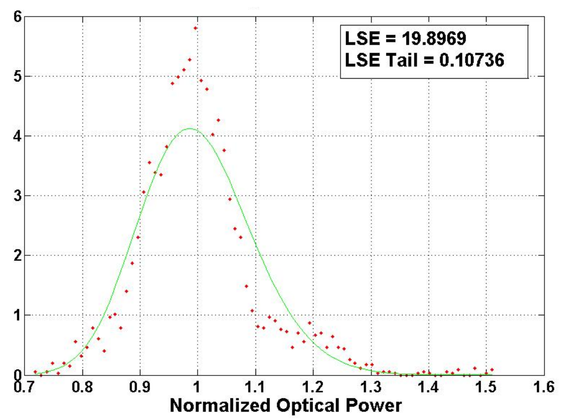

Figure 16: Lognormal distribution for Simulated Severn Water (12 PSU) with Temperature Gradient

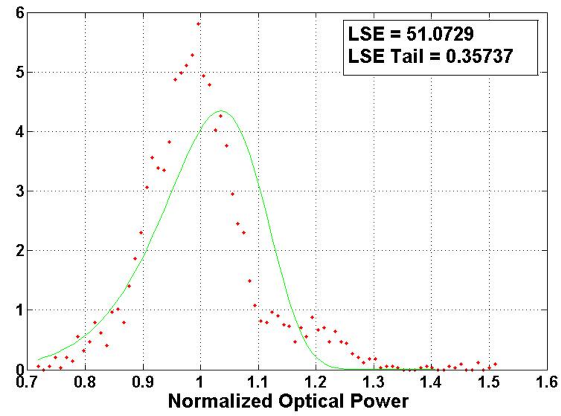

Figure 17: Weibull distribution for Simulated Severn Water (12 PSU) with Temperature Gradient

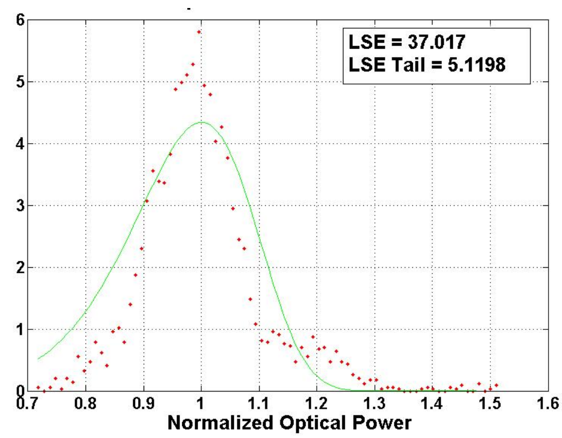

Figure 18: Exponentiated Weibull distribution for Simulated Severn Water (12 PSU) with Temperature Gradient

These results closely mimic those seen with actual Severn water, with Lognormal still being the most effective, but Weibull and exponentiated Weibull showing promise.

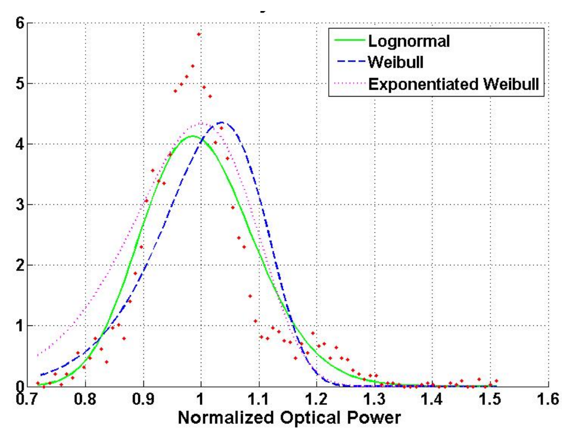

Figure 19: Overlay of 3 PDFs for Simulated Severn Water with a Temperature Gradient

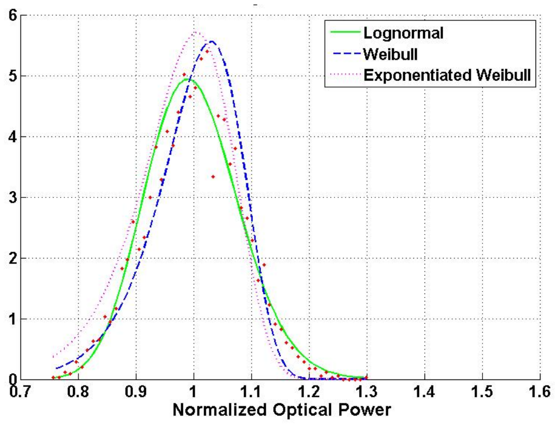

Figure 20: Overlay of 3 PDFs for Actual Severn Water

Figures 19 and 20 show all 6 PDF models for both the simulated Severn water and actual Severn water on the same scale. Figure 20 is a repeat of Figure 14, shown here again for clarity. Qualitatively, these functions and their associated histograms are very similar. The histograms for both the emulator and the actual Severn water are nearly identical. The PDFs are very similar in shape, but the emulator model shows slight deviation from the histogram values due to possible pinching localized to that particular data run.

V. Conclusions

In general, the Lognormal distribution appears to show the most promise for underwater propagation. The Weibull and exponentiated Weibull distributions performed well in some cases, but are not well suited to the modeling of extremely low turbulence situations.

However, both the Weibull and exponentiated Weibull distributions show promise for application in high turbulence situations, most notably when dealing with an introduced temperature gradient. Additionally, the Weibull and exponentiated Weibull distributions are often as effective or more effective than the Lognormal distribution in the tail. The parameters derived in [12] were derived for the free space channel, and with modifications towards the underwater channel could prove effective in more accurately predicting beam propagation through water. This is one area of potential future investigation.

Acknowledgment

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