Matched field processing under posterior probability constraints *

Guolei Zhu, Yingmin Wang, Qi Wang
College of Marine Engineering
Northwestern Polytechnical University
Xi'an, China

***Abstract*—When under uncertain ocean environment, the environmental model of matched field processing (MFP) is different from the real world. The performance of MFP decreases largely beacuse of the environmental mismatch. In order to improve its robustness to the environmental mismatch, a MFP with posterior probability constraints (MFP-PPC) is proposed. The algorithm derives the posterior probability density function (PDF) of the source locations from Bayesian Criterion, then the main lobe of AMFP is protected by posterior PDF, so the MFP-PPC has not only the merit of high resolution as AMFP, but also the advantage of robustness. We use experimental data to evaluate the algorithm, the results shown that in the uncertain ocean environment the MFP-PPC is robust not only to the moored source, but also to the moving source. Meanwhile, the localization and tracking is consistent with the trajectory of the moving source.**

Keywords—acoustics; matched field processing; posterior probability density; robustness

I. INTRODUCTION

Matched field processing (MFP) technology can combine the physical characteristics of underwater acoustic channel and the traditional signal processing algorithm successfully, thus is widely used in the underwater target passive location and also used in the underwater acoustic parameters inversion, etc. The main method is to use sound propagation model [1](such as the normal mode model, the parabolic equation, and the ray, etc.) to build a replica vector of the field in the observation sea area, and then match the measured field with the replica to estimate the location of the source or the channel information [2]. MFP is mainly divided into two categories: one is the conventional matching field processor (Conventional MFP, CMFP), also known as the Bartlett processor; another is adaptive matched field processor (Adaptive MFP, AMFP). Usually, CMFP is robust than AMFP, but AMFP provides maximum array gain and location accuracy in theory.

MFP, however, has the intrinsic defects of poor robustness, when the environment parameters are uncertain, i.e. when the underwater acoustic channel model with which we calculate the replica is different with the actual conditions, result in the MFP worse or even failure. Real shallow water environment influenced by the wind and waves, the internal waves and suspended matter in sea water, is a complex sound propagation channel with time-varying and space-varying. Especially in shallow water environment, the underwater terrain has more complex characteristics, and the artificial factors such as ships make the underwater acoustic channel become more complex and changeable, these uncertainties are greatly restricted the application of MFP.

This paper was partially supported by the Northwestern Polytechnical University Foundation for Fundamental Research (NPU-JC20110211).

In view of the uncertainty of shallow water environment, the domestic and foreign scholars have proposed many tolerance MFP method, such as: Multiple Constraint Method(MCM) [3], the Optimum in all Field Processor(OUFP) [4], Sector Focusing(SF) [5], etc.

MCM is equivalent to linear constrained minimum variance beamformer in array signal processing, it's a kind of AMFP with adding multiple linear constraint to the weight vector, to provide protection of the main lobe when environmental mismatch or array perturbation. The Minimum Variance-Neighborhood Location Constraints (MV-NLC) MFP and Minimum Variance-Environmental Perturbation Constraints (MV-EPC) MFP are derived from the MCM.

They use two way to improve the robustness of the algorithm: one is constructing perturbed environmental parameters of the model in a certain range to protecting the main lobe for MFP; another way to obtain more robust performance is by doing the probability estimation under the environment parameter perturbation model.

Therefore, in this article firstly we make signal vector model under uncertain environment, then using Bayesian criterion to deduce posterior probability estimates of location parameters, which combined the CMFP and environmental perturbation AMFP together by the posterior probability constraints, and obtained good tolerance in uncertain environment. Finally, through the processing and analysis with the ocean experimental data from SACLANT research center in 1993, making comparison of the CMFP, AMFP and MFP-PPC location capability, verifing the validity and robustness of the MFP-PPC algorithm in the uncertain environment.

II. ROBUST MFP WITH POSTERIOR PROBABILITY CONSTRAINTS

The data vector of measurement field which is received by vertical linear array of N sensors in the frequency domain can be expressed as

$$x(\omega) = a(\omega)s(\omega, m, \psi) + n \quad (1)$$

When doing MFP, we start with calculating the sampling covariance matrix by measuring field data.

$$R = E\{XX^H\} \quad (2)$$

According to the bayesian criterion, the posterior PDF of the sound source location parameters can be expressed as:

$$p(m \mid x) = \frac{p(x \mid m)p(m)}{p(x)} \quad (3)$$

Where, $p(m)$ and $p(x)$ are respectively, the probability density function(PDF) of the sound source location m and the PDF of the measurement field x. When the measurement field x is known, conditional probability density $p(x \mid m)$ is the function of the location parameter m.

For the data model shown in Eq.(1), because the noise of every element of the vertical linear array of N sensors is additive white gaussian noise with independent identically distribution, so we can got the conditional probability density $p(x \mid m)$, that is the likelihood function can be expressed as below:

$$p(x \mid m) = \frac{1}{(\sqrt{2\pi}\sigma_n)^N} \exp\left[-\frac{|x - ad(m)|^2}{2\sigma_n^2}\right] \quad (4)$$

Where, $d(m)$ is the replica vector when locate at m.

Let $\partial p(x \mid m)/\partial a = 0$ we got the maximum likelihood estimation of a shown as below:

$$\hat{a} = \frac{x^H d(m)}{|d(m)|^2} \quad (5)$$

Substituting Eq.(5) into Eq.(4) yields the likelihood function:

$$p(x \mid m) = \frac{1}{(\sqrt{2\pi}\sigma_n)^N} \exp\left[-\frac{[1 - B_C(m)]|x|^2}{2\sigma_n^2}\right] \quad (6)$$

Where:

$$B_C(m) = \frac{|x^H d(m)|^2}{|x|^2 \cdot |d(m)|^2} = \frac{d^H R(x) d}{|x|^2 \cdot |d(m)|^2} \quad (7)$$

The $B_C(m)$ in Eq.(7) is normalized linear Bartlett correlation.

Now, we can get the posterior PDF by bayesian criterion:

$$p(m \mid x) = \frac{1}{(\sqrt{2\pi}\sigma_n)^N} \exp\left\{-\frac{[1 - B_C(m)]|x|^2}{2\sigma_n^2}\right\} \cdot \frac{p(m)}{p(x)} \quad (8)$$

In white gaussian noise environment, from the frequency snapshot model shown in Eq.(1), because:

$$\frac{|x|^2}{\sigma_n^2} = \frac{(as + n)^H \cdot (as + n)}{\sigma_n^2} = \frac{\sigma_s^2 + \sigma_n^2}{\sigma_n^2} = SNR + 1 \quad (9)$$

Where $SNR = \sigma_s^2 / \sigma_n^2$, σ_s^2 and σ_n^2 are respectively, the power of signal and noise. Substituting Eq.(9) into Eq.(8) yields:

$$p(m \mid x) = \frac{p(m)/p(x)}{(\sqrt{2\pi}\sigma_n)^N} \exp\left\{-\frac{[1 - B_C(m)](SNR + 1)}{2}\right\} \quad (10)$$

Assume that in the observation sea area, the appearance probability of the target is equal, that is $p(m)$ is constant. When the prior PDF $p(x)$ is unknown, in order to ensure the integral of the posterior PDF equal to 1, we defined a normalization constant C_x, then the Eq.(8) can be rewritten as:

$$p(m \mid x) = C_x \exp\left\{-\frac{[1 - B_C(m)] \cdot (SNR + 1)}{2}\right\} \quad (11)$$

Clearly, Eq.(11) shows that the posterior PDF is the function of sound source position parameter m and it's monotonically increasing when the output of the MFP is increasing.

Because of the posterior PDF $p(m \mid x)$ represents the appearance probability of the target at position m, so after we got the $p(m \mid x)$ by measurement field and priori knowledge, we can construct MFP as below, named as MFP with posterior probability constraints(MFP-PPC):

$$B_{PPC} = p(m \mid x) \cdot B_A = C_x \exp\left\{-\frac{[1 - B_C(m)] \cdot (SNR + 1)}{2}\right\} \cdot B_A \quad (12)$$

Where B_A is the output power of the MV-EPC:

$$B_A = \frac{1}{d^H R^{-1} d} \quad (13)$$

III. VERIFICATION BY OCEAN EXPERIMENTAL DATA

The ocean near the north island of Elba of the west coast of Italy is a typical shallow water environment, On October 26 and 27, 1993, the SACLANT research center doing ocean experiment for two days at the ocean[6]. in this paper, we use the experimental data to do processing and analysis. The array and environment parameters for the experiment is described in figure 1. The bathymetry (125.5-128.5m); upper sediment sound-speed (1450-1550m/s); lower sediment sound-speed (1500-1600m/s); sediment density (1.2-2.2 g/cm3); sediment attenuation (0.0-0.4dB/λ); sediment thickness (0.0-0.6m); sub-bottom sound-speed (1550-1650m/s); sub-bottom density (1.2-2.2 g/cm3); sub-bottom attenuation (0.0-0.4dB/λ).

During the experiment on October 26, sound source moored in 5500±200m distance from the receiving array, the depth is about 79m. On October 27, sound source was towed by a tugboat with 3kn speed, navigation in a straight line. the depth is about 65m, the initial distance from the receiving array is about 5.9km. The experiment collect the signals receiving from the linear array for 10 minutes. During the two days experiment, the acoustic signal model are slightly different, on October 26, the signal is continuous signal, on October 27, the signals were not continuous, they were on for

30 out of every 60 sec. we use the experimental data to verify the performance of the MFP-PPC.

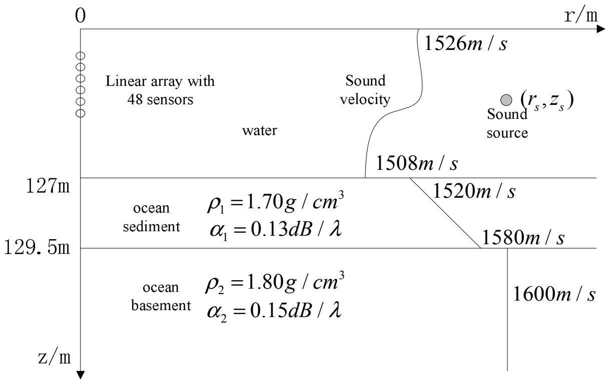

Fig. 1. Environmental model in shallow water

According the Eq.(2), we use the data of first 25 seconds to make 25 snapshots, the average of these snapshots is the maximum likelihood estimation of sampling covariance matrix. The length of the snapshot is 2 seconds, the overlap between the two snapshots is 50%, for 1kHz sampling rate, each snapshot contains 2000 sampling points.

The location result of the moored source at the first minute is shown in figure 2. The rectangle area is the position of the peak amplitude on the ambiguity surface, it is also the location result of the MFP, the ellipse area is the position of real target, we only draw a rectangle when the ellipse and rectangle overlapped. If the peak amplitude position of the ambiguity surface is near the real target(depth around ±10m, distance around ±500m), that suggests the localization success.

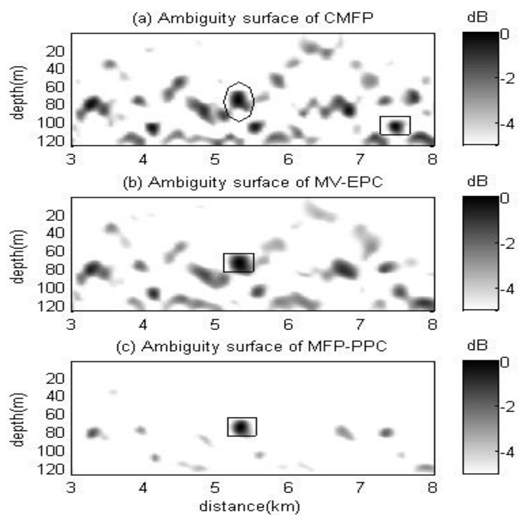

Fig. 2. Ambiguity surfaces of moored source at the first minute

It is clearly that the location result of CMFP is wrong in figure 2(a), the error result is (104.7m, 7450m), there also has peak at the real position. The location result (73.5m,5300m) of MV-EPC is shown in figure 2(b), it is correct, but the amplitude of the sidelobe is a little bit higher, only around 1dB lower than the mainlobe. Figure 2(c) is the ambiguity surface of MFP-PPC, we can clearly find the target and there is no confusion between the target and sidelobe.

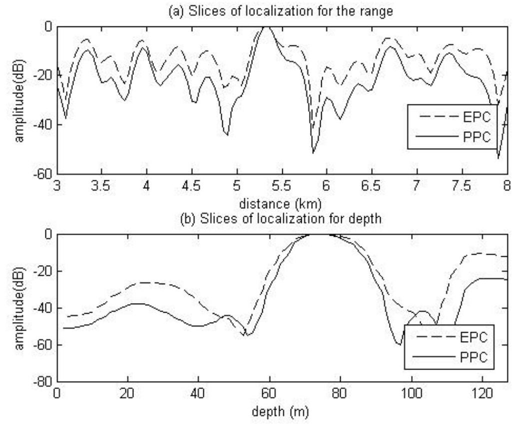

Fig. 3. Slices of the moored source at the first minute

The slices of the moored source at the first minute is shown in figure 3, under the real uncertain environment, the CMFP is failed to localization, thus we only compare the results of MV-EPC and MFP-PPC. From the figure, it is clear that the MFP-PPC has the more narrow mainlobe and lower sidelobe. The location results of 2 to 10 minutes are similar to the result of first minute, the main difference is that the CMFP give the correct results at 2,5,7,8 minutes, the location results are all around(76.1m,5300m). Both MV-EPC and MFP-PPC give the correct results during the 10 minutes, the results are same as in first minute.

In order to make quantitative evaluation for the three kinds of MFP, we introduce two parameters for performance evaluation, one is signal to interference noise ratio(SINR), the other is peak to background ratio(PBR). The SINR quantize the differentiation between the mainlobe and sidelobe, it will be easier to distinguish mainlobe from sidelobe when the SINR is bigger; the PBR quantize the differentiation between the mainlobe and background, it will be easier to find out the mainlobe when the PBR is bigger.

We make quantitive comparison for the location results of the three kinds of MFP at the second minute in tabel 1. For the depth localization, the result of CMFP is more close to the target depth, but the result difference to the other two processor is only a depth of a grid. For the distance localization, the results are same for the three processor, but the SINR and PBR of the MV-EPC are about 1dB higher than the CMFP, and the SINR and PBR of the MFP-PPC are about 2.5dB higher than the MV-EPC. From the numerical results of the moored source at the second minute, we can see that MFP-PPC has optimal localization performance, then the MV-EPC, the CMFP is the worst.

TABLE I. RESULTS OF THE MOORED SOURCE AT THE SECOND MINUTE

Processor	**Location results**			
	z(m)	***r(m)***	***SINR(dB)***	***PBR(dB)***
CMFP	76.1	5300	10.70	5.41
MV-EPC	74.8	5300	11.57	6.49
MFP-PPC	74.8	5350	14.32	9.16

The curve of localization results of the moving source in 10 minutes on October 27 is shown in figure 4. During the 10 minutes, the CMFP give the correct results at 2,3,5,6,7,8 minutes, other time failed, the error localization results are not shown in figure 4. The MV-EPC and MFP-PPC give the correct results in the whole 10 minutes, we can see that the motion curve of MFP-PPC more like a uniform linear motion as shown in the figure 4, the slope of the curve of MFP-PPC is about 1.6m/s, the MFP-PPC is the best consistent with the speed and trajectory of the moving source.

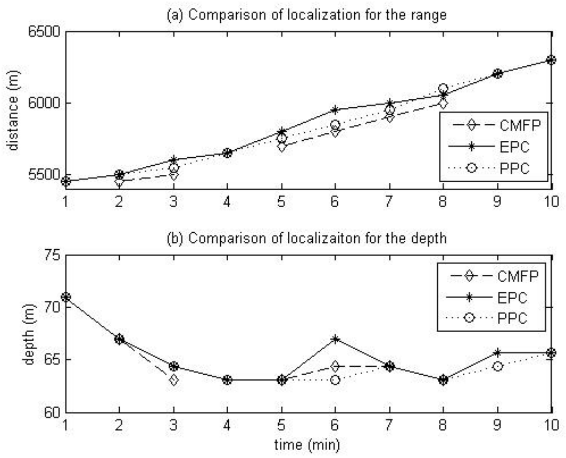

Fig. 4. Tracking of the moving source in the whole ten minutes

The distance localization result is around 500m closer than the real target as shown in figure 4(a), it is perhaps that the water depth around source is 13m deep than 127m, it is discussed in detail in the article[6], the distance localization result of this article is equal to the article[6]. In figure 4(b), the depth localization results of the three processors at first minute are around 71m, 6m error, at 6th minute, the error of MV-EPC is about 3m, the error of the other two processor is 1m, at the other time, the depth localization error of the three processors are all within 2m. Because the moving source is towed behind the ship, for the first and second minutes, at the beginning of the movement, the tension of the cable which connect the source is too small and unstable, thus causes the deep source. During this period, the depth localizaiton results of the three processors are same, so we can consider that the depth of source is about 71m. For the 6th and 9th minute, the tugboat moving smoothly, the depth localization difference of the three processors is mainly due to the ups and downs of sea waves; the uncertainty of the environment and array element position error. Under this condition, the localization performance is heavily depends on the robustness of the algorithm. It is clearly that the localization result of MFP-PPC is more accurate than MV-EPC, the CMFP is failed at 9th minute.

The SINR curve for the moving source in the 10 minutes is shown in figure 5(a), when give the correct localizaiton results, the SINR of MV-EPC is about 1.5dB higher than CMFP, the SINR of MFP-PPC is about 3dB higher than MV-EPC. The PBR curve for the moving source in the 10 minutes is shown in figure 5(b), the PBR of MV-EPC is about 1.7dB higher than CMFP, the PBR of MFP-PPC is about 3dB higher than MV-EPC. So not only it is easiest to distinguish mainlobe from sidelobe by the MFP-PPC, but also the mainlobe and background difference is the largest of MFP-PPC. It is clearly that the MFP-PPC has the best localization performance as shown in figure 5.

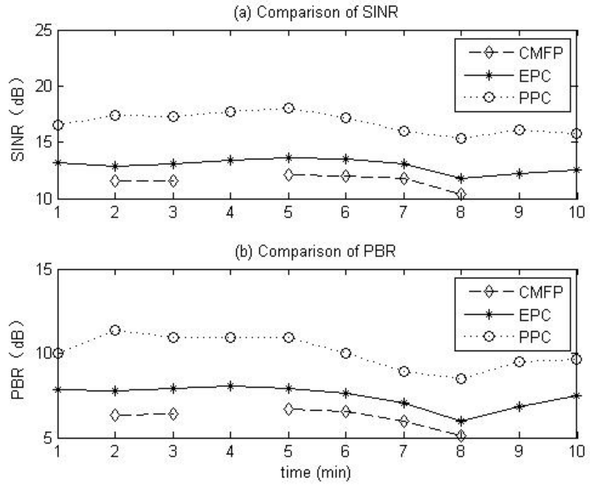

Fig. 5. Localization performance for the moving source

IV. CONCLUSIONS

Because of the uncertain of priori environment information of the shallow sea, such as array element position ups and downs with the waves and tides and target motion, these make mismatch between the constructed environment model and the real environment occur easily, finally lead to serious loss of performance of the matched field processing. In order to solve the above problem this paper proposes a posterior probability constraints matched field processing method. From the formulation of the algorithm, the posteriori probability estimation of the position parameters is a monotone increasing function of the power output of the conventional linear matching field processor, thus the essence of the algorithm is to use linear matching field processor to provide protection of the main lobe for adaptive matched field processor. Simultaneously, posteriori probability constraints can be used widely, using linear matching field processor to restrict the environmentally perturbed matching field processor is just a special case of the algorithm in this paper.

For typical uncertain shallow environment, the analysis result of ocean experimental data of the moving sound source showed that: MFP-PPC can restrain location errors which are caused by physical factors like the wind and waves, the array element position, and movement of the source, especially in the process of tracking for 10 minutes, the range and position curve accord well with the moving trail of the sound source.

REFERENCES

[1] Porter M B, The KRAKEN Normal Mode Program, SACLANT Undersea Research Centre, La Spezia Italy, 1992.

[2] Baggeroer A B, Kuperman W A, Mikhalevsky P N. An overview of matched field methods in ocean acoustics [J]. Oceanic Engineering, IEEE Journal of, 1993, 18(4): 401-424.

[3] Schmidt H, Baggeroer A B, Kuperman W A, et al. Environmentally tolerant beamforming for high-resolution matched field processing: Deterministic mismatch [J]. The Journal of the Acoustical Society of America, 1990, 88(4): 1851-1862.

[4] Richardson A M, Nolte L W. A posteriori probability source localization in an uncertain sound speed, deep ocean environment [J]. The Journal of the Acoustical Society of America, 1991, 89(5): 2280-2284.

[5] Chandler H A, Feuillade C, Smith G B. Sector-focused processing for stabilized resolution of multiple acoustic sources [J]. The Journal of the Acoustical Society of America, 1995, 97(4): 2159-2172.

[6] Krolik J L. Robust matched-field beamforming with benchmark shallow-water acoustic array data. Acoustics, Speech, and Signal Processing, 1996. ICASSP-96. Conference Proceedings., 1996 IEEE International Conference on, 1996, 1185-1188 vol. 2.