Reconstruction of acoustic fields based on the least squares for short vertical linear array*

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***Abstract*—In order to sample the underwater acoustic fields sufficiently the receiver hydrophones array used must cover the entire water column. For short arrays the samples are usually insufficient, so an approach called reconstruction of acoustic fields based on the least squares algorithm for short vertical linear array is proposed. This method decomposes the received fields of short array into depth function matrixes and amplitudes by the normal mode propagation model in the shallow water, then the mode amplitudes are estimated using the least squares in the sense of minimum norm, after that the received fields of shallow water are reconstructed which contain more environmental information. In the end, some computer simulations which use different short vertical linear arrays are processed in the classical Pekeris waveguide. Especially, the mode amplitudes of short arrays estimated by the least squares are compared with the ideal ones, which are the basis of fields reconstruction. The results show that the reconstruced ones with short arrays are much like the fields received with the long array, and the sample deficient problem of short vertical linear array is addressed properly.**

Keywords—acoustic fields; short arrays; shallow water; normal mode; reconstruction

I. INTRODUCTION

In recent years, the algorithms likes matched field processing, virtual time reversal mirror and so on have achieved high performace in numerous domains[1, 2]. But as we all know that these algorithms which make use of the acoustic fields usually need to sample the acoustic fields sufficiently, that is the receiver hydrophones array used to sample the fields must cover the whole water column. Even if the environment is shallow water, the array is still about one hundred meters. Such large layout of hydrophones array hinders its engineering application seriously[3].

To overcome this shortcoming, the normal mode propagation model in the shallow water is used to describe the underwater acoustic fields received by receiver arrays. Then, analysing the components of acoustic fields, we assume that the acoustic fields can be decomposed into the depth function and mode amplitudes. Actually, this is a modal decomposition method in assumption that both the magnitude and phase of all the mode amplitudes can be measured from the pressure field data received on an hydrophone array, in other words, either the real or imaginary parts of mode amplitudes are implied in the received pressure fields[4]. At this point, we make some work on extracting the mode amplitudes from the received fields of short arrays by the least squares. After the mode amplitudes are estimated, the fields of the entire water column can be reconstructed, and an approach called reconstruction of acoustic fieldsbased on the least squares algorithm is proposed.

In principle, the key point of the proposed one is extracting the mode amplitudes. As previous studies, the conventional extracting method is the least squares, by which the mode amplitudes have been estimated by computing the pseudoinverse directly, but it can find the best fit only when the number of hydrophones is lager than the mode number, i.e., it can only work well for overdetermined systems. Yang made some efforts by the eigenvector mode decomposition method[4], in his works, the mode amplitudes were divided into two parts by the orthonormal eigenvecotrs, one part was projected into the mode signal space that generally correspond to the lower order modes, and the second part was projected into the mode noise space that was related to the higher order modes, his method is reliable for long array and for the case where the higher order modes are negligible. In parallel, Wilson et al. also had done an extensive amount of work to extract mode amplitudes by a QR orthogonal decomposition of the mode function matrix[5], their technique was more accurate than conventional ones which computed the pseudoinverse for solving underdetermined systems, but was also not reliable and applicable for short arrays.

The proposed one in this article uses the least squares in the sense of norm to extract the mode amplitudes, and it is different with the conventional least squares, meanwhile it is suitable for short arrays. The last parts of this article are organized as follows: Section II formulates the acoustic fields by the normal mode, and reconstructs the received fields of short arrays, Sec.III estimates the proposed method using simulate data, and discusses the results, Sec.IV is a summary.

II. THEORETICAL FORMULATION

In the past several decades, in order to discuss the sound propagation through the ocean, some propagation models have been proposed by researchers in the world, like the ray theoretical models, the normal mode solutions, the multipath expansion techniques, the fast field theory, and the parabolic approximation approach[6-9]. All of these models require lots of environmental parameters as input, and each of them has a unique domain of applicability that is restricted by terms of acoustic frequency and other environmental complexity. In this paper, the environment considered is a classical range independent shallow water, and the acoustic source is low frequency, so the normal

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mode is adopted to model the sound propagation in the water[10].

A. Normal model in shallow water

At the beginning, the nomal mode is used to describe the oceanic acoustic fields in the shallow water, by which the fields at each frequency can be expressed as a sum of individual modal contributions:

$$p(r,z)=\sum_{m=1}^{M}p_m(r,z)=\frac{je^{-j\pi/4}}{\rho(z_s)\sqrt{8\pi}}\sum_{m=1}^{M}\frac{\varphi_m(z)\varphi_m(z_s)e^{j\xi_m r}}{\sqrt{\xi_m r}} \quad (1)$$

Where $p(r,z)$ is the acoustic pressure field at the location (r,z), and $\rho(\bullet)$ is the density of the water, z_s is the depth of source, $\varphi_m(\bullet)$ is the mode depth function of the mth mode, ξ_m is the horizonal wave number, M is the number of normal mode.

The Eq.(1) can be written simply as:

$$p(r,z)=\sum_{m=1}^{M}d_m(r,z_s)\varphi_m(z) \quad (2)$$

Where $d_m(r,z_s)$ is the mode amplitudes of the mth mode:

$$d_m(r,z_s)=\frac{je^{-j\pi/4}}{\rho(z_s)\sqrt{8\pi\xi_m r}}\varphi_m(z_s)e^{j\xi_m r} \quad (3)$$

By this way, the received acoustic fields are decomposes into depth function and amplitudes of mode. Particularly, we found that no matter which receiver arrays are adopted, the amplitudes of normal mode in a range independent ocean waveguide are all the same. So the received acoustic pressure fields of a long array can be reconstructed with that of short vertical linear array.

B. Acoustic field reconstruction

For a receiver array of N hydrophones which covers the whole water column, the fields as Eq.(2) can be expressed as follows:

$$\boldsymbol{p}_N=\boldsymbol{\varphi}_{NM}\boldsymbol{d}_M \quad (4)$$

Where $\boldsymbol{p}_N$ represented the pressure fields of N hydrophones. When N is larger than M, the mode amplitudes $\boldsymbol{d}_M$ can be estimated by computing the pseudoinverse directly using the least squares:

$$\boldsymbol{d}_M=(\boldsymbol{\varphi}_{NM}^H\boldsymbol{\varphi}_{NM})^{-1}\boldsymbol{\varphi}_{NM}^H\boldsymbol{p}_N \quad (5)$$

This method of computing the mode amplitudesworks well for overdetermined systems. For a short array of L hydrophones,the equation (4) is underdetermined when $L<M$. Meanwhile, considering the perturbation of environment and received fields, the problem can be described as follows:

$$\min_{\Delta\boldsymbol{\varphi}_{LM},\Delta\boldsymbol{p}_L,\boldsymbol{d}_M}\left\|[\Delta\boldsymbol{\varphi}_{LM},\Delta\boldsymbol{p}_L]\right\|_2^2 \text{ s.t. } \boldsymbol{p}_L+\Delta\boldsymbol{p}_L=(\boldsymbol{\varphi}_{LM}+\Delta\boldsymbol{\varphi}_{LM})\boldsymbol{d}_M \quad (6)$$

Here, $\|\bullet\|$ denotes the Frobenius norm, $\Delta\boldsymbol{p}_L$ and $\Delta\boldsymbol{\varphi}_{LM}$ are perturbation matrixes that consist of noise and environmental mismatch in an uncertain ocean waveguide.

For simplification, Eq.(6) is expressed as:

$$(\boldsymbol{B}+\boldsymbol{D})\boldsymbol{x}=\boldsymbol{0} \quad (7)$$

Here, $\boldsymbol{B}=[\boldsymbol{\varphi}_{LM},\boldsymbol{p}_L]$ is the data augmented matrix, $\boldsymbol{D}=[\Delta\boldsymbol{\varphi}_{LM},\Delta\boldsymbol{p}_L]$ is the augmented matrix correction or the perturbation matrix. It is obvious that whether $\boldsymbol{B}$ or $\boldsymbol{D}$ is $L\times(M+1)$ matrix, $\boldsymbol{x}=[\boldsymbol{d}_M^T,-1]^T$ is $(M+1)\times 1$ vector. The problem that Eq.(6) described is to find a perturbation matrix $\boldsymbol{D}\in C^{L\times(M+1)}$ whose norm square is minimize, which involves $\boldsymbol{B}+\boldsymbol{D}$ in rank deficiency. So the optimization problem of Eq.(6) can be written to be a standard least squares form:

$$\min\|\boldsymbol{B}x\|_2^2,\ \boldsymbol{x}^H\boldsymbol{x}=2 \quad (8)$$

Now, we decompose the augmented matrix $\boldsymbol{B}$ by the singular value decomposition (SVD) method:

$$\boldsymbol{B}=\boldsymbol{U\Sigma V}^H \quad (9)$$

Where $\boldsymbol{\Sigma}=\mathrm{diag}(\sigma_1,\sigma_2,\cdots,\sigma_{M+1})$, $\sigma_1\geq\sigma_2\geq\cdots\geq\sigma_{M+1}$ are the singular values, $\boldsymbol{v}_1,\boldsymbol{v}_2\cdots,\boldsymbol{v}_{M+1}$ are the right singular vectors.

Next, Let $\boldsymbol{V}_1=[\boldsymbol{v}_{k+1},\boldsymbol{v}_{k+2},\cdots,\boldsymbol{v}_{M+1}]$, $\bar{\boldsymbol{v}}_1$ denotes the first row of $\boldsymbol{V}_1$, the total least squares in the sense of minimum norm is used to estimate the mode amplitudes:

$$\hat{\boldsymbol{d}}_M=\frac{\boldsymbol{V}_1\bar{\boldsymbol{v}}_1^H}{\bar{\boldsymbol{v}}_1\bar{\boldsymbol{v}}_1^H}=\alpha^{-1}\boldsymbol{V}_1\bar{\boldsymbol{v}}_1^H \quad (10)$$

Then substituting Eq. (10) into Eq. (4), and the fields of the whole water can be reconstructed with the received fields of the short vertical linear array as:

$$\hat{\boldsymbol{p}}_N = \boldsymbol{\varphi}_{NM}\hat{\boldsymbol{d}}_M = \alpha^{-1}\boldsymbol{\varphi}_{NM}\boldsymbol{V}_1\bar{\boldsymbol{v}}_1^H \tag{11}$$

It can be seen that the reconstructed fields contain more environmental information.

C. *Evaluation model*

To evaluate the similarity between the reconstructed ones and the real pressure fields, the generalized cosine-squared between two vectors is defined as:

$$\cos^2(\boldsymbol{p}_N, \hat{\boldsymbol{p}}_N) = \frac{(\boldsymbol{p}_N^H \hat{\boldsymbol{p}}_N)(\hat{\boldsymbol{p}}_N^H \boldsymbol{p}_N)}{(\boldsymbol{p}_N^H \boldsymbol{p}_N)(\hat{\boldsymbol{p}}_N^H \hat{\boldsymbol{p}}_N)} \tag{12}$$

When the whole acoustic fields are reconstructed, the performance of matched field processing or other algorithm with short array is improved.

III. SIMULATIONS

In order to evaluate the performance of proposed reconstruction method, lots of simulations with short vertical arrays are processed in the classical Pekeris waveguide[11], the nominal baseline environmental parameter values are shown in Fig.1. The underwater acoustic sourece is at the depth of 80m and at the range of 7000m away from the receiver arrays, and the frequency tone is 500Hz, thus the most order mode number is 22 in the Pekeris waveguide.

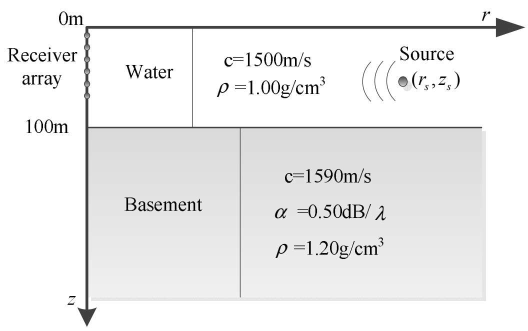

Fig. 1. The Pekeris environment features

In the simulations several vertical linear hydrophone arrays are located near by the water surface. Fig.2 is the layout of all the receiver arrays of which the elements are equally spaced 1m from each other, espically 1# array with 99 hydrophone elements is a long array which covers all the water column, and it is used for the reference, 2#, 3#, and 4# are short vertical linear arrays which have 24, 20, 16 elements, respectively. In these short arrays, the length of 2# which is maximum is only a quarter of the water depth, and is far less than the water depth. In the last, we will evaluate the reconstruction performance of these short arrays.

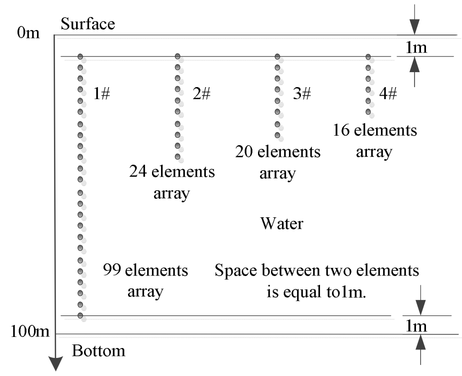

Fig. 2. The layout of the receiver arrays. 1# array is a long vertical array, 2#, 3#, and 4# arrays are all short vertical linear ones.

Fig.3 is the mode amplitudes estimated using Eq.(10). In Fig.3 we can see that the estimation of real part is more accurate than that of imaginary part, and the estimation of low order mode is more accurate than that of high order mode. Also, it can be seen that the array with more hydrophones can estimate the amplitudes more accurately.

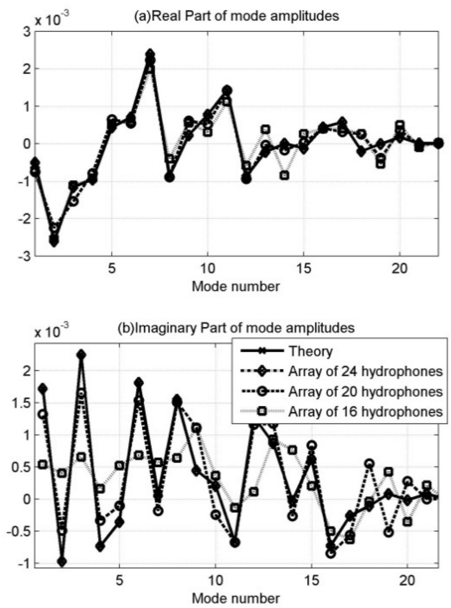

Fig. 3. Mode amplitudes estimated with different short arrays

When the amplitudes is estimated successfully, we can reconstruct the acoustic fields using equation (11). Table 1 shows the similarity of reconstructed fields, it can be seen that the reconstructed ones of short arrays are more similar to the reference than the original ones, and the array with more hydrophones can reconstructed the acoustic fields more accurately. We can also know that for 2# array whose elements are more than the modes, the fields reconstruction is an overdetermined problem, and for 3# and 4#, are the underdetermined problem, all of these can work well, so the proposed method is robust for short arrays with different hydrophones.

TABLE I. THE SIMILARITY BETWEEN THE RECONSTRUCTED FIELDS AND THE FIELDS RECEIVED BY THE LONG ARRAY

Array Number	*Elements of hydrophones*	*Cosine-squared (Original fields)*	*Cosine-squared (Reconstructed fields)*
1#	99	1.0	1.0
2#	24	0.3073	0.9917
3#	20	0.2286	0.7705
4#	16	0.1353	0.6581

Fig.4 shows the comparison results between the generalized cosine-squared of the original fields and that of the reconstructed fields versus water depth for 3# array. It can be seen that although the generalized cosine-squared of the original fields and reconstructed fields can vary along with the source depth in the water, the improvement of fields reconstruction using the proposed least squares is clear and obvious. Other short arrays has the similar results like Fig.4.

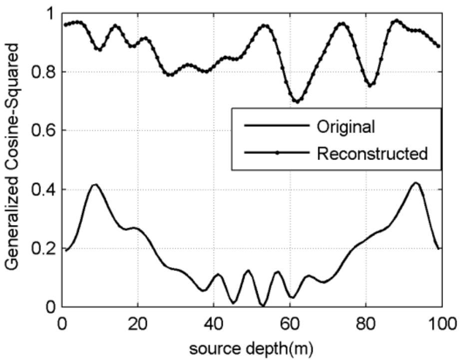

Fig. 4. Comparison between the generalized cosine-square of original fields and that of reconstructed fields for 3# array.

Fig.5 shows the improvement results using the proposed reconstruction method for 2#, 3#, and 4# array. It is obvious that the improvement of 2# array is the largest, and that of 4# is the smallest at all the depth. So we can conclude that the more elements of short array has, the more similar the fields reconstructed are with that of the long array which is used for reference. Specially, the fields reconstructed by 1# array is almost the same with the reference, so the value of generalized cosine-square is always equal to one versus the source depth.

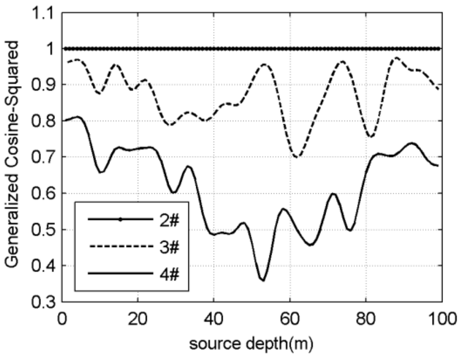

Fig. 5. Comparison among the generalized cosine-square of reconstructed fields for 2#, 3#, and 4# array.

All the results show that the reconstruction can be successfully processed with short vertical array, and when the received fields are reconstrued using the least squares, the generalized cosine-squared is larger obviously, that is the reconstructed ones contain more environmental information, so the performance of matched field passive localization or other method using acoustic fields with short array is improved.

IV. CONCLUSIONS

In summary, it can be seen that the reconstruction of acoustic fields based on the least squares for short vertical linear array can work well in the Pekeris waveguide. In principle, extracting the mode amplitudes by the least squares in the sense of minimum norm plays an important part in reconstruction which contains more environmental information. All the simulations in this paper shows that the proposed method can proceeded successfully even when the hydrophones of short arrays are less that the modes in the shallow water, and it is useful for engineering application of short vertical linear arrays.

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