Biangunilateration using azimuth, elevation, and depth difference to localize submerged assets

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***Abstract*—Most methods to estimate the position of a submerged asset rely on measuring the two-way travel time of an acoustic signal to and from a transponder mounted on that asset. Some alternative methods use one-way travel times and synchronized clocks, and others use multiple direction-only measurements from different locations. The method presented here estimates the location of a submerged asset using a single one-way acoustic ping, without synchronized clocks. The tracker must be able to estimate the direction of the incoming acoustic signal, and the submerged asset must be able to encode its own depth in the transmission. Simple trigonometry then gives the location of the submerged asset relative to the tracker.**

I. Introduction

When trying to estimate the location of an object, most people think of triangulation – that is, computing a position from three angles. Underwater applications usually use at least one range in computing position. Long baseline (LBL) acoustic positioning needs at least three sides (trilateration). Other methods like short baseline (SBL) and ultra-short baseline (USBL) use direction and range. Using a terminology mashup, with the direction expressed as two angles, this would be called biangunilateration. Like traditional USBL, the method proposed here also uses two angles and one side, but instead of a range it uses the depth difference between the two platforms.

This method could be applied on an autonomous surface vessel to track a submerged autonomous underwater vehicle (AUV), buoyancy glider, profiling float, or seafloor platform. It could also be inverted so that the submerged asset can localize itself by tracking the surface platform. Since the method does not require two-way travel times, it has lower overhead for multi-platform operations, and the observations do not have inherent measurement delay. Since it does not require precise synchronized clocks, it can be used to track any asset with a depth sensor and acoustic modem – many existing oceanographic assets already have these systems.

The rest of this section briefly reviews existing methods to acoustically localize submerged assets. Section II introduces the new method with three specific examples. Section III examines the uncertainty in the relative position estimate for given uncertainties in the input measurements. Finally, section IV discusses practical issues to consider when applying the new method in the field, and comments on how the new method can compliment existing methods using synchronized clocks or two-way travel times when those resources are available.

A. Related work

Estimating position underwater usually involves some kind of acoustic measurement. The Global Positioning System (GPS) works well on the surface and in the air, but the radio waves carrying GPS signals do not travel far through water.

Computing a position fix within a LBL transponder net uses two-way travel times to and from multiple fixed transponders at known locations [1]. Ranges to at least three transponders are required for a unique solution (trilateration) and more ranges can be used when available to compute overconstrained solutions (multilateration).

SBL and USBL use an array of hydrophones at the receiver to compute the direction of the incoming signal (see, e.g., [2]–[4]), but typically still use the two-way travel time to compute range. This gives position relative to the receiver, which can be added to the position of the receiver if it is known.

A typical USBL measurement cycle consists of three one-way acoustic transmissions. First the tracker sends a query to the submerged asset, then the asset responds. The USBL uses the relative phases of the incoming signal on each of its hydrophones to estimate the direction of the incoming signal, and the two-way travel time to estimate the range to the contact. The tracker combines this range and direction with its own position and orientation, then sends the computed position back down to the submerged asset. This works fine when tracking a small number of assets and when the acoustic channel is not being used for a lot of communication, but the overhead becomes more limiting with more submerged assets and more communication.

Acoustic transmissions through water have inherent delays – on the order of seconds to tens of seconds over ranges of kilometers to tens of kilometers. Observations that rely on multiple one-way transmissions also have inherent measurement delays, which should be handled explicitly in order to avoid imprecise estimates or overconfident estimators [5]. This applies to traditional USBL downlinks as well as to LBL, where each two-way range is a partially-delayed measurement [6].

Reducing the number of transmissions required for positioning means less overhead and more time to send important data, track more assets, or track one asset more closely. One could reduce some of the overhead by intelligently interleaving the query and the computed position downlink, and also

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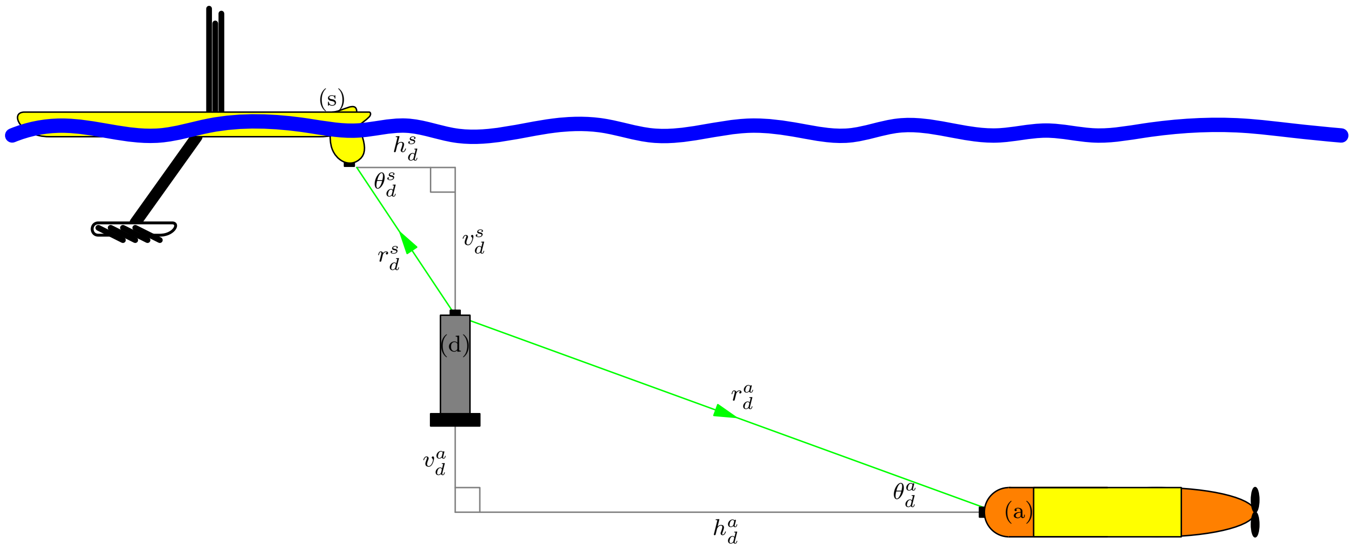

Fig. 1. A surface vehicle (s) and AUV (a) can both track the same profiling drifter (d) using the same one-way acoustic broadcast transmission.

interleaving with other communication. If each submerged asset has a precision clock, ranges can be computed from one-way travel times (see, e.g., [7]–[9]). Low-drift clocks are becoming smaller, cheaper, and more energy efficient, but legacy systems will not always have them. The method proposed here works with a single one-way transmission and does not rely on syncronized clocks.

II. Localizing with Azimuth, Elevation, and Depth Difference

The tracker must be able to estimate direction of incoming acoustic signals (e.g., using USBL or SBL). The platform being tracked must be able to send acoustic signals encoding depth (for submerged platforms) or latitude and longitude (for surface platforms).

The real benefit of only needing one-way transmissions comes when working with multiple platforms, so let's illustrate with a small team of three platforms:

(a) a submerged AUV,
(s) a surface vehicle, and
(d) a profiling drifter.

This is a small team, but enough to illustrate the three main scenarios. It's also big enough to achieve useful science goals – for example: the drifter can park at the depth of a chlorophyll patch and drift with it, while the AUV tracks, circles, and yoyos up and down through the water column at a specified radius collecting contextual data, and the surface vehicle follows while relaying the positions of all team members to operators and scientists on shore. This is a science mission we have done at Monterey Bay Aquarium Research Institute during field experiments for the Controlled, Agile, and Novel Observing Network (CANON) initiative. So far, we are using traditional USBL with two-way transmissions, but the method proposed in this paper would be more scalable to large teams and for sharing more data acoustically.

Assume all team members are capable of sending messages encoding depth and other variables via acoustic modems. Also assume the surface vehicle and AUV are both able to measure the direction ($\hat{u}$) of an incoming acoustic signal using a USBL or SBL array of receiving hydrophones. That direction can be expressed as two angles: azimuth (ψ) and elevation (θ). The azimuth angle describes how far to port or starboard the signal came from, while the elevation describes how far above or below. Solving the triangle in the vertical plane using measurements of elevation and depth difference will give the horizontal range h – that is the goal of the method presented here. Once the horizontal range is known, the azimuth angle gives the distances to the contact along x and y axes. In a reference frame with x positive forward and y positive starboard:

$$x = h\cos\psi$$
$$y = h\sin\psi$$

This is the same as for traditional USBL. Another way to approach it – without losing generality – is to rotate the reference frame in the horizontal plane to align with $\hat{u}$, making $\psi = 0$. This leaves the focus on the vertical plane and solving for the horizontal range.

A. *Surface platform tracking submerged drifter*

This is the simplest configuration – and most similar to how USBL acoustic positioning is typically used in the field today. The surface platform is tracking a submerged asset – consider the drifter, though this applies equally well to tracking the AUV.

The submerged drifter sends an acoustic signal to the surface platform. The signal has the drifter's depth encoded in it, perhaps with other status information or measurements from its science sensors. The drifter may send this signal on a fixed interval, but it can also be completely asynchronous.

The surface platform receives the signal and estimates the direction it came from, producing θ_d^s, the elevation angle of the drifter, as observed by the surface platform. The surface platform also decodes the depth of the drifter z_d from the message, and subtracts the depth of its USBL transducer to calculate the vertical range to the drifter.

$$v_d^s = |z_d - z_s|$$

The surface platform can now calculate the horizontal range to the drifter by solving the trigonometry problem:

$$h_d^s = \frac{v_d^s}{\tan \theta_d^s} = \frac{|z_d - z_s|}{\tan \theta_d^s}. \tag{1}$$

If the drifter needs to know its global position, the surface platform will still need to send that via a second acoustic transmission, the same as the downlink in traditional USBL. As far as the drifter is concerned, that will be a delayed measurement. So if the drifter is moving quickly relative to the measurement cycle the delay should be explicitly accounted for. Practically speaking, this is more likely to be a concern for a faster submerged platform like an AUV.

The method proposed here works based on measuring the elevation angle θ_d^s and the depth difference v_d^s, where traditional USBL based on two-way acoustic transmision measures elevation angle θ_d^s and slant range r_d^s. Since the triangle in the vertical plane is fully constrained, the slant range can also be calculated:

$$r_d^s = \frac{v_d^s}{\sin \theta_d^s} = \frac{|z_d - z_s|}{\sin \theta_d^s}. \tag{2}$$

This could be useful for comparing to traditional USBL, for example, or for estimating the time offset between clocks on the two platforms. It would also be useful for calculating the relative position of the contact without additional rotations or trigonometry, i.e.:

$$p_d^s = r_d^s \hat{u}_d^s. \tag{3}$$

B. AUV tracking the same submerged drifter

If the drifter from the previous example is broadcasting its depth on a channel that the AUV can listen on, the AUV can track it without any additional acoustic transmissions. This could be particularly useful when many platforms are coordinating to achieve a common goal.

$$h_d^a = \frac{v_d^a}{\tan \theta_d^a} \tag{4}$$

This scales up trivially with the number of trackers. Although some planning would be necessary to avoid overlapping broadcast signals, the lower overhead for one-way transmissions means this also scales better than two-way methods when increasing the number of platforms being tracked.

C. AUV updating its own position by tracking surface platform

This configuration is useful for deployments where a single surface vehicle will be tending several submerged AUVs, serving as a navigation aid and communications gateway.

Instead of the submerged platform broadcasting a signal encoding its depth, suppose that the surface platform broadcasts a signal encoding its latitude longitude, and the depth of the transducer. When the AUV receives the signal and uses its USBL to estimate the direction the signal came from, it can calculate its position relative to the surface platform:

$$h_s^a = \frac{v_s^a}{\tan \theta_s^a} \tag{5}$$

From there it is a simple matter to combine the relative position with the GPS position of the surface platform, producing a geographic position fix for the submerged AUV.

For example, if the AUV was 101 m deep when it received a signal from due North at an elevation angle of 20° with the encoded location 36.8° N, 122° W and a transducer depth of 1 m, it could calculate the horizontal distance as $(101 - 1 \text{ m})/(\tan 20°) = 274.7$ m, giving its own location at 36.797524° N, 122° W.

III. Uncertainty analysis

Measurement errors in the elevation angle and in the depth difference will both contribute to errors in the estimated horizontal range. Due to the geometry of the problem, the magnitude of these errors will also vary with the true elevation angle. Consider the surface vehicle tracking the submerged drifter again. Here we will examine the uncertainty in the output for a given uncertainty in each input, using the true value for the other input. Then we will examine the combined effect of uncertainties in both inputs.

Figure 2 shows the uncertainty in estimated horizontal range due solely to noise in the measured elevation angle, $\hat{\theta}$, for two angles – 30°and 60°. The noise is additive and normally distributed with a standard deviation of 1°, so that:

$$\hat{\theta}_d^s = \mathcal{N}\left(\theta_d^s, \sigma_{\theta_d^s}\right) \tag{6}$$

$$\hat{h}_d^s = \frac{v_d^s}{\tan \hat{\theta}_d^s} \tag{7}$$

The histograms in the lower panel clearly illustrate the dependence on actual elevation angle. Although both measurements have $\sigma_{\theta_d^s} = 1°$, the resulting horizontal range estimates have different uncertainties. At shallow angles the range estimate distribution has a lower, broader peak – meaning a less certain estimate. In this 30° example, $\sigma_{\theta_d^s} = 1°$ leads to $\sigma_{h_d^s} = 12.1$ m. At steep angles the distribution becomes taller and narrower, depicting higher certainty in the estimate. In this 60° example, $\sigma_{\theta_d^s} = 1°$ leads to $\sigma_{h_d^s} = 4.04$ m.

The estimated horizontal range is less sensitive to uncertainties in the depth difference measurement. Figure 3 shows that the uncertainty is still greater for shallow elevation angles, but $\sigma_{v_d^s} = 1$ m leads to $\sigma_{h_d^s}$ of only 1.73 m for the 30° example, and 0.58 m for the 60° example. That's rougly an order of magnitude lower than from the 1° uncertainty in the elevation measurement.

Uncertainty due to noise in the elevation measurement dominates in the combined case, as illustrated in Figure 4. Here, $\sigma_{\theta_d^s} = 1°$ and $\sigma_{v_d^s} = 1$ m lead to $\sigma_{h_d^s}$ of 12.2 m and 4.08

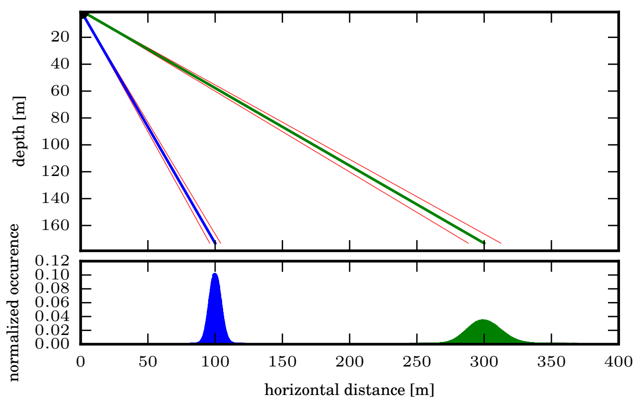

Fig. 2. Uncertainty in horizontal range resulting from uncertainty in measured elevation angle for contacts at 30 °and 60 °elevation. The elevation measurements are corrupted by additive white noise with $\sigma_\theta = 1°$, resulting in errors in the estimate of horizontal range, as shown in the lower panel. Shallower angles result in lower, broader peaks in estimated horizontal range ($\theta = 30°$, green) while steeper angles produce taller, narrower peaks ($\theta = 60°$, blue).

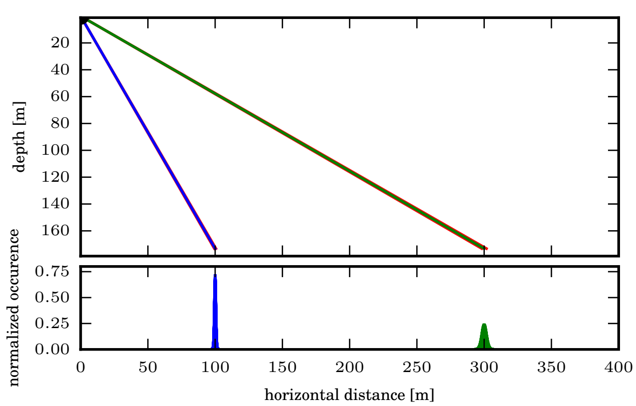

Fig. 3. Uncertainty in horizontal range resulting from uncertainty in measured depth difference for contacts at 30 °and 60 °elevation. The depth difference measurements are corrupted by additive white noise with $\sigma_v = 1$ m, resulting in errors in the estimate of horizontal range, as shown in the lower panel. Shallower angles result in lower, broader peaks in estimated horizontal range ($\theta = 30°$, green) while steeper angles produce taller, narrower peaks ($\theta = 60°$, blue).

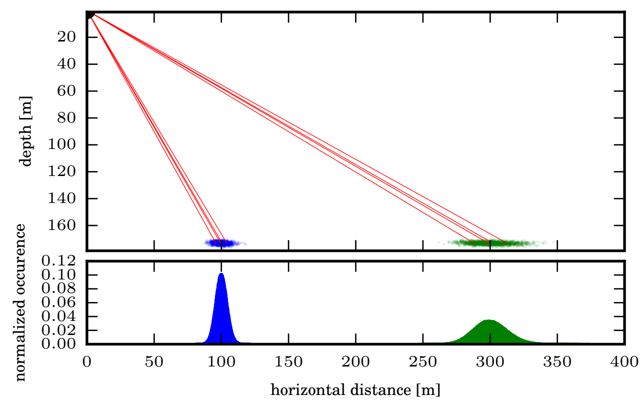

Fig. 4. Uncertainty in horizontal range from combined uncertainty in input measurements. The actual contacts are at 30 °elevation and 300 m horizontal distance (green) and at 60 °elevation and 100 m horizontal distance (blue); both are 173.2 m deeper than the receiver. Elevation measurements are normally distributed with $\sigma_\theta = 1°$ and depth difference measurements are normally distributed with $\sigma_v = 1$ m. The top panel illustrates the estimated relative position of the contact using point clouds (green, blue) and rays (red) to the 1-σ points around the mean estimated position. The bottom panel shows a normalized distribution of the estimated horizontal range, and highlights how it depends on the actual elevation angle.

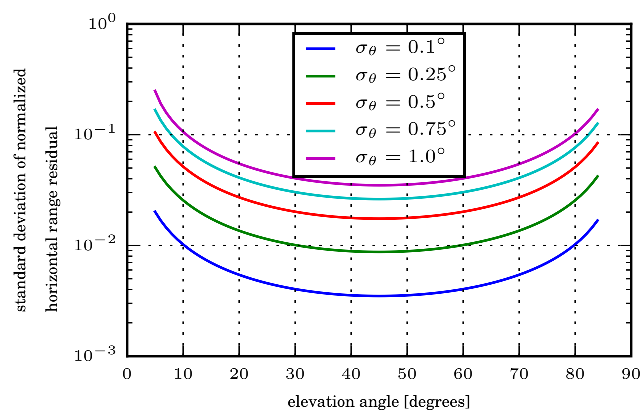

Fig. 5. Standard deviation of the normalized residual in horizontal range over a range of elevation angles for various levels of uncertainty.

m, for the 30° and 60° examples, respectively. That is only a very slight increase over the uncertainty from the elevation angle measurement alone.

Since uncertainty in the elevation angle measurement dominates the uncertainty in the horizontal range estimate, it make sense to examine this closer. Figure 5 shows the standard deviation of the *normalized* horizontal range residual:

$$\operatorname{std}\left(\frac{\hat{h}_d^s - h_d^s}{h_d^s}\right) \tag{8}$$

over a range of elevation angles θ_d^s for uncertainties at several levels $\sigma_{\theta_d^s}$. Many combinations offer estimation uncertainties in the 1-10% range, which may not be sufficient for precision navigation, but serve well enough for tracking and coordinating with nearby targets. At $\sigma_{\theta_d^s} \leq 0.25$, estimation uncertainty is less than 1% of the actual horizontal range. Some USBL systems have published angular accuracies on this order [10], [11], and navigation aiding at this level would be helpful to submerged vehicles dead reckoning based on a magnetic compasse and bottom-lock measurements from a Doppler velocity log (DVL).

Figure 5 also hints at the behavior of this method at the limits – solving the triangle will fail at the extremes. For shallow angles close to 0°, the tangent approaches 0, so (1) divides by zero. The depth difference also approaches zero at the same time. Physically, this means that the method cannot be used when there is no difference in depths, which should come as no surprise. Operationally, this means a tracker would have to switch to two-way ranging, or use synchronized clocks and one-way ranges, to track contacts at the same depth. This wouldn't likely be an issue for platforms on the surface, since they could synchronize with GPS, and would probably share data via radios anyway. For steep angles approaching 90°, the tangent asymptotically approaches infinity. The triangle disappears here again, but that is of little concern because the contact is telling the tracker its depth, and the tracker knows the contact is directly below (or above).

IV. Discussion

Here we have proposed a slight twist to traditional SBL and USBL – using a single one-way acoustic transmission and encoded depth difference rather than a two-way transmission and slant range. This has logistical advantages when dealing with many vehicles because many trackers can use a single broadcast to localize one platform, and because it halves the number of acoustic transmissions required for each tracking fix.

To apply this new method in the field, practical issues like acoustic multipath and ray-bending due to a varying soundspeed profile will need to be addressed. These challenges are not unique to the new method, and existing mitigation techniques should apply. Since the platforms may be profiling continuously through the water column during some field applications, the soundspeed profile may be characterized better than with traditional USBL, leading to improved performance. Since the new method works with a single one-way transmission, the observations have no inherent measurement delay, so that won't need to be handled explicitly. The exception is in the case where a tracker must downlink a position fix to a platform that only has a modem and no USBL, which is similar to the problem for traditional USBL and can be solved the same way.

The new approach does have limitations, especially at shallow elevation angles, but should be sufficient for many applications, and a complement to traditional techniques based on two-way transmissions in other cases. In particular, it would be interesting to study the overconstrained solutions when both depth difference and slant range are readily available (from two-way travel times, or from one-way travel times with synchronized clocks).

Acknowledgment

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