Design and Analysis of a One-Dimensional Sea Surface Simulator Using the Sum-of-Sinusoids Principle

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***Abstract*—Simulators for sea surface waves are useful for many practical applications, such as the construction of offshore structures and ocean surface animations. This paper studies three methods for the design of one-dimensional sea surface waves simulators with given wave spectra using the sum-of-sinusoids (SOS) principle. The wave spectrum provides insight into important statistical properties of the sea surface waves, such as the autocorrelation function (ACF) of the sea surface waves, significant wave height, and the moments of the spectrum. The sea surface simulator is designed by applying the concept of deterministic channel modelling on two main classical wave spectra, namely the Pierson-Moskowitz (PM) model and JOint North Sea WAve Project (JONSWAP) model. For the parametrization of the sea surface simulator, we adopt three well-known parameter computation methods: the L_p-norm method (LPNM), method of equal distances (MED), and the method of equal areas (MEA). A good match between the given sea surface models and the corresponding simulation model is achieved with respect to the main statistical properties of the sea surface waves. It is shown that for a given number of sinusoids, the LPNM and the MEA have almost the same performance, whereas the MED results in a sea surface waves simulator that suffers from a relatively small period.**

***Index terms* — Sea surface modelling, Pierson-Moskowitz spectrum, JONSWAP spectrum, sea surface wave simulator, spectral moments.**

I. INTRODUCTION

Sea surface simulation is a popular research topic in ocean engineering [1] and computer graphics [2]. Sea surface waves greatly enhance the flux of energy and momentum between the atmosphere and the sea. A detailed knowledge of the statistics of the sea waves is important for the construction of coastal works like offshore structures [3] and for animating games and movies [4]. Thus, a realistic simulation model for sea surface waves is of great interest.

Moreover, the temporal elevation of the sea surface causes backscatterer signals influencing the Doppler frequency spectrum of acoustic underwater waves. To design more realistic channel models for acoustic underwater communications, accurate simulation models for the sea surface dynamics are required, which capture the temporal elevation of the sea surface. A proper approach is to model the sea surface waves by a random process with a specified wave spectrum, such as the Pierson-Moskowitz (PM) spectrum [5] and JOint North Sea WAve Project (JONSWAP) spectrum [6].

The kinetic energy of random waves is represented by the wave spectrum, which plays a key role in the analysis of the statistical properties of sea surface wave processes. The energy contained in the waves is defined by the energy density per unit area of the horizontal sea surface. The wave spectrum is of central importance for the characterization of ocean waves and gives insight into the distribution of the wave energy as a function of the wave frequencies. One source of irregularity of sea surface waves is the local wind. The wind-generated waves mix with the swell on the sea surface, which can be identified in the wave spectrum.

A few approaches exist in the literature to simulate the sea surface waves. The approaches can be classified as physical and parametric/spectral methods. In general, physical approaches (e.g., [7]–[9]) aiming to solve the wave equations are too complex to find analytical solutions. In contrast, parametric/spectral approaches (e.g., [2], [10]) are appropriate to model the periodic motion of the sea surface elevation in deep water environments.

In [2], a method has been presented in which an adaptive spectrum sampling technique has been applied for determining the sea surface model parameters. This study considered only a limited frequency range and selected waves that are the most typical representatives of the wave spectrum. A real-time additive sound synthesis method applied to the sea surface wave modelling has been described in [11]. In this paper, the sea waves have been considered as partials of a sound by transposing the frequency and time domains to the wavenumber domain. In [12], a real-time simulation model has been proposed based on the sum-of-sinusoids (SOS) principle for the simulation of sea surface waves characterized by a given wave spectrum.

In this paper, we present three methods to design a one-dimensional sea surface wave simulator based on the SOS principle. This paper aims to exploit some techniques used in the area of the mobile radio channel modelling for simulating sea surface waves. For the design of a sea surface simulator, we need to compute the model parameters, which are the wave amplitudes, wave frequencies, and phase shifts. In this paper, parameters of the sea surface wave simulator have been determined by means of the L_p-norm method (LPNM) [13], method of equal distances (MED) [14], and the method of equal areas (MEA) [14]. Assuming the PM and JONSWAP

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spectra, the main statistical properties of the sea surface waves reference model are compared with those of the sea surface waves simulator. A good fitting between the reference model and the simulation model has been achieved with respect to the ACF, distribution of the sea surface elevation, and the moments of the PM and JONSWAP spectra.

The rest of this paper is organized as follows. In Section II, the PM and JONSWAP spectra are presented. Section III describes the proposed simulation model for sea surface waves. The statistical properties of the reference model and the simulation model are derived in Section IV. In Section V, the utilized parameter computation methods are briefly reviewed and applied on the PM and JONSWAP spectra. The numerical results are illustrated in Section VI. Finally, the conclusions are drawn in Section VII.

II. Parametric Sea Surface Spectra

The PM and JONSWAP models are two non-symmetric parametric models of the sea surface spectrum. From systems theory, it follows that the sea surface spectrum must be real and even, i.e., $S_{\mu\mu}(\omega) = S^*_{\mu\mu}(-\omega)$ to obtain real-valued sea surface waves. However, non-symmetric spectra are preferred in experimental practice [15]. In this paper, we introduce a symmetrical PM (JONSWAP) spectrum by mirroring the original PM (JONSWAP) model with respect to the y-axis. To have the same energy level as the one for the non-symmetric spectra, we divide the symmetrical spectra by two.

In the PM model, the wave spectrum is a function of the wind speed and the wave frequency. For a fully-developed sea, the symmetrical PM spectrum model is given by

$$S_{\mu\mu}(\omega) = \frac{\alpha g^2}{2\,|\omega|^5}\, e^{-\beta\left(\dfrac{g}{\omega U}\right)^4}, \quad -\infty < \omega < +\infty \tag{1}$$

where $\alpha = 0.0081$, $\beta = 0.74$ and the symbol ω stands for the sea wave frequency (in rad·Hz). The parameters U and g denote the wind speed (in m/s) and the gravity acceleration (in m/s^2), respectively. Fig. 1 illustrates the PM spectrum for positive frequencies ω in terms of different wind speeds.

The JONSWAP spectrum is an empirical model, which was established during the joint research project called the joint north sea wave project. The JONSWAP spectrum is an extension of the PM spectrum for fetch-limited seas. The symmetrical JONSWAP spectrum is given by

$$S_{\mu\mu}(\omega) = \frac{\alpha_J g^2}{2\,|\omega|^5}\, e^{-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4} \gamma^{\, e^{-\frac{(|\omega|-\omega_p)^2}{2\sigma_J^2\omega_p^2}}}, \quad -\infty < \omega < +\infty \tag{2}$$

where $\alpha_J = 0.076\left(U^2/Fg\right)^{0.22}$, in which F is the fetch defined as the distance over which the wind blows with constant velocity. The symbol $\omega_p = 22\left(g^2/FU\right)^{1/3}$ is the peak wave frequency, which is also called the modal frequency. The parameter γ is the so-called peak enhancement factor. Its default value is $\gamma = 3.3$. The symbol σ_J is a measure of the width of the spectral peak. Typical values of the parameter σ_J are 0.07 on the low frequency side ($|\omega| \leq \omega_p$) and 0.09 on the high side ($|\omega| > \omega_p$). The JONSWAP spectrum is shown in Fig. 2 for different wind speeds.

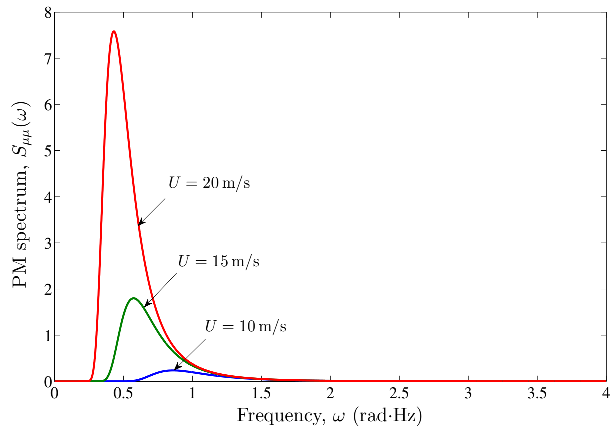

Fig. 1. The symmetrical PM spectrum $S_{\mu\mu}(\omega)$ for $\omega > 0$.

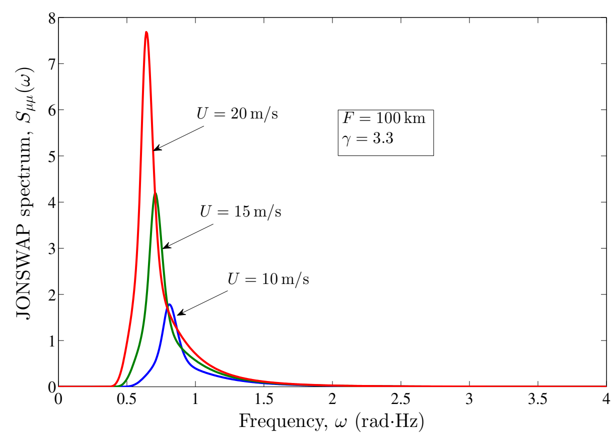

Fig. 2. The symmetrical JONSWAP spectrum $S_{\mu\mu}(\omega)$ for $\omega > 0$.

From systems theory, it follows that the sea surface spectrum must be real and even, i.e., $S_{\mu\mu}(\omega) = S^*_{\mu\mu}(-\omega)$ to obtain real-valued sea surface waves. However, non-symmetric spectra are preferred in experimental practice [15].

III. Proposed Sea Surface Model

By using Rice's SOS [16], [17], the sea surface waves can be modeled as a superposition of sine waves. In this paper, we propose a simulation model for sea surface waves described by a stochastic SOS process $\hat{\mu}(t)$ of the form

$$\hat{\mu}(t) = \sum_{n=1}^{N} c_n \cos(\omega_n t + \theta_n) \tag{3}$$

where N is the number of sinusoids. The symbol c_n stands for the nth wave amplitude (in m). The symbols ω_n and θ_n denote the wave frequency (in rad·Hz) and the phase shift of the nth sea surface wave, respectively. In Section. V, three methods are described for the computation of constant values for the wave amplitudes c_n and the wave frequencies ω_n. The phase shifts θ_n are modelled by independent and identically distributed (i.i.d.) random variables, which are supposed to be uniformly distributed over the interval $(0,\,2\pi]$.

IV. The Statistical Properties of the Sea Surface Waves

In this section, we study the main statistical properties of the PM and JONSWAP spectra, which serve as reference models, and those of the corresponding simulation model.

A. Statistical Properties of the Reference Model

The ACF $r_{\mu\mu}(\tau)$ is obtained by taking the inverse Fourier transform of the (symmetrical) spectrum $S_{\mu\mu}(\omega)$ with respect to the wave frequency ω, i.e,

$$r_{\mu\mu}(\tau) = \frac{1}{\pi}\int_0^{\infty} S_{\mu\mu}(\omega)\cos(\omega\tau)\,d\omega \tag{4}$$

where the symbol τ denotes the time separation variable. In this paper, the sea surface sea process is considered to be a narrowband process, meaning that the energy of the waves spectrum is concentrated within a narrow frequency range. In analogy to [18], the symmetrical PM spectrum can be formulated as

$$S_{\mu\mu}(\omega) = \frac{A}{2\,|\omega|^5}\,e^{-\dfrac{B}{\omega^4}},\quad -\infty < \omega < +\infty \tag{5}$$

where $A = \alpha g^2$ and $B = \beta(g/U)^4$. By using the wave spectrum given in (5), the moments m_j can be expressed by

$$m_j = \int_{-\infty}^{\infty} \omega^j S_{\mu\mu}(\omega)d\omega \tag{6a}$$

$$m_0 = \frac{A}{4B} \tag{6b}$$

$$m_1 = 0 \tag{6c}$$

$$m_2 = \frac{\sqrt{\pi}A}{4\sqrt{B}} \tag{6d}$$

where the zeroth moment m_0 is the area under the spectrum, which is related to the significant wave height H_s. For a narrowband spectrum, the significant wave height H_s is approximately four times the square root of the zeroth moment, i.e.,

$$H_s = 4\sqrt{m_0} = 2\sqrt{\frac{A}{B}}\,. \tag{7}$$

For a narrowband spectrum, the maximum wave height H follows approximately the Rayleigh distribution [19]. Therefore, the distribution $p_H(x)$ of the maximum wave height H can be approximated by [18, Eq. (2.71)]

$$p_H(x) = \frac{2x}{R}\,e^{-x^2/R}, \qquad \text{if} \;\; x > 0 \tag{8}$$

in which $R = 8m_0$. The average wave height $\bar{H}$ can be determined as [18, Eq. (2.72)]

$$\bar{H} = \int_0^{\infty} x\left(\frac{2x}{R}\,e^{-x^2/R}\right)\,dx = \frac{\sqrt{\pi R}}{2}\,. \tag{9}$$

Hence, by using $R = 8m_0 = 2A/B$, the average wave height $\bar{H}$ equals $\bar{H} = \sqrt{\pi\,A/(2B)}$. The ratio of the significant wave H_s height to the average wave height $\bar{H}$ can be computed as

$$H_s/\bar{H} = \sqrt{\frac{8}{\pi}} = 1.6\,. \tag{10}$$

The modal frequency ω_p of the PM spectrum is given by [18, Eq. (2.63)]

$$\omega_p = 0.4\sqrt{\frac{g}{H_s}}\,. \tag{11}$$

The JONSWAP spectrum cannot be formulated like the PM spectrum in (5). Thus, we have to compute the corresponding characteristic quantities numerically. The numerical results are presented in Table. II. However, the total energy of the JONSWAP spectrum can be expressed in closed form by

$$m_0 = 1.67\times 10^{-7}\frac{U^2F}{g}\,. \tag{12}$$

B. Statistical Properties of the Sea Surface Waves Simulator

The ACF $\hat{r}_{\mu\mu}(\tau)$ of the stochastic SOS process $\hat{\mu}(t)$ can be computed as

$$\hat{r}_{\mu\mu}(\tau) = E\left\{\hat{\mu}^*(t)\hat{\mu}(t+\tau)\right\} = \sum_{n=1}^{N}\frac{c_n^2}{2}\cos(\omega_n\tau) \tag{13}$$

where $E\{\cdot\}$ denotes the expected value operator, which has to be applied on the random phases θ_n. It should be noted that the ACF $\hat{r}_{\mu\mu}(\tau)$ in (13) depends on the number of sinusoids N, the wave amplitudes c_n, and the wave frequencies ω_n, but not on the phases θ_n. The spectral density $\hat{S}_{\mu\mu}(\omega)$ of the sea surface waves simulator is obtained computed by taking the Fourier transform of the ACF $\hat{r}_{\mu\mu}(\tau)$ with respect to the variable τ, which results in

$$\hat{S}_{\mu\mu}(\omega) = \frac{\pi}{2}\sum_{n=1}^{N} c_n^2\left[\delta(\omega-\omega_n) + \delta(\omega+\omega_n)\right]. \tag{14}$$

Analogously to (6a), the moments $\hat{m}_j$ of the spectrum $\hat{S}_{\mu\mu}(\omega)$ can be computed by using

$$\hat{m}_j = \int_{-\infty}^{\infty} \omega^j \hat{S}_{\mu\mu}(\omega)d\omega\,. \tag{15}$$

For instance, the first three moments of the spectrum $\hat{S}_{\mu\mu}(\omega)$ are given by

$$\hat{m}_0 = \pi\sum_{n=1}^{N} c_n^2 \tag{16a}$$

$$\hat{m}_1 = 0 \tag{16b}$$

$$\hat{m}_2 = \pi\sum_{n=1}^{N}\left(\omega_n c_n\right)^2. \tag{16c}$$

By replacing m_0 by $\hat{m}_0$ in (7) and (9), the significant wave height $\hat{H}_s$ and the average wave height $\hat{\bar{H}}$ of the sea surface simulator can be expressed as

$$\hat{H}_s = 4\sqrt{\pi \sum_{n=1}^{N} c_n^2} \tag{17}$$

and

$$\hat{\bar{H}} = \sqrt{2\,\pi^2 \sum_{n=1}^{N} c_n^2} \tag{18}$$

respectively.

The distribution $\hat{p}_\mu(x)$ of the stochastic process $\hat{\mu}(t)$ with random phases θ_n is given by [20, Eq. (32)]

$$\hat{p}_\mu(x) = 2\int_0^\infty \left[\prod_{n=1}^{N} J_0(2\pi c_n \nu)\right] \cos(2\pi\nu x) d\nu \tag{19}$$

where $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind. It should be mentioned that the distribution $\hat{p}_\mu(x)$ depends only on the number of sinusoids N and the gains c_n. If the gain c_n are chosen as $c_n = \sqrt{m_0/(\pi N)}$, then it follows form the central limit theorem that the distribution $\hat{p}_\mu(x)$ approaches the normal distribution with zero mean and variance $\hat{\sigma}^2 = \hat{m}_0/(2\pi)$ if N tends to infinity.

V. The Utilized Parameter Computation Methods

In this section, we represent the three utilized parameter computation methods, namely the LPNM [13], MED [14], and the MEA [14]. The objective is to compute the wave amplitudes c_n and the wave frequencies ω_n such that the ACF $\hat{r}_{\mu\mu}(\tau)$ of the sea surface waves simulator matches the ACF $r_{\mu\mu}(\tau)$ of the reference model described by the PM and JONSWAP spectra.

A. LPNM

For the parametrization of the sea surface waves simulator, we first apply the LPNM [13], which is widely used in the area of mobile radio channel modelling. The application of the LPNM on the given parametrization problem requires the minimization of the following error function

$$E_p = \left[\frac{1}{\tau_{\max}} \int_0^{\tau_{\max}} |r_{\mu\mu}(\tau) - \hat{r}_{\mu\mu}(\tau)|^p d\tau\right]^{\frac{1}{p}} \tag{20}$$

where p is a positive integer, and $\tau_{\max}$ defines an appropriate time-lag interval $[0, \tau_{\max}]$ over which the approximation of $r_{\mu\mu}(\tau)$ is of interest. After the minimization of E_p by using numerical optimization techniques, we obtain a set $\{\omega_n^{\text{opt}}\}$ of optimized wave frequencies. In the literature, the sea surface waves have been reported to follow a zero-mean Gaussian process in deep water [18]. For a given finite value of N, it was shown in [20] that the closest match to the Gaussian distribution can be achieved if all wave amplitudes c_n have the same value $c_n = \sqrt{m_0/(\pi N)}$. Therefore, we only need to compute the parameters ω_n by sunstituting (4) and (13) in (20) and optimizing ω_n such that the error norm E_p results in a minimum.

B. MED

By using the MED [14], any pair of adjacent discrete wave frequencies ω_n have the same Euclidean distance, which can be achieved by defining ω_n as

$$\omega_n = \frac{\Delta\omega}{2}(2n-1), \quad n = 1, 2, ..., N \tag{21}$$

where $\Delta\omega = \omega_{\max}/N$ denotes the distance between two adjacent discrete wave frequencies, i.e., $\Delta\omega = \omega_n - \omega_{n-1}$ for $n = 2, 3, ..., N$. The parameter $\omega_{\max}$ stands for the maximum wave frequency, where the wave spectrum is unequal to zero. The wave amplitudes c_n are determined as

$$c_n = \sqrt{\frac{1}{\pi}\int_{\omega \in I_n} S_{\mu\mu}(\omega)\, d\omega} \tag{22}$$

in which the frequency intervals I_n are defined as

$$I_n = \left[\omega_n - \frac{\Delta\omega}{2}, \omega_n + \frac{\Delta\omega}{2}\right), \quad n = 1, 2, ..., N. \tag{23}$$

By substituting (22) in (13), one can easily prove that $\hat{r}_{\mu\mu}(\tau) \longrightarrow r_{\mu\mu}(\tau)$ holds as $N \longrightarrow \infty$. Concerning the periodicity property of $\hat{\mu}(t)$ [21, p. 151], we need to compute the greatest common divisor (gcd) of the discrete wave frequencies, which is equal to

$$\omega_0 = \text{gcd}\{\omega_n\}_{n=1}^{N} = \frac{\Delta\omega}{2} = \frac{\omega_{\max}}{2N}. \tag{24}$$

Thus, by using the MED, $\hat{\mu}(t)$ is periodic with period $T_0 = 2\pi/\omega_0 = 4\pi N/\omega_{\max}$. The fact that the period T_0 is proportional to N is the major disadvantage of the MED.

C. MEA

The MEA, which was first presented in [14], chooses the wave frequencies ω_n such that the area under the wave spectrum $S_{\mu\mu}(\omega)$ is equal to $m_0/(2N)$ within the wave frequency range $\omega_{n-1} < \omega \leq \omega_n$, i.e.,

$$\int_{\omega_{n-1}}^{\omega_n} S_{\mu\mu}(\omega)\, d\omega = \frac{m_0}{2N} \quad \text{for } n = 1, 2, ..., N \tag{25}$$

where $\omega_0 = 0$. To compute the discrete wave frequencies ω_n, we define an auxiliary function $G_\mu(\omega_n)$ as

$$G_\mu(\omega_n) = \int_{-\infty}^{\omega_n} S_{\mu\mu}(\omega)\, d\omega. \tag{26}$$

In the case of a symmetrical wave spectrum, $G_\mu(\omega_n)$ can be expressed in the form

$$G_\mu(\omega_n) = \frac{m_0}{2}\left(1 + \frac{n}{N}\right). \tag{27}$$

If the inverse function of G_μ, denoted by G_μ^{-1}, exists, the wave frequencies ω_n can be computed as

$$\omega_n = G_\mu^{-1}\left[\frac{m_0}{2}\left(1 + \frac{n}{N}\right)\right]. \tag{28}$$

In [21, p. 205], it is shown that by replacing n by $n-1/2$, we can improve the performance of the MEA. Thus, by applying the substitution $n \to n-1/2$, we can rewrite (28) as

$$\omega_n = G_\mu^{-1}\left[\frac{m_0}{2}\left(1+\frac{2n-1}{2N}\right)\right]. \qquad (29)$$

Like the LPNM, the wave amplitudes c_n have the same value, which are given by $c_n = \sqrt{m_0/(\pi N)}$.

VI. Numerical Results

In the following we compute the sets of the model parameter $\mathcal{P} = \{c_n, \omega_n, N\}$ by applying the three parameter computation methods presented in Section V. In our simulation setup, the number of sinusoids has been set to $N = 40$. For the JONSWAP spectrum, the fetch F and the peak enhancement factor γ have been set to $F = 100\,\text{km}$ and $\gamma = 3.3$, respectively.

Figs. 3 and 4 show the resulting ACFs $\hat{r}_{\mu\mu}(\tau)$ of the sea surface waves simulator designed by the MEA for the symmetrical PM and JONSWAP spectra, respectively. As can be seen, there is an excellent fitting between the ACFs of the reference models and those of the simulation model.

Sample functions of the sea surface process $\hat{\mu}(t)$, which have been generated using the sea surface waves simulator designed by using the MEA for the PM and the JONSWAP spectra with $U = 10\,\text{m/s}$ are illustrated in Figs. 5 and 6, respectively.

Figs. 7 and 8 show the distribution $\hat{p}_\mu(x)$ of the sea surface wave process $\hat{\mu}(t)$ designed by using the MEA for the given PM and JONSWAP spectra. Our results show that a good fitting between the analytical model and the simulation model can be achieved with respect to the distribution of the sea surface elevation. The individual curves can hardly be distinguished from each other in Figs. 7 and 8. As can be seen from both figures, the distribution of the simulated sea surface elevation follows closely the Gaussian distribution.

The distribution $p_H(x)$ of the maximum wave height H of the sea surface waves simulator designed by using the MEA assuming the PM spectrum is shown in Fig. 9. With reference to the results illustrated in Fig. 9, one can see that the distribution $p_H(x)$ of the maximum wave height H follows closely the Rayleigh distribution for large values of x. The reason for the poor fitting between theory and simulation for medium and low values of x is that the analytical expression presented in (8) is an approximation for the distribution of the maximum sea-wave heights.

In case of the PM spectrum, Table I compares the statistical properties of the sea surface waves, such as the moments, significant wave height H_s, average wave height $\bar{H}$, and the period T_0 of the reference model with the corresponding statistical quantities of the simulation model designed by the different parameter computation methods. The same comparison is shown in Table II assuming the JONSWAP spectrum. Although all methods have a relatively good performance in terms of the metrics presented in the tables, they have some specific advantages and disadvantages. As mentioned

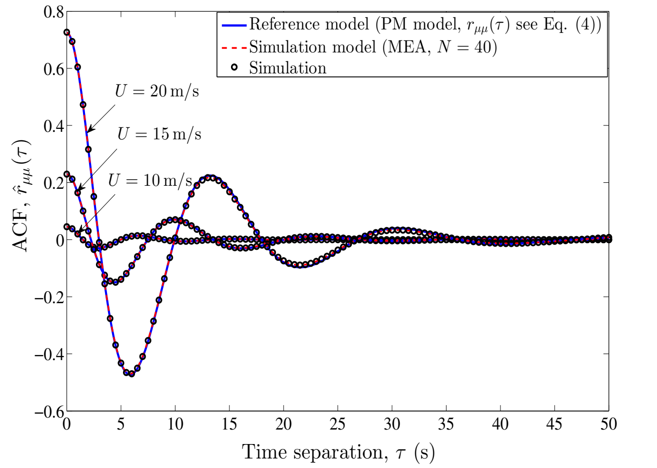

Fig. 3. The ACF $\hat{r}_{\mu\mu}(\tau)$ of sea surface waves simulator assuming the PM spectrum designed by the MEA ($N = 40$) in comparison with the ACF $r_{\mu\mu}(\tau)$ of the reference model.

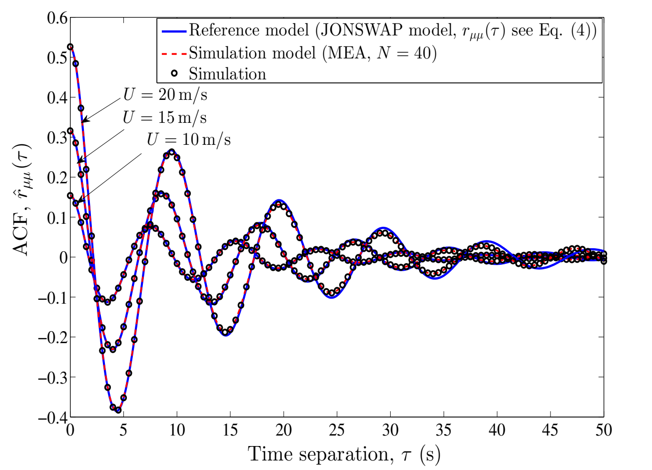

Fig. 4. The ACF $\hat{r}_{\mu\mu}(\tau)$ of sea surface waves simulator assuming the JONSWAP spectrum designed by the MEA ($N = 40$) in comparison with the ACF $r_{\mu\mu}(\tau)$ of the reference model.

in Section V-B, the relatively low period T_0 of $\hat{\mu}(t)$, which increases linearly with N, is the main drawback of the MED. Nevertheless, it must be noted that the MED has a low computational complexity, and can be easily adopted to determine the sea surface model parameters. The main disadvantage of the MEA is the computation of the inverse function G_μ^{-1} in (28), which does not exist for the PM and JONSWAP spectra. Thus, we have to determine the wave frequencies ω_n numerically. As can be observed from the tables, the performance of the LPNM is slightly better than that of the other methods, but, the price to be paid for this achievement is a considerable increase in numerical complexity.

VII. Conclusion

In this paper, we have proposed three methods for the design of a sea surface waves simulator based on the SOS

TABLE I
COMPARISON OF THE SEA SURFACE WAVE PROPERTIES OF THE SIMULATION MODEL DESIGNED BY THE LPNM, MED, AND MEA BY USING THE PM SPECTRUM ($U = 15\,m/s$).

Parameters	PM spectrum	Simulation model, LPNM ($N = 40$)	Simulation model, MED ($N=40$, $\omega_{\max}=8$ rad·Hz)	Simulation model, MEA ($N = 40$)
Zeroth moment, m_0	1.4405	1.4405	1.4405	1.4405
First moment, m_1	0	0	0	0
Second moment, m_2	0.9388	0.9380	0.9366	0.9327
Forth moment, m_4	1.8970	1.8990	1.9149	1.9189
Significant wave height, H_s	4.8008 m	4.8008 m	4.8008 m	4.8008 m
Average wave height, $\bar{H}$	3.0084 m	3.0084 m	3.0084 m	3.008 m
Ratio of the significant wave height to the average wave height, $H_s/\bar{H}$	1.6	1.6	1.6	1.6
Modal frequency ω_p	0.5717 rad·Hz	0.5717 rad·Hz	0.5717 rad·Hz	0.5717 rad·Hz
Period, T_0	∞	Quasi ∞	$4\pi/\Delta\omega = 62.84\,s$	Quasi ∞

TABLE II
COMPARISON OF THE SEA SURFACE WAVE PROPERTIES OF THE SIMULATION MODEL DESIGNED BY THE LPNM, MED, AND MEA BY USING THE JONSWAP SPECTRUM ($U = 15$ M/S, $F = 10$ KM, $\gamma = 3.3$).

Parameters	JONSWAP spectrum	Simulation model, LPNM ($N = 40$)	Simulation model, MED ($N = 40$, $\omega_{\max}=8$ rad·Hz)	Simulation model, MEA ($N = 40$)
Zeroth moment, m_0	1.9794	1.9794	1.9794	1.9794
First moment, m_1	0	0	0	0
Second moment, m_2	1.5882	1.5888	1.5894	1.5896
Forth moment, m_4	3.6413	3.6464	3.6626	3.6665
Significant wave height, H_s	5.6276 m	5.6276 m	5.6276 m	5.6276 m
Average wave height, H	3.5266 m	3.5266 m	3.5266 m	3.5266 m
Ratio of the significant wave height to the average wave height, $H_s/\bar{H}$	1.6	1.6	1.6	1.6
Modal frequency ω_p	0.7056 rad·Hz	0.7056 rad·Hz	0.7056 rad·Hz	0.7056 rad·Hz
Period, T_0	∞	Quasi ∞	$4\pi/\Delta\omega = 62.84\,s$	Quasi ∞

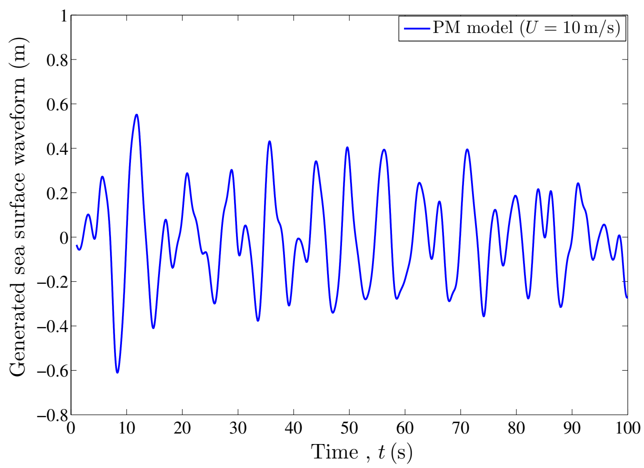

Fig. 5. Sample function of the sea surface waves process $\hat{\mu}(t)$ designed by using the MEA ($N = 40$) assuming the PM spectrum ($U = 10$ m/s).

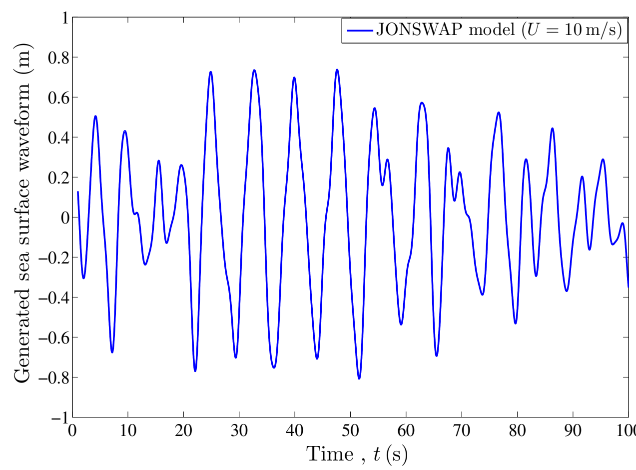

Fig. 6. Sample function of the sea surface waves process $\hat{\mu}(t)$ designed by using the MEA ($N = 40$) assuming the JONSWAP spectrum ($U = 10$ m/s).

concept. We have employed three well-known parameter computation methods, namely the LPNM, MED, and the MEA, which have widely been used in the area of mobile radio channel modelling. These methods have been applied to compute the parameters of a sea surface waves simulator for the case of the PM and JONSWAP spectra. The results of the assessment show an excellent agreement between the main statistical properties of the sea surface waves simulator designed by the three parameter computation methods and those of the given reference model. It has been shown that the LPNM and the MEA have almost the same performance, whereas the MED results in a sea surface waves simulator that has a relatively small period.

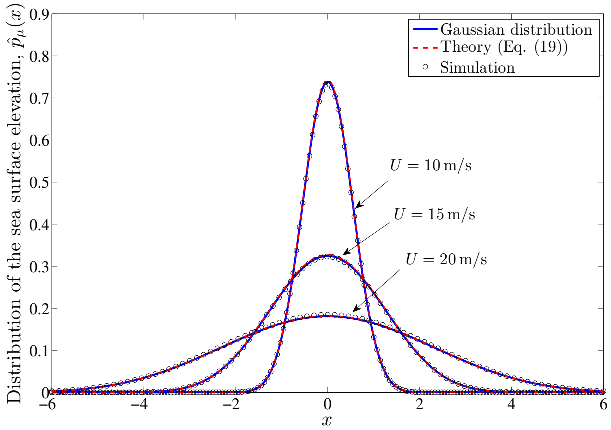

Fig. 7. The distribution $\hat{p}_\mu(x)$ of the sea surface waves process $\hat{\mu}(t)$ designed by using the MEA ($N = 40$) assuming the PM spectrum in comparison with the Gaussian distribution.

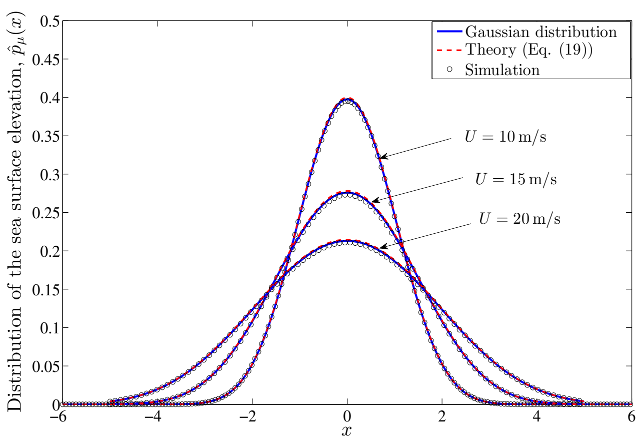

Fig. 8. The distribution $\hat{p}_\mu(x)$ of the sea surface waves process $\hat{\mu}(t)$ designed by using the MEA ($N = 40$) assuming the JONSWAP spectrum in comparison with the Gaussian distribution.

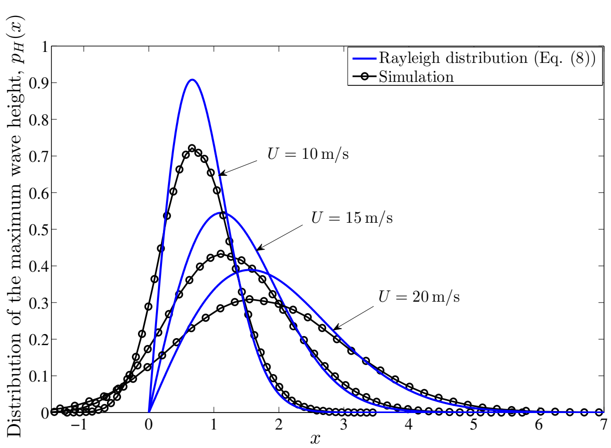

Fig. 9. The distribution $p_H(x)$ of the maximum wave height H designed by using the MEA ($N = 40$) assuming the PM spectrum in comparison with the Rayleigh distribution.

References

[1] X. Ma, Z. Chen, and G. Shi, "Real-time ocean wave motion simulation based on statistic model and GPU programming," in *Proc. 2nd Int. Conf. on Information Science and Engineering (ICISE)*, Dec 2010, pp. 3876–3880.

[2] J. Fréchot, "Realistic simulation of ocean surface using wave spectra," in *Proc. 1st Int. Conf. on Computer Graphics Theory and Applications (GRAPP 2006)*, Seyúbal, Portugal, Feb. 2006, pp. 76–83.

[3] R. Luppez, B. Düz, H. J. L. Van Der Henri P. Van Der Plas, and A. E. p. Veldman, "Numerical simulation of extreme wave impact on offshore platforms and coastal constructions," in *Proc. 5th Int. Conf. on Computational Methods in Marine Engineering (MARINE)*, 2011.

[4] Y. Kryachko, "Using vertex texture displacement for realistic water rendering," in *Proc. Int. Conf. on GPU Gems*, 2005, pp. 283–294.

[5] W. J. Pierson and L. Moskowitz, "A proposed spectral form for fully developed wind seas," *Journal of Geophysical Research*, vol. 69, pp. 5181–5203, 1963.

[6] K. Hasselmann et al., "Measurement of wind wave growth and swell decay during the joint North Sea Wave Project (JONSWAP)," vol. 12, pp. 1–95, Jan. 1973.

[7] A. Arakawa and V. R. Lamb, "A potential enstrophy and energy conserving scheme for the shallow water equations," *Monthly Weather Review*, vol. 109, no. 1, pp. 18–36, Jan. 1981.

[8] D. Enright, S. Marschner, and R. Fedkiw, "Animation and rendering of complex water surfaces," *ACM Trans. Graph.*, vol. 21, no. 3, pp. 736–744, July 2002.

[9] R. Sadourny, "The dynamics of finite-difference models of the shallow-water equations," *J. Atmos. Sci.*, vol. 32, no. 1, pp. 680–689, Apr. 1975.

[10] E. Darles, B. Crespin, D. Ghazanfarpour, and J. C. Gonzato, "A survey of ocean simulation and rendering techniques in computer graphics," *Computer Graphics Forum*, vol. 30, no. 1, pp. 43–60, 2011.

[11] M. Robine and J. Fréchot, "Fast additive sound synthesis for real-time simulation of ocean surface," in *Proc. Int. Conf. on Systems, Signals and Image Processing, IWSSIP*, Budapest, Hungary, Sept. 2006, pp. 223–226.

[12] L. M. Lachman, "An open programming architecture for modeling ocean waves," in *Proc. of the IMAGE Conference*, Scottsdale, AZ, USA, Jul. 2007, pp. 1–13.

[13] M. Pätzold, U. Killat, F. Laue, and Y. Li, "A new and optimal method for the derivation of deterministic simulation models for mobile radio channels," in *Proc. IEEE 46th Veh. Technol. Conf., VTC'96*. Apr./May 1996, vol. 3, pp. 1423–1427, Atlanta, Georgia, USA.

[14] M. Pätzold, U. Killat, and F. Laue, "A deterministic model for a shadowed Rayleigh land mobile radio channel," in *Proc. 5th IEEE Int. Symp. Personal, Indoor and Mobile Radio Commun., PIMRC'94*, Hague, Netherlands, 1994, vol. 4, pp. 1202–1210.

[15] S. R. Massel, *Ocean Surface Waves: Their Physics and Prediction*, vol. 11, World Scientific, 1st edition, 1996.

[16] S. O. Rice, "Mathematical analysis of random noise," *Bell Syst. Tech. J.*, vol. 23, pp. 282–332, July 1944.

[17] S. O. Rice, "Mathematical analysis of random noise," *Bell Syst. Tech. J.*, vol. 24, pp. 46–156, Jan. 1945.

[18] M. K. Ochi, *Ocean Waves: The Stochastic Approach*, Cambridge University Press, 1st edition, 2005.

[19] D. E. Cartwright and M. S. Longuet-Higgins, "The statistical distribution of the maxima of a random function," *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, vol. 237, no. 1209, pp. 212–232, Oct. 1956.

[20] M. Pätzold, U. Killat, F. Laue, and Y. Li, "On the statistical properties of deterministic simulation models for mobile fading channels," *IEEE Trans. Veh. Technol.*, vol. 47, no. 1, pp. 254–269, Feb. 1998.

[21] M. Pätzold, *Mobile Fading Channels*, Chichester: John Wiley & Sons, 2nd edition, 2011.