Constrained Model Predictive Control Design for Dynamic Positioning of a Supply Ship

Guoqing Xia, Ju Liu, Chengcheng Pang, Jingjing Xue
College of Automation, Harbin Engineering University
Harbin, China
liuju1127@hrbeu.edu.cn

***Abstract*—This paper researches how to design a constrained model predictive control (MPC) algorithm based dynamic positioning (DP) control system of a supply ship considering system constraints. First, a linear low-frequency motion model with three degrees of freedom necessary for the MPC predictive model is established. Secondly, an MPC algorithm based DP controller is designed considering input constraints on both amplitude and rate. Finally, the simulation is carried out to verify the effectiveness of the constrained MPC controllers. The simulation result shows effectiveness of the constrained MPC controller shows better performance due to its direct constraint-handling ability.**

Keywords—MPC; constrained model predictive control; DP; dynamic positioning; supply ship

I. INTRODUCTION

With the development of the sea resources exploration, the dynamic positioning system (DPS) becomes an advanced piece of technology which is widely used in various offshore applications including offshore service vessels, drilling rigs (semisubmersibles) and ships, shuttle tankers, cable and pipe layers, floating production, storage and off-loading units (FPSOs), crane and heavy lift vessels, geological survey vessels, rescue vessels, multipurpose construction vessels, cruise ships, yachts, fishing boats, navy ships, tankers and other vessels[1]. A positioned vessel or platform continuously exposed to environmental disturbances such as wind, waves, and current, automatically maintains its position and heading by applying thrusters. As the core of the structure of a DPS, the control system has become increasingly important. The proportion integration differentiation (PID) control law based controller is the first generation DPS controller. The second generation DPS controller uses linear-quadratic Gaussian (LQG) control technology. Nowadays, the neural network control and fuzzy control technologies are widely used in the DPS[2]. Although, the control performance is satisfying during some mild or moderate sea condition, the control performance would deteriorate due to various system limitations. Considering the ability to directly handle system constraints[3, 4], the model predictive control algorithm is introduced to the DP field, which is as well promising in ship motion discipline. Recently, more and more attention has been paid to the MPC method for the control systems of ships, and other marine structures. In research literature, the term receding horizon control (RHC) has occasionally been used and MPC is the conventional name of this technique. Model predictive control is an advanced control scheme, which includes three essential modules: predictive model, online optimization and constraints handling. MPC optimizes a cost function at each sampling time through the use of a quadratic programming algorithm. The optimization produces a sequence of control actions in order to attain optimal performance. To compare the control performance, two MPC algorithm based effective control methods - unconstrained and constrained control methods - are designed to solve the DP problem, meanwhile the performance improvement of the constrained MPC is shown by comparison between both control methods. In this paper, MPC scheme is based on a linear motion model of the supply ship with the amplitude and the rate of input constraints taken into account. The simulations are provided to illustrate the effectiveness and advantages.

This paper is organized as follows: The mathematic characteristics of the controlled plant is described in section 2. In section 3, the MPC algorithm is adopted to develop the constrained MPC controllers. The simulations to verify the proposed MPC algorithm based DP controllers are carried out in section 4, where the simulation results are analyzed as well. Finally, the conclusion is summarized in section 5.

II. DYNAMIC POSITIONING SHIP MODEL

Considering the natural robustness of the MPC method, high fidelity model for controller design is not must. Although the establishment of a proper model for the controlled plant is of great importance for good control performance. Two coordinates are commonly utilizing to denote a sufficiently detailed ship motions. They are earth-fixed coordinate which is also known as North-East-Down frame (NED frame), and body-fixed frame (B frame). The NED frame is defined as a Cartesian coordinate system, whose coordinate origin is fixed at some point on the earth, with three axes point to north, east and down to the Geocentric respectively, denoted as $X_E Y_E Z_E$ in Figure 1. The B frame is defined such that the coordinate origin is fixed at somewhere on the ship (which is usually consistent with the center of gravity) and three axes point to the bow, starboard and the bilge keel respectively, denoted as XYZ in Figure 1. They are both right-handed coordinate systems. The variables denote ship motion are described in Table 1.

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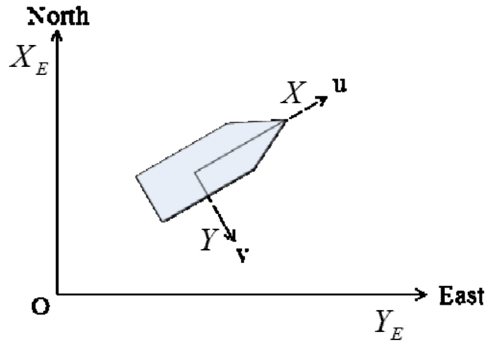

Fig. 1. The coordinate for DP motion description

TABLE I. VARIABLE DENOTATION AND DESCRIPTION

u :	surge velocity, linear velocity projected to X axis in B frame relative to NED frame.
v :	sway velocity, linear velocity projected to X axis in B frame relative to NED frame.
r :	yaw rate, angular velocity.
n :	displacement of the ship from NED frame origin along X_E axis.
e :	displacement of the ship from NED frame origin along Y_E axis.
ψ :	heading of the ship.

For surface ships, the ship motion is usually described by the three-degree motion components in surge, sway and yaw. According to [5], the general three-degree surface vessel equations of motion can be written in a vector setting as follows:

$$\dot{\boldsymbol{\eta}} = \mathbf{R}(\psi)\mathbf{v} \tag{1}$$

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}(\mathbf{v})\mathbf{v} = \boldsymbol{\tau} + \mathbf{w} \tag{2}$$

where $\boldsymbol{\eta} = [n\, e\, \psi]^{\mathrm{T}}$ represents ship position and heading; $\mathbf{v} = [u\, v\, r]^{\mathrm{T}}$ represents ship linear velocities and angular rate; $\boldsymbol{\tau} = [\tau_X\ \tau_Y\ \tau_N]^{\mathrm{T}}$ represents general control forces in three degree freedoms ; $\mathbf{w} = [w_1\ w_2\ w_3]^{\mathrm{T}}$ represents disturbances; $\mathbf{R}(\psi)$ is a transformation matrix related to heading information; $\mathbf{M}$ is the inertia matrix; $\mathbf{C}(\mathbf{v})$ is centripetal matrix and $\mathbf{D}(\mathbf{v})$ is damping matrix. For dynamic positioned ships, the ship motion are very slow. A linear model is usually adopted to describe the DP motion. Hence, the ship dynamics can be represented as

$$\dot{\boldsymbol{\eta}} = \mathbf{R}(\psi)\mathbf{v} \tag{3}$$

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{D}\mathbf{v} = \boldsymbol{\tau} + \mathbf{w} \tag{4}$$

where

$$\mathbf{R}(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{M} = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_g - Y_{\dot{r}} \\ 0 & mx_g - Y_{\dot{r}} & I_z - N_{\dot{r}} \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & -Y_r \\ 0 & -N_v & -N_r \end{bmatrix}.$$

Here, for a given ship, $\mathbf{M}$ and $\mathbf{D}$ are constant matrices.

III. MPC BASED DP CONTROL SYSTEM DESIGN

The DP control system usually includes three modules: an estimator, a controller and the constraint handling module as depicted in Figure 2. Control increment command $\Delta\mathbf{u}$ is computed by a DP controller according to reference information $\mathbf{y}_d$ and system state information $\mathbf{x}$ which is estimated by the estimator using system output information $\mathbf{y}$ measured by sensors. In practice, the control information is modified indirectly by the constraint handling module before being applied into actuator system for security and economics. Although this handling approach suits for many maneuvering scenes, the control performance would deteriorate in other maneuvering scenes. Thus the constrained control schemes which can directly deal with system constraints attract the attention from both researchers and engineers. As a classic constrained control scheme, the MPC algorithm based constrained DP controller is developed in this paper. To verify the control performance, the unconstrained MPC algorithm based constrained DP controller is designed too.

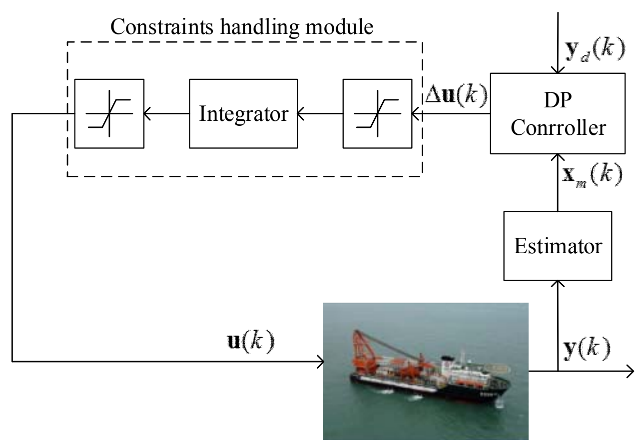

Fig. 2. The sketch of a common dynamic positioning control system

For DP control, the reference trajectory is constant position and heading information. The basic MPC algorithm is outlined in Algorithm 1. The idea of MPC scheme is: at each sampling time t, the predicted future behavior is optimized over the predictive horizon $k = 0,1,...,N_P - 1$ as an optimal control sequence, and then the first element is taken as the feedback control law for the next sampling interval.

TABLE II. CONSTRAINED MPC ALGORITHM FOR DP

Algorithm 1. MPC algorithm for DP controller
For each sampling time t_i ($i = 0,1,\ldots$, sampling instant):
(1) Measure the system state $\mathbf{x}(i)$.
(2) Set $\mathbf{x}_0 = \mathbf{x}(i)$,
(3) solve the optimal control problem(OCP)
minimize $J = \frac{1}{2}\sum_{k=0}^{N_P-1}(\mathbf{y}_d - \mathbf{y})^{\mathrm{T}} Q(\mathbf{y}_d - \mathbf{y}) + \frac{1}{2}\sum_{k=0}^{N_C-1}\Delta\mathbf{u}(k)^{\mathrm{T}} R\Delta\mathbf{u}(k)$
subject to
$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\Delta\mathbf{u}(k)$
$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k)$
$\mathbf{x}(k) \in \mathbb{X}$
$\mathbf{u}(k) \in \mathbb{U}$
then, the obtained control sequence is denoted as $\{\Delta\mathbf{u}\}_{N_C}^{*}$.
(4) The MPC control law is defined as the first element of the optimal control

To design MPC based DP controller, the predictive model, the objective function and the OCP solution is completed below.

A. Predictive model for MPC

Given that the supply ship is positioned at fixed heading ψ_0, Equation 3 is linearized near the ship heading ψ_0 as follows:

$$\dot{\boldsymbol{\eta}} = \mathbf{R}_{\psi_0}\boldsymbol{\nu} \tag{5}$$

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{D}\boldsymbol{\nu} = \boldsymbol{\tau} + \mathbf{R}_{\psi_0}^{\mathrm{T}}\mathbf{b} \tag{6}$$

For simplicity, $\mathbf{x}_m = \left[\boldsymbol{\nu}^{\mathrm{T}}\ \boldsymbol{\eta}^{\mathrm{T}}\right]^{\mathrm{T}}$ is chosen to be a new state variable vector. Thus, the ship motion model is reformed as the following continuous state-space model

$$\dot{\mathbf{x}}_m = \mathbf{A}_c\mathbf{x}_m + \mathbf{B}_c\mathbf{u} \tag{7}$$

$$\mathbf{y} = \mathbf{C}_c\mathbf{x}_m \tag{8}$$

where

$$\mathbf{u} = \boldsymbol{\tau} + \mathbf{R}_{\psi_0}^{\mathrm{T}}\mathbf{b}$$

$$\mathbf{A}_c = \begin{bmatrix} -\mathbf{M}^{-1}\mathbf{D} & \mathbf{0}_{3\times3} \\ \mathbf{R}_{\psi_0} & \mathbf{0}_{3\times3} \end{bmatrix}, \mathbf{B}_c = \begin{bmatrix} \mathbf{M}^{-1} \\ \mathbf{0}_{3\times3} \end{bmatrix}, \ \mathbf{C}_c = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \end{bmatrix}.$$

Considering that MPC can deal naturally with the discrete-time system. The DP system is firstly discretized as following:

$$\mathbf{x}_m(k+1) = \mathbf{A}_m\mathbf{x}_m(k) + \mathbf{B}_m\mathbf{u}(k) \tag{9}$$

$$\mathbf{y}(k) = \mathbf{C}_m\mathbf{x}_m(k) \tag{10}$$

where k represents the k-th time instant of the discrete-time system. Denoting

$$\mathbf{x}(k) = \begin{bmatrix} \Delta\mathbf{x}_m(k) \\ \mathbf{y}(k) \end{bmatrix},$$

the predictive model is eventually obtained as below

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\Delta\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) \end{aligned} \tag{11}$$

where $\mathbf{A} = \begin{bmatrix} \mathbf{A}_m & \mathbf{0}_{6\times3} \\ \mathbf{C}_m\mathbf{A}_m & \mathbf{I}_{3\times3} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{B}_m \\ \mathbf{C}_m\mathbf{B}_m \end{bmatrix}, \mathbf{C} = \begin{bmatrix} \mathbf{0}_{3\times6} & \mathbf{I}_{3\times3} \end{bmatrix}$.

B. Objective function for MPC

The objective function is defined in quadratic form as below

$$J = \frac{1}{2}\sum_{k=0}^{N_P-1}(\mathbf{y}_d - \mathbf{y})^{\mathrm{T}} Q(\mathbf{y}_d - \mathbf{y}) + \frac{1}{2}\sum_{k=0}^{N_C-1}\Delta\mathbf{u}^{\mathrm{T}} Q\Delta\mathbf{u} \tag{12}$$

Assuming that, at the sampling instant k , the current ship motion information $\mathbf{x}(k)$ is available through measurement and/or estimation. The future control trajectory is defined as

$$\Delta U \triangleq \left[\Delta\mathbf{u}_k^{\mathrm{T}}\ \Delta\mathbf{u}_{k+1}^{\mathrm{T}} \ldots \Delta\mathbf{u}_{k+N_C-1}^{\mathrm{T}}\right]^{\mathrm{T}} \tag{13}$$

Similarly, we define the output vector as

$$Y \triangleq \left[\mathbf{y}_{k+1|k}^{\mathrm{T}}\ \mathbf{y}_{k+2|k}^{\mathrm{T}} \ldots \mathbf{y}_{k+N_P|k}^{\mathrm{T}}\right]^{\mathrm{T}} \tag{14}$$

The superscript $^{\mathrm{T}}$ indicates matrix transpose. The subscript $_k$ indicates the k-th sampling instant, and the subscript $_{k+i|k}$ indicate the i-th prediction information upon the k-th sampled information of ship motion, where $i = 1,\ldots,N_P$. Then, the predicted output trajectory can be calculated by

$$Y = F\mathbf{x}(k) + \Phi\Delta U \tag{15}$$

where

$$F = \begin{bmatrix} \mathbf{CA} \\ \mathbf{CA}^2 \\ \mathbf{CA}^3 \\ \vdots \\ \mathbf{CA}^{N_P} \end{bmatrix},$$

$$\Phi = \begin{bmatrix} \mathbf{CB} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \cdots & \mathbf{0}_{3\times3} \\ \mathbf{CAB} & \mathbf{CB} & \mathbf{0}_{3\times3} & \cdots & \mathbf{0}_{3\times3} \\ \mathbf{CA}^2\mathbf{B} & \mathbf{CAB} & \mathbf{CB} & \cdots & \mathbf{0}_{3\times3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{CA}^{N_P-1}\mathbf{B} & \mathbf{CA}^{N_P-2}\mathbf{B} & \mathbf{CA}^{N_P-3}\mathbf{B} & \cdots & \mathbf{CA}^{N_P-N_C}\mathbf{B} \end{bmatrix}.$$

Assuming the set-point reference information is denoted by vector $\mathbf{y}_d(k)$ defined by

$$Y_d \triangleq \underbrace{\left[\mathbf{I}_{3\times3} \ldots \mathbf{I}_{3\times3}\right]}_{N_P\ elements\ of\ \mathbf{I}_{3\times3}}{}^{\mathrm{T}}\ \mathbf{y}_d(k) = D\mathbf{y}_d(k) \tag{16}$$

Finally, the objective function is eventually reformed as

$$J = \frac{1}{2}(Y_d - Y)^{\mathrm{T}} Q (Y_d - Y) + \frac{1}{2}\Delta U^{\mathrm{T}} R \Delta U \tag{17}$$

C. MPC considering amplitude and increment constraints

Usually encountered system limits in DP control system are input constraints and system constraints. Here in this paper, we only focus on the amplitude and increment constraints on control input. As hard constraints, increment constraints on control input is specified as following:

$$\begin{bmatrix} -I \\ I \end{bmatrix} \Delta U \le \begin{bmatrix} -\Delta U_{min} \\ \Delta U_{max} \end{bmatrix} \tag{18}$$

where ΔU_{min} and ΔU_{max} are column vectors with N_C elements of $\Delta \mathbf{u}_{min}$ and $\Delta \mathbf{u}_{max}$. As for the constraints on amplitude of control input, traditionally, they are imposed for all future instants. The future input trajectory is:

$$\begin{bmatrix} \mathbf{u}(k) \\ \mathbf{u}(k+1) \\ \mathbf{u}(k+2) \\ \vdots \\ \mathbf{u}(k+N_C-1) \end{bmatrix} = C_1 \mathbf{u}(k-1) + C_2 \begin{bmatrix} \Delta\mathbf{u}(k) \\ \Delta\mathbf{u}(k+1) \\ \Delta\mathbf{u}(k+2) \\ \vdots \\ \Delta\mathbf{u}(k+N_C-1) \end{bmatrix} \tag{19}$$

where

$$C_1 = \begin{bmatrix} I \\ I \\ I \\ \vdots \\ I \end{bmatrix}, \; C_2 = \begin{bmatrix} I & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ I & I & \mathbf{0} & \cdots & \mathbf{0} \\ I & I & I & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I & I & I & \cdots & I \end{bmatrix}.$$

Thus, the amplitude constraints on control input are specified as

$$-C_1 \mathbf{u}(k-1) - C_2 \Delta U \le -U_{min} \tag{20}$$

$$C_1 \mathbf{u}(k-1) + C_2 \Delta U \le U_{max} \tag{21}$$

where U_{min} and U_{max} are column vectors with N_C elements of $\mathbf{u}_{min}$ and $\mathbf{u}_{max}$.

Finally, the constrained MPC problem is transferred a minimization problem to find the optimal ΔU that minimizes

$$J = \frac{1}{2}(D\mathbf{y}_d(k) - F\mathbf{x}(k))^{\mathrm{T}} Q (D\mathbf{y}_d(k) - F\mathbf{x}(k)) - \Delta U^{\mathrm{T}} \Phi^{\mathrm{T}} Q (D\mathbf{y}_d(k) - F\mathbf{x}(k)) + \frac{1}{2}\Delta U^{\mathrm{T}} (\Phi^{\mathrm{T}} Q \Phi + R) \Delta U \tag{22}$$

subject to

$$\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \Delta U \le \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \tag{23}$$

where

$$M_1 = \begin{bmatrix} -C_2 \\ C_2 \end{bmatrix}, \; N_1 = \begin{bmatrix} -U_{min} + C_1 \mathbf{u}(k-1) \\ U_{max} - C_1 \mathbf{u}(k-1) \end{bmatrix}, \; M_2 = \begin{bmatrix} -I \\ I \end{bmatrix},$$
$$N_2 = \begin{bmatrix} -\Delta U_{min} \\ \Delta U_{max} \end{bmatrix}.$$

For compactness, generally, the constrained MPC problem is reformulated as a standard quadratic programming (QP) problem

$$J = \frac{1}{2}\Delta U^{\mathrm{T}} E \Delta U + \Delta U^{\mathrm{T}} F + J_0 \tag{24}$$

$$M \Delta U \le \gamma \tag{25}$$

where

$$E = \Phi^{\mathrm{T}} Q \Phi + R, \; F = -\Phi^{\mathrm{T}} Q (D\mathbf{y}_d(k) - F\mathbf{x}(k)),$$

$$J_0 = \frac{1}{2}(D\mathbf{y}_d(k) - F\mathbf{x}(k))^{\mathrm{T}} Q (D\mathbf{y}_d(k) - F\mathbf{x}(k)),$$

$$M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}, \; \gamma = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}.$$

D. Online optimization for MPC

As for unconstrained MPC, according to the minimum principle

$$\frac{\partial J}{\partial \Delta U} = -\Phi^{\mathrm{T}} Q (D\mathbf{y}_d(k) - F\mathbf{x}(k)) + (\Phi^{\mathrm{T}} Q \Phi + R)\Delta U = 0 \tag{26}$$

the optimal solution of the control input is obtained assuming that $(\Phi^{\mathrm{T}} Q \Phi + R)^{-1}$ exists:

$$\Delta U = (\Phi^{\mathrm{T}} Q \Phi + R)^{-1} \Phi^{\mathrm{T}} Q (D\mathbf{y}_d(k) - F\mathbf{x}(k)) \tag{27}$$

As for constrained MPC problem, adding a Lagrange multiplier λ, the optimization problem is equal to

$$J = \frac{1}{2}\Delta U^{\mathrm{T}} E \Delta U + \Delta U^{\mathrm{T}} F + J_0 + \lambda^{\mathrm{T}} (M\Delta U - \gamma) \tag{28}$$

$$\lambda^{\mathrm{T}} (M\Delta U - \gamma) = 0 \tag{29}$$

The necessary conditions for this problem are the Kuhn-Tucker Conditions:

$$\begin{aligned} E\Delta U + F + M^{\mathrm{T}}\lambda &= 0 \\ M\Delta U - \gamma &\le 0 \\ \lambda^{\mathrm{T}}(M\Delta U - \gamma) &= 0 \\ \lambda &\ge 0 \end{aligned} \tag{30}$$

The procedure of minimization is to find the active sets of constraints. Supposing that the active constraints are satisfied, then M_{act} and λ_{act} related to active constraints are known. The optimization solution is

$$\lambda_{act} = -(M_{act}E^{-1}M_{act}^{\mathrm{T}})^{-1}(\gamma_{act} + M_{act}E^{-1}F) \tag{31}$$

$$\Delta U = -E^{-1}(F + M_{act}^{\mathrm{T}}\lambda_{act}) \tag{32}$$

E. Stability analysis

The standard form of the MPC based state feedback control law is:

$$\begin{aligned}\Delta\mathbf{u}(k) &= K(\Phi^{\mathrm{T}}Q\Phi + R)^{-1}\Phi^{\mathrm{T}}Q(D\mathbf{y}_d(k) - F\mathbf{x}(k)) \\ &= K_y\mathbf{y}_d(k) - K_{mpc}\mathbf{x}(k)\end{aligned} \tag{33}$$

where $K = \left[\mathbf{I}_{3\times3}\ \mathbf{0}_{3\times3} \ldots \mathbf{0}_{3\times3}\right]$, $K_y = K(\Phi^{\mathrm{T}}Q\Phi + R)^{-1}\Phi^{\mathrm{T}}QD$ and $K_{mpc} = K(\Phi^{\mathrm{T}}Q\Phi + R)^{-1}\Phi^{\mathrm{T}}QF$. Considering the special structures of $\mathbf{C}$ and $\mathbf{A}$, we can write $K_{mpc} = \left[K_x\ K_y\right]$, where K_x corresponds to the feedback gain vector related to $\Delta\mathbf{x}_m(k)$, and K_y corresponds to the feedback gain related to $\mathbf{y}(k)$ .The closed-loop system is obtained as below

$$\mathbf{x}(k+1) = (\mathbf{A} - \mathbf{B}K_{mpc})\mathbf{x}(k) + \mathbf{B}K_y\mathbf{y}_d(k) \tag{34}$$

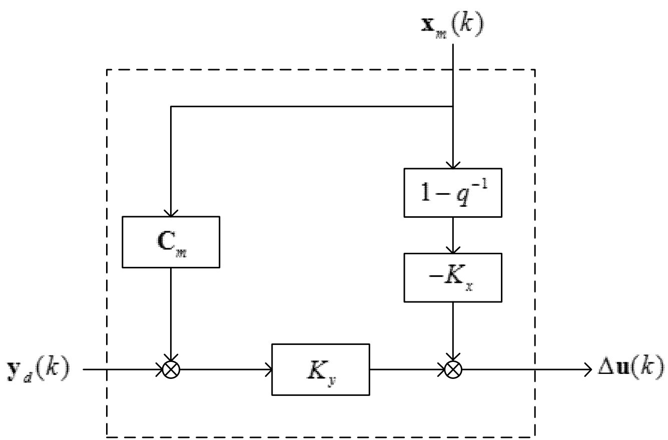

Fig. 3. The block diagram of the discrete-time MPC controller

The block diagram of the MPC controller is obtained as in Fig. 3. The eigenvalues of the closed-loop system can be calculated through the characteristic equation

$$\det\left[\lambda I - (\mathbf{A} - \mathbf{B}K_{mpc})\right] = 0 \tag{35}$$

Supposing that the given system $\left(\mathbf{A},\mathbf{B},\mathbf{C}\right)$ is controllable, which is reasonable for dynamic positioning systems, the closed-loop eigenvalues can be manipulated, thus, into a satisfying range through tuning the weight matrices Q and R .According to Equation 38, the adopted incremental control is

$$\Delta\mathbf{u}(k) = K_y\mathbf{y}_d(k) - K_{mpc}\mathbf{x}(k) - K_0 \tag{36}$$

where $K_0 = (\Phi^{\mathrm{T}}Q\Phi + R)^{-1}M_{act}^{\mathrm{T}}\lambda_{act}$ is a correction term due to the active constraints. The eigenvalue can be manipulated into a satisfying zone to stabilize the control system if there exists the satisfying zone.

IV. SIMULATION AND RESULTS

Suppose that the sampling time of the DP system is $\Delta T = 0.3$ sec. The MPC controller is designed with control horizon $N_C = 5$ and predict horizon $N_P = 30$. The limits on the amplitude and the rate of change on the control signal are specified as

$$\begin{bmatrix}-980000\\-417000\\-18000000\end{bmatrix} \le \begin{bmatrix}\tau_X\\\tau_Y\\\tau_N\end{bmatrix} \le \begin{bmatrix}1240000\\417000\\18000000\end{bmatrix} \begin{bmatrix}-32736\\-11008\\-475200\end{bmatrix} \le \begin{bmatrix}\Delta\tau_X\\\Delta\tau_Y\\\Delta\tau_N\end{bmatrix} \le \begin{bmatrix}32736\\11008\\475200\end{bmatrix}$$

The initial position coordinates and heading information of the DP ship are $(1.2m, 1.3m)$ and 0° respectively. The ship is maneuvered to the target point $(0m, 0m, 0^\circ)$ by the constrained MPC algorithm based DP controller. Common constraint handling method is utilizing in the unconstrained DP controller for contrast. The ocean environment is simulated as low-frequency sinusoidal forces and moment in three degrees of freedom respectively, which are approximations of the actual ocean effects on the ship, reflecting the dynamic external forces and moment onto the ship to some extent. The trajectory information is shown in Figure 4. We can see that the controller is effective to keep the ship nearly stationary within the 1-meter precision circle, and the constrained MPC control system maneuvers the ship more smoothing on transient stage, and smaller steady error on steady stage, which is quantitatively shown in Figure 5. The DP precision, general control command/actual forces and their increments are shown in Figure 5, 6 and 7 respectively. In figure 6 and 7, the black dashed lines represent the rate constraints and blue lines represent the actual input rate. In Figure 7, the input rate command is the same as the actual input rate.

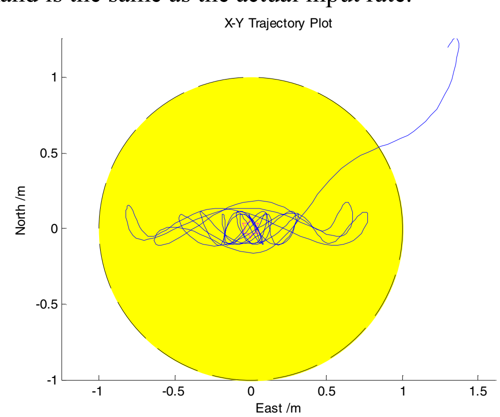

Fig. 4. The positioned ship trajectory under constrained MPC controller.

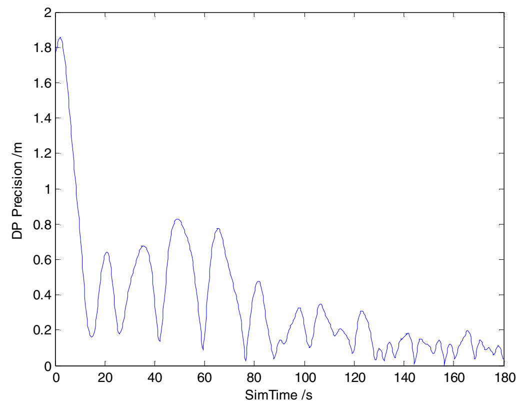

Fig. 5. The DP precision under constrained MPC controller.

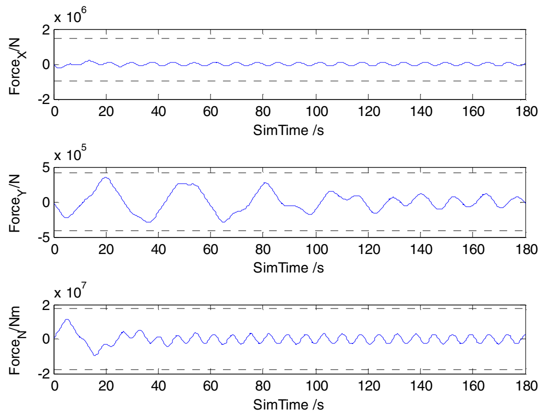

Fig. 6. The general control force/moment under constrained MPC controller.

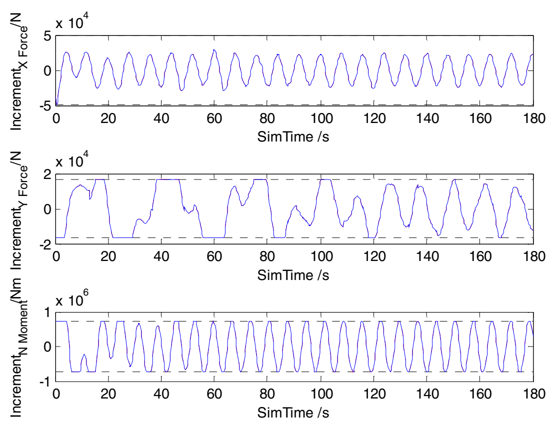

Fig. 7. The increments of general control force/moment under constrained MPC controller.

V. CONCLUSIONS

In this paper, we have designed constrained MPC algorithm based DP controllers for a supply ship. Meanwhile, the simulation is carried out to verify the effectiveness of the controller and to verify the advantage of direct constraint handling in controller design. Because of the constraint handling ability, the constrained MPC controller performs smoothly and precisely according to the simulation results. This paper finally shows the advantages of the MPC algorithm applying to dynamic positioning systems, which is as well promising in the application of the other ship motion control field.

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