

Observations of Predictive Skill for Real-Time Deterministic Sea Waves from the WaMoS II

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Abstract—OceanWaveS GmbH has been developing a prototype system, based on standard non-coherent X-band navigational radars, capable of predicting future sea surface elevations. Combined with a vessel hydrodynamic simulator, this system will forecast ship motions. Applications for such a system include offshore operations, e.g. crane lifts, LNG, cargo and personnel transfers. The forecasts may be assimilated into automated control systems; e.g. floating wind turbines or dynamic positioning systems, or used within Decision Support Systems. This article presents correlation analysis results between the predicted sea surface elevations and vessel-mounted reference sensors for three independent sea trials. Despite neglecting vessel hydrodynamics, correlations of forecasts to measured references of 80% are achieved for T+60 seconds predictions for all sea trials. The prediction horizon is observed up to 180 seconds of forecast.

I. INTRODUCTION

This article presents initial results from a real-time prototype wave prediction system developed by OceanWaves GmbH, using the Wave and Surface Current Monitoring System (WaMoS II). The utility of wave prediction is primarily vessel motion prediction. Specific applications include helicopter landings, liquid nitrogen gas transfers, maritime construction, small craft recovery, and crane operations. These activities are limited in their operation time due to wave induced motions which damage equipment and endanger personnel. The reactive control systems of wave energy converters and floating wind turbines may benefit from predictive wave information, via reductions in control surface variance and wear. Simulations of a point wave energy converter show a two-fold increase in energy output for a control scheme using Deterministic Sea Wave Prediction (DSWP) [1].

The analysis herein includes empirical results collected from three field trials. A correlation analysis is used to demonstrate the predictive skill of the WaMoS II derived sea surfaces against independent measurements of vessel motion. Observations of the effective Prediction Region are reported. Vessel hydrodynamics have been omitted from the analysis.

The prototype system is capable of real-time 3-dimensional Sea Surface Elevation (SSE) predictions (Fig. 1). An example feature is the Decision Support GUI (Fig. 3), which visualizes in real-time both the predicted vessel heave and the historical predictions relative to the measured vessel heave. Coloured indicators inform the operator when limiting criteria are exceeded.

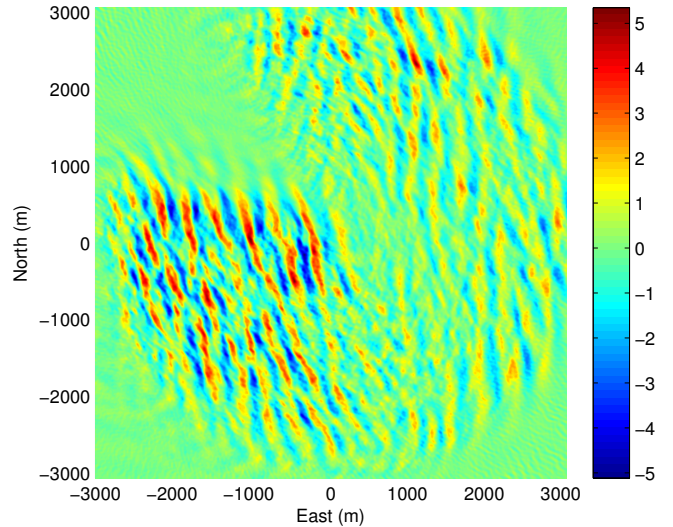


Fig. 1. An example WaMoS II sea surface elevation (SSE) field derived from the NSRS sea trial data, with the color scale indicating elevation in meters. The prototype algorithm uses the full sampling domain of the radar, just over 3 km for this example. The dominant wave direction is propagation towards the North-East. Characteristics of the EM: ocean wave measurement principle remain in the derived SSE, including a directional bias along the dominant wavevector, and absence of signal due to vessel shadowing [North-West].

A. WaMoS II

The WaMoS II derives both statistical sea state parameters and 3-dimensional sea surface elevation maps from nautical X-band radars. The measurement principle is based on sea surface modulation of the backscattered electromagnetic waves [2]–[6]. A technique for deriving the 3-dimensional unambiguous ocean wave spectrum from a series of radar images was developed by [7]. The method is based on the spectral analysis of radar data via a 3-dimensional (Fast-) Fourier Transform. See [8] for a description of the WaMoS II processing method.

A deterministic system requires sufficient information of the sea state in real-time. The appeal of WaMoS II is its ability to measure the sea surface with high temporal and spatial resolution, combined with a large observation domain, in real-time. This contrasts with conventional oceanographic wave buoys, sonars, and pressure sensors. These instruments measure at a spatial point or small region, and do not provide information over a sufficiently large spatial domain. They are

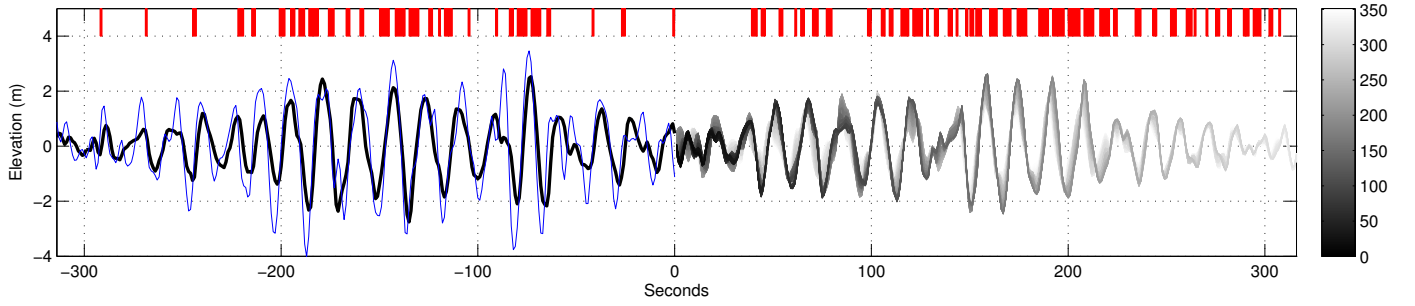


Fig. 3. Example of the Decision Support GUI with data from the NSRS sea trial. The WaMoS II derived sea surface elevation predictions are displayed in grey scale, from real-time [black] to 350 seconds forecast [white]. A motion reference unit [blue] provides feedback on the prediction skill. An (arbitrary) limiting criterion of less than 0.5 m/s velocity is displayed at the top [red]. The operator chooses a sufficiently long interval absent of predicted limiting criteria, from time [0, +30] seconds in this example.

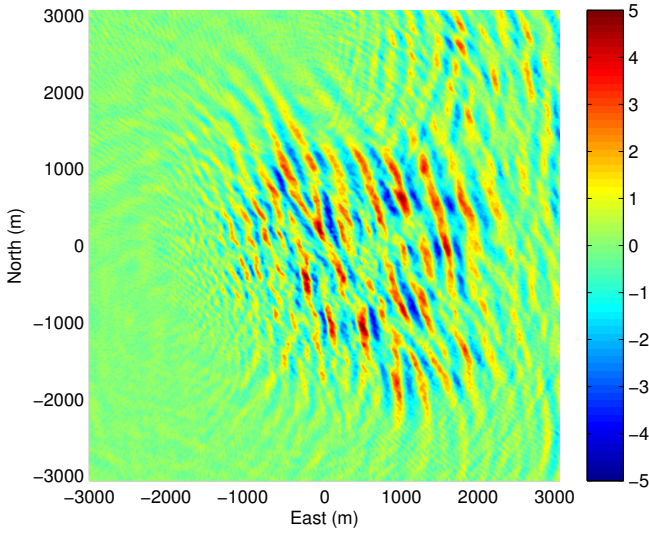


Fig. 2. An example T+180 second SSE prediction. Waves that were well-resolved within the South-West region of the now-cast (Fig. 1) have propagated to the North-East. Peak wavenumber at this time was 0.0197 rad/m. The apparent propagation distance of 1.9 km agrees well with the corresponding group speed of 11.15 m/s.

also infeasible for moving vessels.

Similar DSWP technologies include AGI's Landing Pad Designator [9] and DRDC's Flight Deck Motion System. The commonality between these systems is the use of a Motion Reference Unit (MRU) to obtain real-time vessel orientation, in conjunction with vessel hydrodynamic models, to predict the immediate vessel response. The prediction horizon of these systems is deterministically limited by the non-spatial measurement of the MRU. Thus only the kinematic and potential energy of the vessel in conjunction with a historical representation of the sea state is available for derivation of predicted motions. As discussed in the proceeding sections, measurements of the sea state at a distance of $O(100)$ meters are necessary to extend the prediction horizon past $O(10)$ seconds. LIDAR is an alternative technology with a measurement domain comparable to X-Band radar, but does not have the large existing hardware base of navigational X-Band radar, which are often mandatory for vessels exceeding a given tonnage or size. [10] reports on the use of LIDAR for sea

surface measurements.

B. Deterministic Sea Wave Prediction

Deterministic Sea Wave Prediction implies phase-resolved short-term (30-90 sec) predictions of the sea surface elevation to the order of centimeters and seconds accuracy and precision. In this sense, it differs from traditional statistical descriptions of the sea state. This imposes < 10 sec computation time and update rate for DSWP systems.

In recent years, the field of Deterministic Sea Wave Prediction has received increasing attention. Successful prediction of SSE and wave induced ship motions from experimental data was reported by [11]. [12] evaluated two methods for linear DSWP using both simulations and empirical sea trial data from the WaMoS II. Simulations of the fixed-point solution method resulted in correlation values with median values $R \approx [0.7, 0.8]$ for T+30 second predictions. Correlations values from the sea trial data ranged from $R = [0, 0.8]$.

[13] evaluated the accuracy of SSE derived from the WaMoS II via comparison to a TriaxysTM wave buoy. After removing registration errors, the majority of correlation values exceed 0.87, leading to conclude the WaMoS II was sufficiently accurate for input to a DSWP system. The newly developed, Environmental and Ship Motion Forecasting (ESMF) system is an operational DSWP, with empirical correlations exceeding 80% for T+30 second predictions over 30 minute intervals [14]–[16]. It applies a least-squares inversion method to solve for the sea state, and a Reduced Order Model for ship motion forecasting.

C. Prediction Region

In the context of DSWP, the Prediction Region is defined as the space-time region where predictions are possible, i.e. causally deterministic, as determined by the space-time measurement domain and the propagation speeds of the dominant waves [17], [18]. [19] simulated the prediction zone using the Pierson-Moskowitz empirical sea model, and found that triangular prediction zones roughly approximate the prediction region, but wave propagation velocities were insufficient to fully describe the prediction region boundaries. Using linear wave theory for simulated wave fields combined with laboratory experiments, [20] addressed the debate whether group or phase speed is the controlling velocity, and concluded

the group speed adequately indicates the predictable zone. Given the nominal 1-4 km range of usable radar signal, the prediction horizon is of order 1-2 minutes. Consequently, DSWP algorithms must have exceedingly short output latency, i.e. less than 10 seconds.

II. ALGORITHM

The short computation time combined with the inherent complexity of the sea leads to practical limitations for DSWP algorithms. The linearity assumption is used to reduce the computational burden. Although resolving non-linear wave interactions may lead to improved accuracy, the propagation distances (1-4 km) are not far enough for non-linearities to be significant [21], [22].

The prototype algorithm used herein follows the general outline described in [19]. Summarized here:

- 1) Measure the sea surface over a spatial-temporal domain, hereafter the observation domain
- 2) Apply pre-processing methods
- 3) Compute the magnitude and phase of wavevector coefficients
- 4) Phase shift the wavevector coefficients to propagate, i.e. predict, the sea surface profile at space-time offsets

Item (1), at the lowest level, is digitized radar signal from a navigational X-Band radar. This is the hardware component of the WaMoS II. A summary of the WaMoS II measurement properties is given in [13]. The maximum sampling resolution is 3.5 meters and 1.25/2.50 seconds, depending on the radar model. The range varies depending on environmental conditions, nominally (1-4) km. The native sampling is polar coordinates (range and angle) relative to the vessel, and helical in space-time due to the antenna rotation. Vessel rotation and translation further complicate the observation domain, leading to non-uniform Lagrangian measurements of the ocean surface. Assimilation of GPS coordinates and gyroscopic heading register the data to an Earth-fixed Eulerian reference frame. Errors in these registration data sources can result in prohibitive inaccuracies in the derived sea surface elevations [13]. For DSWP, the location of the derived sea surface is equally important as elevation. Such registration errors should be included in DSWP simulations. Primary error sources identified include random 5-15 m GPS positioning error, and gyroscope latency.

Following linear wave theory, wave propagation is achieved via phase shifts of the wavevector coefficients corresponding to the space-time translation between the measurement domain and the prediction domain. Thus, a given instantaneous set of coefficients may be used to predict SSE at any arbitrary space-time position, although the accuracy on any such prediction is limited by the Prediction Region. Estimation of the wavevector coefficients via the 3D-FFT forces joint domain periodicity on the predictions. See [23], [24] for an analysis of the suitability of the 3D-FFT method for surface wave prediction. [13] used the 3D-FFT method combined with phase propagation to predict SSE, and found no significant correlation to a vessel mounted MRU. [25] describes a method for predicting SSE using a 2D-FFT. Simulations resulted in correlation values of $R = 0.91$ for real-time SSE, and $R = 0.84$ for T+135 second predictions.

Most X-Band radars do not provide a direct measurement of the sea surface elevation. Rather, they provide a measurement in volts of the backscattered EM intensity. The WaMoS II calibrates the derived wave spectra against references following the Signal-to-Noise Ratio (SNR) method of [7]. A variety of alternative methods exist for addressing this fundamental scaling issue. Measurements of the wave-induced motions of the vessel, e.g. via a Motion Reference Unit (MRU), provide a collocated reference. [26] [27] developed a Shipboard Wave Data Fusion (SWDF) system which calibrates the WaMoS II 2-D wave spectrum (non-phase-resolved) using MRU measurements and vessel Response Amplitude Operators (RAOs). [14] addressed the scaling issue with a custom coherent radar; obtaining Doppler measurements of wave orbital velocity in addition to EM backscatter intensity. [28] has proposed a novel scaling method based on the statistical properties of EM shadowing by ocean waves. [29] used a neural network approach based on SNR, peak wave length, and mean wave period to improve significant wave height estimates. [25] proposes a scaling method using historical RMS ratios of the predicted sea surfaces to MRU measurements.

With computation time the primary limiting factor, the prototype algorithm used herein was designed to be "running" or "sliding" over a coherent interval. The matrices representing the current state of the sea surface are modified incrementally; with new data appended and old data removed, analogous to a FIFO buffer. The equations are then re-evaluated, with all efforts made to reduce redundant computation. The consequence of this algorithm design is a relatively constant computational load imposed on the processor, and a large random-access memory requirement, as the historical state of matrices must be retained over varying coherent intervals. This design allows for continuous wave prediction output at rates exceeding 1 Hz. Conversely, the 3D-FFT algorithm of [7] necessitates intermittent output, with computation time $O(10)$ seconds detracting from the prediction horizon. Overlapping 3D-FFT analyses in a "sliding" fashion is currently computationally prohibitive.

The prototype algorithm is not adaptive; defined as an internal variable or function that varies depending on the input data. This excludes iterative solution methods. Thus the algorithm may be conceptualized as a linear time invariant operator. This design choice was made to simplify analysis of the algorithm during the initial testing phase, and to isolate input variability from algorithm-induced variability. Once the limiting factors and guiding principles of DSWP methods are well known, more sophisticated (adaptive) algorithmic methods will be implemented. The prototype algorithm assimilates all the raw radar data, over the full sampling domain, at rates exceeding 2.1 Msps, with a dynamic range of 12 bits, corresponding to a bit rate of 34.1 MiBps. [13] found that 90% of the WaMoS II derived SSE variance could be accounted for with 12,500 wavevector coefficients. Results herein were calculated using a fixed domain of 22,860 coefficients (Fig. 4).

III. SEA TRIALS

Data from three prior sea trials was used to test the predictive skill of the prototype algorithm:

- The 2008 Joint Industry Project, On board Wave and Motion Estimator (OWME), [30] was conducted in the

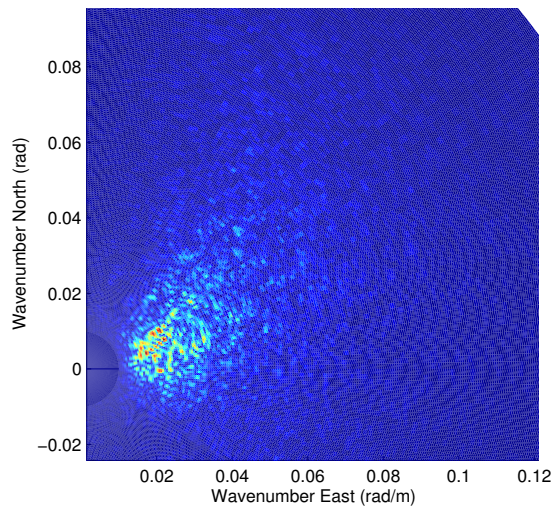


Fig. 4. An example WaMoS II prototype 2D wave vector spectrum, corresponding to the SSE in Fig. 1 and Fig. 2. The full range of the spectrum is ± 0.12 rad/m.

Gullfaks oil field off the coast of Norway, at a water depth of approximately 127 meters, hereafter 2008 OWME trial. An offshore support vessel with dynamic positioning enabled maintained a nearly constant position and heading, under mild sea conditions, from 2008-09-22 to 2008-09-26. Significant wave height ranged from 1 to 3 meters.

- In support of the Seakeeping Operator Guidance project, a sea trial was conducted aboard the Canadian Forces Auxiliary Vessel "Quest"; directed by Defence Research and Development Canada (DRDC). Measurements were taken in November 2012 on the Scotian slope, southeast of Halifax, Canada. The location was deep water, under large wave conditions; significant wave heights within 2-6 meters and peak wave periods within 8-12 seconds. The Quest operated under steady (non-inertial) headings and speed, with a regular manoeuvring pattern about 4 moored wave buoys.
- As part of the NATO Submarine Rescue System (NSRS) program [31], sea trials were conducted off the Western shelf of Scotland near the Stanton banks in November of 2014. The NSRS involves a mother ship, typically a vessel of approximately 10,000 tons, equipped to launch and recover a 30 ton rescue mini-submarine [32]. The vessel operated under steady headings and speed, with only occasional manoeuvring to conduct operations.

IV. CORRELATION ANALYSIS

The predictive skill metric used in this analysis is the correlation coefficient between the WaMoS II derived sea surface elevation (SSE) and the MRU derived vessel heave. All three aforementioned sea trials were equipped with vessel-mounted MRUs. Although information from the MRU could be used to improve or adjust the sea surfaces, such merging of information has been explicitly avoided in this analysis.

Thus the SSE and vessel heave are independent measurements. Correlations values stated in this article are calculated using Pearson's linear correlation [33], commonly denoted R . Results presented herein are from a prototype algorithm recently developed, and applied to historical sea trial data.

The MRU measures accelerations in a device-relative reference frame. Nine degrees of freedom are provided as 3-dimensional linear accelerations, rotational velocities, and magnetic field magnitude. Conversion to an Earth-relative (East,North,Up) reference frame was performed using Attitude and Heading Reference System (AHRS) equations. The Earth-relative linear accelerations were then spectrally integrated to linear displacements, retaining only the vertical component as vessel heave. The spectral integration was performed in post-processing, allowing for zero-phase distortion. For systems with native acceleration or velocity measurements, e.g. accelerometer or Doppler-GPS wave buoys, real-time causal integration filters introduce frequency-dependent phase delays which must be accounted for. No correction for the spatial offset between the radar and MRU mounting locations was applied. Vessel hydrodynamics were entirely omitted. Thus reduced correlation values are expected due to the vessel transfer function.

Because the WaMoS II and MRUs were acquired by different systems, their clocks have a time offset. The clock offset was evaluated at the beginning and end of each trial. The value of this time offset was determined by a sliding correlation between the SSE and MRU. Sliding correlation is analogous to standard lagged correlation in time. Whereas lagged correlation wraps values unphysically, sliding correlation uses earlier or later time samples. For 2 of the 3 sea trials, significant clock offsets were found; 11 and 47 seconds for the OWME and NSRS trials, respectively. Once clock offset was known, the time registration is corrected and no further adjustments are made. The clocks are taken to be correct without drift. For most modern hardware, this is a valid assumption for periods of weeks to months.

The timeseries of cross-correlation between the SSE predictions and the MRU is then calculated using sliding correlation. Correlation values were generally highest for the 2008 OWME trial, regularly exceeding $R > 0.8$ for T+60 second predictions, presumably due to the dynamic positioning of the vessel (Fig. 5). For all trials, the correlation values vary between $R = [0, 0.8]$. Histograms of the correlations provide a convenient summary of the varying performance (Figs. 8, 9, 10).

V. CONCLUSION

This article evaluates the predictive skill of the Wave and Surface Current Monitoring System WaMoS II, combined with a prototype algorithm for deriving deterministic sea surface elevations. The high spatial and temporal resolution of navigational X-Band radars, combined with the large observation area, satisfy the fundamental requirement of sufficient information for determinism.

The definition of sufficiently accurate sea surface predictions depends on the application. The correlation analysis results herein demonstrate that deterministic predictions are

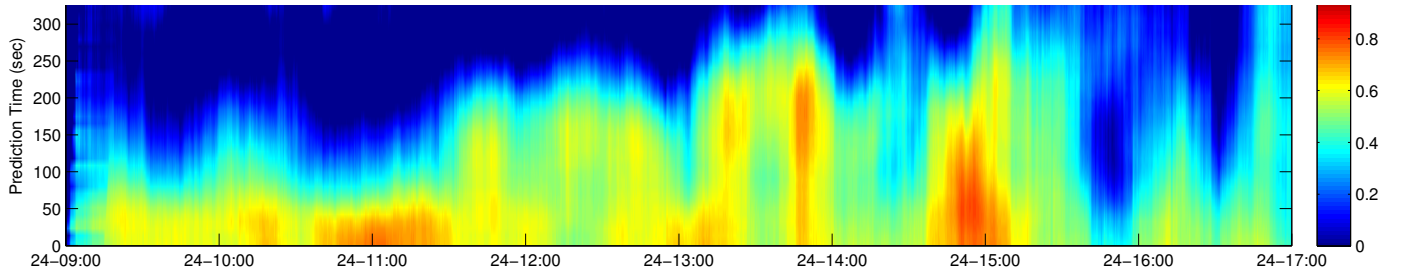


Fig. 5. Timeseries of correlation between predicted SSE and MRU for the 2014 NSRS trial. The horizontal axis is 8 hours of real time, the vertical axis is seconds of prediction time, and the color scale is correlation coefficient $R = [0, 1]$. The correlation interval is a sliding 15 minutes. Intervals of both high and low predictive skill are observed, with continuous transition.

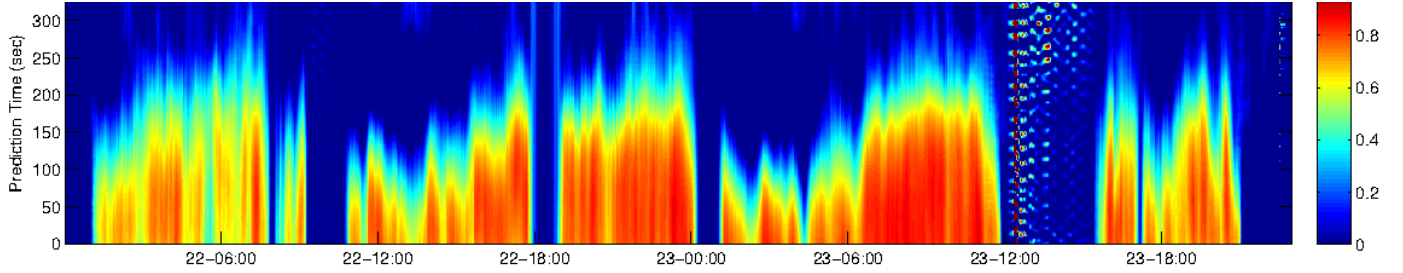


Fig. 6. Timeseries of correlation between predicted SSE and MRU for the 2008 OWME trial. The horizontal axis is 48 hours of real time, the vertical axis is seconds of prediction time, and the color scale is correlation coefficient $R = [0, 1]$. The correlation interval is a sliding 15 minutes. Intervals of zero correlation correspond to no data from either instrument.

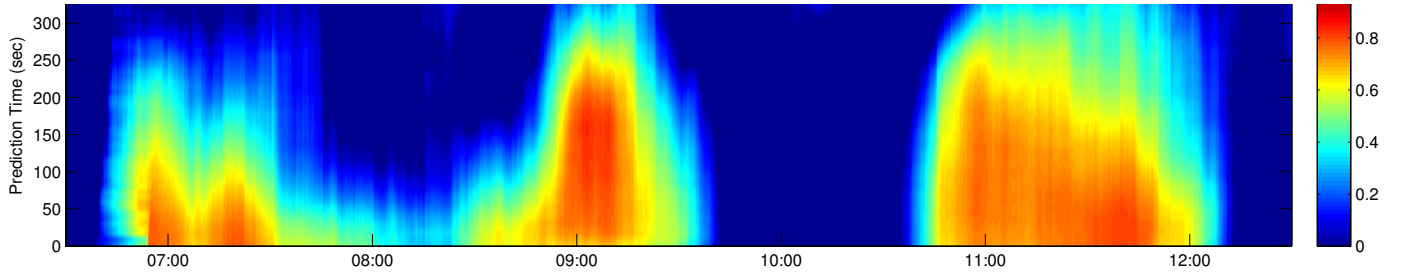


Fig. 7. Timeseries of correlation between predicted SSE and MRU for the 2012 DRDC trial. The horizontal axis is 6 hours of real time, the vertical axis is seconds of prediction time, and the color scale is correlation coefficient $R = [0, 1]$. The correlation interval is a sliding 15 minutes. Intervals of zero correlation correspond to no data from either instrument.

possible, and that linear wave propagation is sufficient for DSWP.

The issue of scaling radar derived SSE has been avoided in this analysis, as cross-correlation is invariant to a scale factor. The use of a 15 minute correlation interval ensures the scaling factor is a smooth continuous function. The linear time-invariant design of the algorithm allows for identification of the causes to varying predictive skill. Scaling methods necessitate consideration of vessel hydrodynamic response functions, which have been entirely omitted in this analysis. Despite this, significant correlation is demonstrated. Incorporation of vessel hydrodynamics and geometry should greatly improve the vessel motion prediction skill. Future work will seek to quantify the prediction zone based on the dominant wave modes and thus arrive at environmental limits to DSWP.

ACKNOWLEDGMENTS

The authors would like to thank Defence Research and Development Canada for the generous use of data and results from the 2012 sea trial.

The authors would like to thank the NATO Submarine Rescue System program for the generous use of data and results from the 2014 sea trial.

Portions of this research were conducted within the PROMISED Operations project (PRediction Of wave induced Motions and forces In Ship, offshore and Dredging Operations), funded by Agency NL, a department of the Dutch Ministry of Economic Affairs, Agriculture and Innovation and co-funded by TU Delft, University of Twente, MARIN (MARitime Research Institute Netherlands), Ocean Waves GmbH, Allseas, Heerema Marine Contractors and IHC.

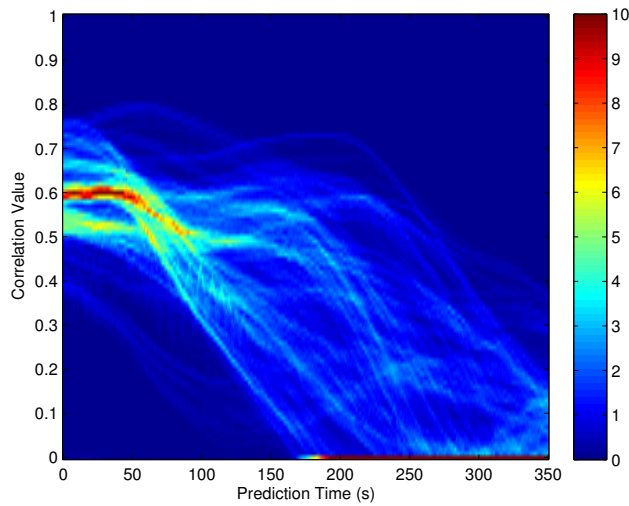


Fig. 8. Histogram of correlation between predicted SSE and MRU for the 2014 NSRS trial. The horizontal axis is prediction time, the vertical axis is correlation coefficient, and the color scale is percentile of all results.

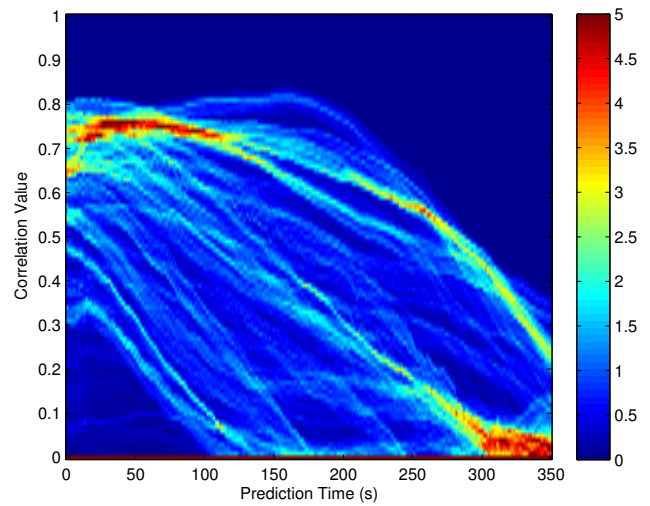


Fig. 10. Histogram of correlation between predicted SSE and MRU for the 2012 DRDC trial. The horizontal axis is prediction time, the vertical axis is correlation coefficient, and the color scale is percentile of all results.

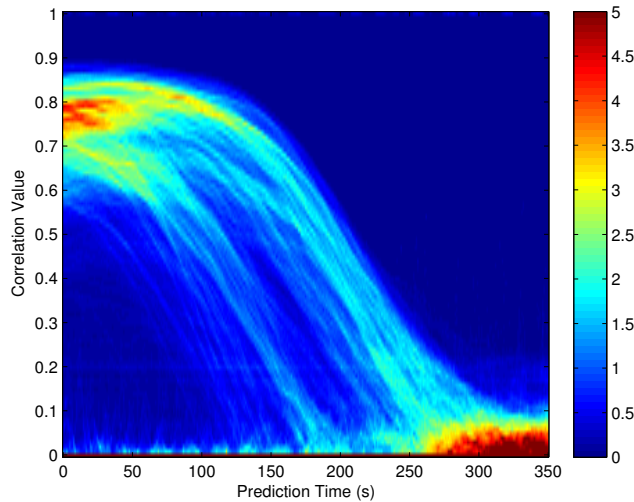


Fig. 9. Histogram of correlation between predicted SSE and MRU for the 2008 OWME trial. The horizontal axis is prediction time, the vertical axis is correlation coefficient, and the color scale is percentile of all results.

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