Conceptual View of Reflection and Transmission of a Tsunami Wave at a Step in Bathymetry

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***Abstract*—Recently published data from a WERA phased array radar operating in Chile after the Great Tohuku Earthquake in 2011 show that the tsunami lost a significant amount of energy as it shoaled towards the coast. This experimental result was contrary to that from the linear theory often used to estimate amplitude growth during shoaling. This paper evaluates the reflection of energy from a bathymetric step that might represent the edge of a continental shelf. A geometric wave approach is used to evaluate coefficients of reflection and transmission at the refractive index boundary. It is shown that reflection can explain some, but not all of the difference between linear theory and observations.**

Keywords—tsunami, reflection coefficient, transmission coefficient, continental shelf, coastal impact

Tsunamis are shallow water gravity waves, even in the deepest oceans. They are normally generated by earthquakes, but can be initiated by any abrupt large-scale disturbance that affects the whole water column of water. Landslides and even severe meso-scale weather events have created tsunamis. For earthquake genesis the global seismic network gives a reliable first alert, and often the DART (Deep-ocean Assessment and Reporting of Tsunamis) buoys can give calibration of the timing and amplitude in deep water. Full wave models like the MOST model (Method of Splitting Tsunamis) [1] can help to make forecasts of timing and amplitude at coastal sites. The MOST model is a computational burden that is often worked-around by running a library of scenarios so that the nearest one can be selected for specific alerts. Previously published work has shown that HF ocean radars are generally able to observe surface current close to the coast [2] and one site in Chile used a phased array system to make measurements beyond the edge of the continental shelf following the Great Tohuku Earthquake in 2011, [3]. These data were the first observations by any instrument that we know of that documented details of dynamical changes to the group of tsunami waves as they crossed the edge of the continental shelf. The data show a significant loss of energy from the wave during this transition. We need to understand the mechanisms involved in this process so that more accurate forecasts of coastal impact can be given.

This paper takes a geometrical approximation for the propagating wave in order to gain insights into the reflection and transmission of tsunami wave energy at a linear step, like the edge of a continental shelf. This theoretical approach is compared with the HF radar data from the site in Chile, and is used to comment on how different bathymetric profiles will modulate the energy from an approaching tsunami. This contributes to calculating coastal impact from a deep ocean DART buoy calibration, and will improve on the linear wave (energy conserved) model widely used.

I. The Linear Model

A tsunami propagates as a shallow water gravity wave, even in the deepest of oceans, and the phase velocity is given by

$$v_p = \sqrt{gd} \qquad (1)$$

where d is the water depth [4].

If there is a reference depth, D, where the disturbance elevation of the tsunami $h(D)$ is known (for example at a DART buoy), then by first order linear theory [5] at other depths, d, the disturbance elevation, a, is

$$a(d) = a(D)\ (D/d)^{1/4} \qquad (2)$$

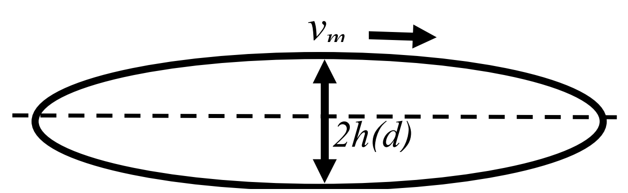

Figure 1. Schematic of the tsunami wave profile. The disturbance elevation from the mean sea level (dash line) is $h(d)$ and the maximum orbital velocity, which the radar measures is $v_m(d)$ for a shallow water wave in water depth, d.

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where symbols are defined in Fig. 1, and the maximum orbital velocity is

$$v_m(d) = v_m(D)\ (D/d)^{3/4}. \qquad (3)$$

In deep oceans, big tsunami events have disturbance elevations of the order 30 – 50 cm, and maximum orbital velocities of 2 – 4 cm/s. Using linear theory, a tsunami with amplitude 0.40 m in a depth of 3,000 m will have an amplitude of 1.2 m when the water depth is 40 m. The corresponding numbers for maximum orbital velocity would be 0.023 m/s at 3000 m depth and 0.59 m/s at 40 m depth.

II. WERA HF Radar Data

Following the Great Tohuku Earthquake in Japan on 11 March 2011, and before the arrival of the tsunami in Chile, the Rumena radar was configured to record time series of phase and amplitude from individual receive antennas in successive 5-minutes duration, operating at 22 MHz and using the full bandwidth of 500 kHz [6]. Post-processing was done on windowed 133 s time series, sliding in 33 s steps across each 5-minute coherent block of data. The 500 kHz bandwidth corresponds to a range resolution of about 0.3 km when the effects of the Hamming window for FFT analysis is included.

A summary of results shown in Fig. 2 clearly show that the linear theory did not work. When the tsunami amplitude was normalised at 880 m depth (solid line), the projection using linear theory gave amplitudes that exceeded the observations (red asterisks) by a factor of about 3 when the wave arrived at a depth of 40 m. Clearly, there is a significant loss of energy from the tsunami between depths of 880 m and 160 m as shown in Fig. 2. The prime contenders for the loss of energy are bottom friction, wave breaking, and reflection/refraction. This paper examines the latter effect by considering a model based on the principles of geometrical optics.

III. The Geometric Model

For an incident wave with amplitude a_i and wavenumber k_i as shown in Fig. 3 the reflected and transmitted waves have amplitudes a_r and a_t with wavenumbers k_r and k_t respectively. When we write the equations we put $k_i = k_r = k_1$ for the wavenumber in medium 1, and $k_t = k_2$ in medium 2, so that

$$a_i = a_0 \exp - j(k_1 \cdot r - \omega t) \qquad (4)$$
$$a_r = a_0\, r \exp - j(k_1 \cdot r - \omega t) \qquad (5)$$
$$a_t = a_0\, t \exp - j(k_2 \cdot r - \omega t) \qquad (6)$$

Phases of the three components must match at each point along the boundary, which means

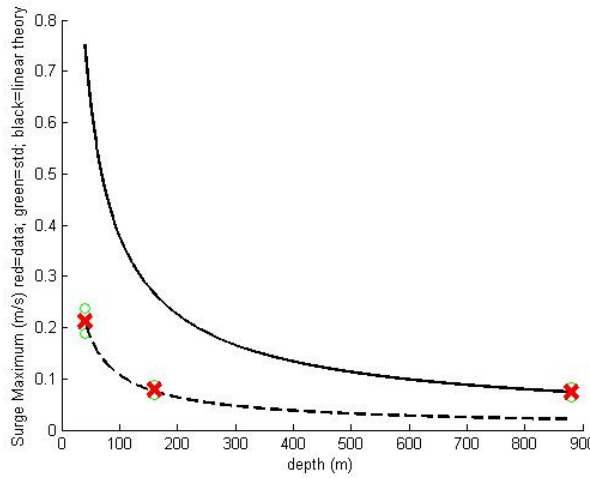

Figure. 2. The mean maxima in wave surge velocities are shown as red crosses (**X**) with the standard deviations indicated with green circles (**O**). The dashed line is produced by linear theory normalised to the observed value at a depth of 40 m. The solid line is produced by linear theory normalised to the observed value at a depth of 880 m. (after Dzvonkovskaya et al., 2014)

$$k_i \sin(\theta_1) = k_r \sin(\theta_1) = k_t \sin(\theta_2)$$

And this leads to the appropriate form of Snell's Law

$$\frac{\sin(\theta_1)}{\sqrt{d_1}} = \frac{\sin(\theta_2)}{\sqrt{d_2}} \quad . \qquad (7)$$

Now we consider only normal incidence. Energy per wavelength is conserved across the boundary and this means

$$\tfrac{1}{2}\rho g a_i^2 + \tfrac{1}{2}\rho g a_r^2 = \tfrac{1}{2}\rho g a_t^2 \qquad (8)$$

and *with* $a_i = a_0$*;* $a_r = a_0\, r$*; and* $a_t = a_0\, t$ where r and t are the Fresnel coefficients for reflection and transmission. It follows that

$$t^2 = 1 + r^2 \qquad (9)$$

An amplification occurs across the boundary governed by the bathymetry according to (2)

$$a(d_2) = \left(\frac{d_1}{d_2}\right)^{1/4} a(d_1) \qquad (10)$$

where d_1 and d_2 are the water depths on each side of the boundary. The amplitude transition across the boundary is then represented by

$$a_0 + a_0 r = a_0 t \left(\frac{d_2}{d_1}\right)^{1/4} \qquad (11)$$

and

$$1 + r = t\left(\frac{d_2}{d_1}\right)^{1/4} \qquad (12)$$

Solving (9) and (12) for r and t, the reflection and transmission coefficients:

$$r = -\frac{\sqrt{d_1} - \sqrt{d_2}}{\sqrt{d_1} + \sqrt{d_2}} \qquad (13)$$

and

$$t = \frac{2\,(d_1 d_2)^{1/4}}{\sqrt{d_1} + \sqrt{d_2}} \qquad (14)$$

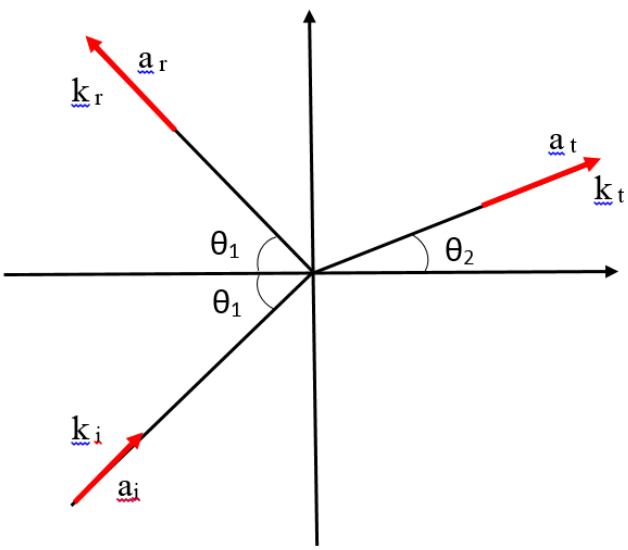

Figure 3. Ray path schematic for a shallow water gravity wave at an abrupt step in bathymetry. The y-axis is the boundary between two depths (shallower on the right side as drawn). This is effectively a refractive index discontinuity.

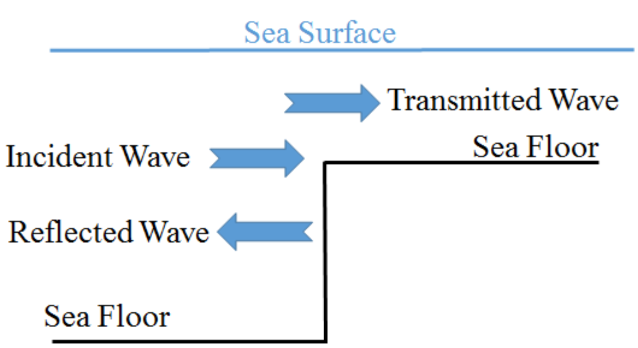

Figure 4. Schematic of reflection and transmission of a shallow water gravity wave in the sea.

These concepts apply to a tsunami wave encountering an abrupt step in the bathymetry as described in Figure 4, where if the incident wave has unit amplitude, then the refracted and transmitted waves have amplitudes r and t respectively.

Since the only variables in the expressions for these coefficients are d_1 and d_2 we can investigate the variability by plotting r and t using a fixed d_1 and let d_2 vary from $d_2 = d_1$ (no step) to $d_2 = 0$ (complete block). The results are shown in Fig. 5.

Because equations (13) and (14) involve the depths only as the ratio d_1/d_2 the variability can be visualised by fixing d_1 and using d_2/d_1 as the abscissa.

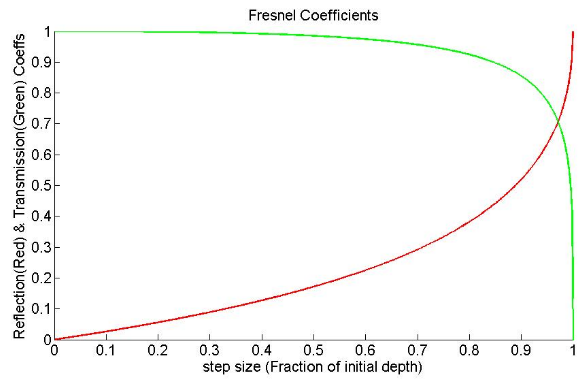

Figure 5. Reflection (red) and transmission (green) coefficients vary with the relative size of the step.

IV. Comparing With Data

The only data set that we know about that can be used to compare with this theoretical approach is that acquired on the WERA radar on the coast at Rumena in Chile during the approach of the tsunami generated by the Great Tohuku Earthquake off the coast of Sendai on 11 March 2011. The phased array radar picked up a clear signal from the tsunami in water deeper than 900m, and a time series was obtained as the succession of waves in the group propagated across the edge of the continental shelf into shallow water. These results have been described previously [3] and are summarised in Fig. 2. From examination of these field results in detail we extracted the data shown in Table 1. We selected the interval where the experimental data in Fig. 2 departed from the upward trend of the linear theory at 500 m depth, and the point at which the observations resumed the trend of linear theory at 200m depth, albeit at a reduced amplitude level. Table 1 shows that the bathymetric change was 300m over a distance of about 7km. The mean wave period, measured from the WERA time series is invariant with depth and equal to 708 seconds.

Table 1. Data from the WERA observations at Rumena, Chile. (from Dzvonkovskaya et al., [3].

Depth	v_m	Elevation anomaly	Distance from shore	Wavelength
500m	0.11 cm/s	0.82m	25km	50km
200m	0.09 cm/s	0.41m	18km	9km

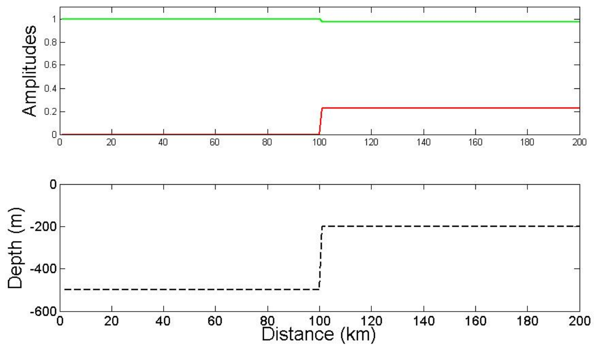

Figure 6. For a step in the bathymetry from 500m to 200m depth (lower panel), the reflected amplitude (red) and transmission amplitude (green) (upper panel) respond with step changes.

Fig. 6 shows the response of wave amplitudes to a step from 500m depth to 200m depth. The amplitude of the transmitted wave is normalised to unity on the left of the step, and becomes 0.97 after the step. The amplitude of the reflected wave is normalised to equal unity if the step were the whole 500m of water column. At the step, the reflected amplitude changes from 0.0 to 0.23.

The amplitude of the transmitted wave in the observations in table 1 changed to 50%, which is significantly more than the change to 97% given by the geometric theory.

V. DISCUSSION

This work was carried out to investigate a significant loss of energy from the tsunami wave as it approached the coast at Rumena, Chile. The data are summarised in Fig. 2 where the maximum surge velocity reduced as the wave shoaled towards the coast. This result by Dzvonkovskaya et al. [3] is highly significant and challenges the linear model of Green [5] for estimating the impact of a tsunami on the shore.

In the case study reported here, the wave amplitude was reduced to 97% across the bathymetric step, compared with an increase to 126% using linear theory, and compared with a reduction to 50% in the observation.

One unvalidated approximation in this case study is that the bathymetric change in the geometric approach is a single step, while the real tsunami experienced the change of 300m in depth over a propagation distance of about 7km. The reason that the step assumption was taken is that the wavelength varied from 50km at 500m depth to 9km at 200m depth. Thus the complete change in depth occurred within a wavelength which, normally in wave theory, is effectively a step. The OCEANS presentation will show the result of a calculation over a series of incremental steps to better simulate the slope in the bathymetry.

This work is suggesting that there are phenomena other than reflection that contribute to the departure of the observations from the linear theory at a bathymetric step.

ACKNOWLEDGMENT

Reference to different HF radar systems in this paper are based on actual data. Readers are advised to consult relevant manufacturers' specifications for performance details of their products.

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