Cooperative Navigation and Control: the EU MORPH project

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***Abstract*—The EU MORPH project advances the use of a formation of five autonomous marine vehicles for mapping and scientific surveying of challenging, unstructured underwater environments as a means to overcome the limitations imposed by current single-vehicle based technology. We describe the test scenarios envisioned in the project and the requirements and constraints that they impose on the cooperative navigation and control systems that enable the concerted operation of the vehicle formation. We then provide a high-level description of each of the blocks that compose the final navigation and formation control architecture, developed at ISR/IST. We highlight how the systems take into account specific mission-related requirements and applications (e.g., typical trajectories employed for mapping and surveying). We describe the results of several trials at sea (involving multiple partners in the project and a group of heterogeneous vehicles) that illustrate the applicability of the MORPH concept in real operational scenarios as well as the limitations of the systems developed.**

Keywords*—Autonomous marine vehicles, underwater surveying, formations of autonomous vehicles, cooperative navigation and control.

I. Introduction

Seabed mapping and surveying has been an area of significant interest in the last decades. This interest stems mostly from the number and variety of applications that benefit from the ability to produce detailed maps and/or measure numerous features of the ocean. Specifically, scientific research in the fields of marine biology and geology is greatly dependent on the technology available to perform these surveys in areas of interest. In this context, unmanned marine vehicles have naturally become widespread tools, mainly because they enable scientists to see and collect data from otherwise inaccessible areas at ever-decreasing costs. The subset of autonomous underwater vehicles (AUVs) has recently assumed a particularly important role by further enabling longer surveys that cover larger areas while reducing the need for continuous human monitoring or piloting from support vessels - which can drastically reduce costs. Yet, many of the interesting underwater environments from a scientific standpoint are unstructured, including for example steep slopes and various complex, tridimensional terrain features. In this kind of scenarios, current technology may prove ineffective for a number of reasons. For example, Doppler Velocity Logs (DVLs), the standard sensors for measuring velocity underwater that often play a key role in AUV navigation, may not perform reliably over rugged terrain, compromising the performance of navigation systems and increasing the risk of operating in these areas. Furthermore, for operations close to underwater cliffs it is certainly not advisable to use a single bulky vehicle carrying all the assets required for navigation, control, and mapping. It is against this backdrop of ideas that the MORPH project sets forth the concept of a virtual supra vehicle that emerges out of bringing together a number of small vehicles with distributed assets, capable of cooperating towards the execution of challenging scientific and commercial missions and eschewing the problems that may arise from the use of a single vehicle. In summary, the MORPH project [1] aims to develop a tool for underwater habitat mapping using a formation of five underwater vehicles with different roles and equipped with complementary sensors, so that reliable navigation and mapping can be achieved. Additionally, the distribution of a number of sensors among the vehicles makes them lighter and cheaper, inherently decreasing the risk of the operations when compared to using a single, bulky, heavily-equipped AUV.

Overall, the MORPH project involves 9 partners: Atlas Elektronik, Center for Maritime Research and Experimentation (CMRE), Consiglio Nazionale delle Ricerche (CNR), IFREMER, Ilmenau University of Technology (IUT), IMAR, Instituto Superior Técnico (IST), Jacobs University, and University of Girona (UdG). A total of 8 vehicles from these partners have been involved in the sea trials so far. The five vehicles in the MORPH formation are divided into two groups, the upper and lower segments. Figure 1 shows the formation flying over flat terrain. The upper segment comprises three vehicles that operate closer to the surface, at a safe distance from the terrain.

- The *Surface Support Vehicle* (SSV) operates at the surface, and its goal is to provide drift-free global navigation to the formation via GPS. This allows the formation to follow a pre-planned path using geographical coordinates and to geo-reference features.
- The *Local Navigation and Sonar Vehicle* (LSV) carries a profiler sonar scanning the horizontal plane for path planning, and a multi-beam sonar mounted on a pan and tilt mechanism looking downwards for terrain mapping and obstacle detection. Being the vehicle with better capability to sense the environment ahead, the LSV is the leader vehicle of the formation that the others should follow. The vehicle follows a preplanned path using the algorithm described in [2]; this path should be re-planned online given the information obtained by the sonars. The online path re-planning based on the environment was developed by UdG.

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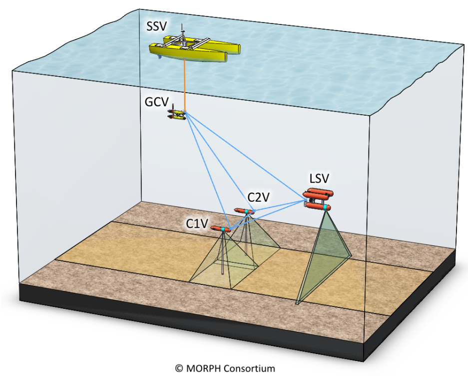

Fig. 1: MORPH formation flying over flat terrain.

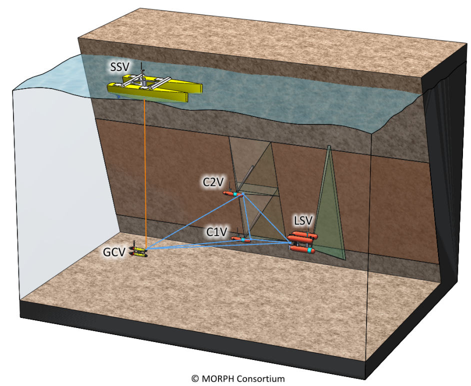

Fig. 2: MORPH formation surveying a negative-slope cliff.

- The *Global Navigation and Communications Vehicle* (GCV) ensures that the formation remains connected, i.e. that there is always a way to transmit information between any pair of vehicles in the formation. As such, it attempts to remain within acoustic visibility of the four other vehicles, acting as a communication relay among them. Due to its central position to the formation, it also carries an Ultra-Short Baseline (USBL) allowing it to determine the position of all the other vehicles relative to itself. This role is especially important in scenarios where, for example, the acoustic link between the SSV and the LSV can be blocked, such as the negative-slope cliff illustrated in Figure 2.

The lower segment comprises two vehicles that operate closer to the terrain and as such should carry a minimal set of equipment: the *Left and Right Camera Vehicles* (C1V and C2V in the figures). The goal of these vehicles is to acquire high-definition imagery using cameras. The images obtained by the two vehicles should have some overlap to allow easier post-mission reconstruction of the environment. As shown in figs. 1 and 2, the line connecting the two CVs should be parallel to the terrain to be surveyed (i.e. horizontal for flat terrain, at an angle for walls) so as to maximize the information obtained from the cameras. This implies that the position of the CVs in the formation should adapt to the angle of the terrain, which can be sensed by the LSV. We refer to this mechanism later in the paper as *formation roll*, since the line between the CVs 'rolls' about the direction of motion of the leader (LSV).

Since only the LSV has considerable environment-sensing capabilities, the remaining vehicles mainly rely on precise formation control to track a trajectory similar to LSV, so as to avoiding obstacles while attempting to follow the pre-planned mission. This is the main problem tackled in this paper: *given the requirements of the project, appropriate assumptions about the dynamics of the vehicles and the mission characteristics, and the limitations inherently imposed by the underwater environment (e.g. frequency and bandwidth of acoustic communication and localization channels), devise a strategy to allow all vehicles to appropriately track the leader vehicle (LSV), while avoiding collisions with the terrain.*

Previous work on formations of marine vehicles for real applications can be found in a number of references. For example, in [3], a leader-follower formation control strategy is employed to achieve a solution for pipeline inspection using multiple vehicles. The work in [4] concerns the use of another leader-follower strategy for controlling a formation of AUVs for performing tedious tasks such as sweeping large areas. Many of these and other references, however, do not provide extensive results of practical experiments (resorting instead to simulations) or do not illustrate the performance of the proposed strategies when the vehicles have to operate at close distances and near the terrain - an important requirement for the MORPH project. A number of papers have been published under the scope of the work developed for MORPH, providing approaches attempted previously or more detailed descriptions of subsystems currently in use - see [5], [6], [7], [8].

The main contribution of this paper is twofold: 1) a navigation and control architecture that enables the follower vehicles to track the leader with satisfactory performance, considering the goals of the project, and 2) a definition of the set of variables to be broadcast by each vehicle in the formation that allows the system to work reasonably well even under severe constraints on the communication network. These algorithms were developed at ISR/IST in the scope of its allocated tasks within the MORPH project.

The remainder of this paper is organized as follows. In section II, we clearly define the problem tackled and identify the system requirements, available sensors, and reasonable assumptions that can be made regarding the vehicle dynamics and typical mission characteristics. Section III starts by examining the navigation and control architecture running in the follower vehicles, including brief technical descriptions of each subsystem. It also clarifies how this general architecture for leader-tracking can be adapted specifically to all the follower vehicles in the MORPH formation by introducing the proposed inter-vehicle communications scheme. In section IV we illustrate and discuss the performance with experimental results obtained in sea trials with the MORPH partners in the Azores, September 2014 and in Girona, June 2015. We finalize with some concluding remarks in section V.

II. Problem Statement

The system to be controlled is a formation of autonomous marine vehicles tracking one leader vehicle. In this setup, the necessary variables to completely define the state would be the attitude of each vehicle and their position and velocity vectors, in appropriate local tangent plane North-East-Down

(NED) coordinates, with the origin defined in a convenient point close to the mission site. This coordinate system will be hereon referred to as 'earth-fixed frame'.

A. Actuators

The available actuators vary from vehicle to vehicle. All of the vehicles are underactuated, since no thrusters to provide pure lateral forces are available. Also, all vehicles have longitudinal thruster(s), and rotation torques are provided either by control surfaces (e.g. SeaCat from Atlas Elektronik) or by differential forces provided by two parallel thrusters (e.g. MEDUSAs from IST). However, given that the vehicles involved have considerable differences in terms of dynamics and actuation, we make some simplifying assumptions:

- all the vehicles are already equipped with inner-loop controllers that can track references in heading angle ψ, surge speed u, and depth z;
- at the low speeds at which the missions are typically performed and given the stability characteristics of the vehicles, 1) the sway motion is negligible, and 2) the motion in horizontal and vertical planes is reasonably decoupled. If this does not apply to some vehicles, we assume additionally that those vehicles are equipped with inner-loops to achieve this decoupling (e.g. minimizing roll and pitch);
- all inner-loops react sufficiently fast, so that we can consider that they track the references in heading, surge speed, and depth perfectly. In practice, as we will see later, this imposes a limitation on how fast the references are allowed to vary.

From these assumptions, we can redefine our actuation mechanism in a simplified, unified way that works with the different kinds of vehicles available. This is done by considering our 'virtual' actuators to be the references for the three inner-loop controllers, ψ_d, u_d and z_d, where the subscript d denotes the desired value for these variables. Besides these controlled inputs, we consider additionally that the vehicles may be subject to disturbances, mainly the ocean current. We model it as a constant velocity vector in NED coordinates, that can be added to the velocity of the vehicle with respect to the water to yield the velocity over ground.

B. Sensors

Concerning the sensing capabilities, at low-level all vehicles need to be able to measure or estimate their attitude angles, the surge speed u, and the depth z, that must be driven to their reference commands. Considering the assumptions stated above, we assume this information is already integrated in the inner-loops of each vehicle, and focus on higher-level sensing capabilities that are relevant to formation control:

- the LSV carries a multibeam echosounder and a profiler that allow it to reasonably sense the environment;
- the SSV is at the surface and equipped with GPS, so that its position in an earth-fixed frame is available;
- the GCV carries an USBL and all other vehicles carry modems and transponders; thus, the GCV can obtain the position of each vehicle relative to itself;
- the CVs can measure the ranges (distances) to the LSV and the GCV and the 'distance to the environment', at least on an angular section of the $Y - Z$ plane of the body of the vehicle (using a mechanically rotating imaging sonar).

Additionally, bidirectional acoustic communication links are available from the GCV to all the other vehicles. Links between other pairs of vehicles may be blocked by the environment, so we consider that they are generally unavailable. In a realistic operational scenario, however, these links should of course be used when available.

To accomplish formation control, each follower vehicle needs information that allows it to compute, either directly or indirectly, 1) its position relative to the leader vehicle, and 2) the velocity vector of the leader and its turning rate. As we will see later, these are the variables needed for a follower to track the leader vehicle with the proposed solution. However, it should be clear by now that the follower vehicles do not have access to this information directly from their sensors. In section III, we start by describing the system designed for a follower vehicle to track the leader *assuming it has access to all this information*. We then clarify how this can be applied to the specific case of the MORPH formation by defining what information each vehicle should broadcast, so that all the followers obtain the data needed to track the leader vehicle.

C. Simplified model

In summary, all the assumptions made above allow us to describe the simplified horizontal-plane model of each vehicle as:

$$\dot{\boldsymbol{p}} = \frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = R(\psi)\begin{bmatrix} u \\ v \end{bmatrix} + \boldsymbol{v}_c = R(\psi_d)\begin{bmatrix} u_d \\ 0 \end{bmatrix} + \boldsymbol{v}_c \quad (1)$$

and the vertical-plane model (depth) as:

$$z = z_d \quad (2)$$

where $\boldsymbol{p}$ denotes the position of the vehicle in the horizontal plane, $\boldsymbol{v}_c$ is the ocean current velocity vector, and $R(\cdot)$ is a 2D rotation matrix parametrized by the rotation angle that transforms the velocity with respect to the water from the vehicle's body frame to the earth-fixed frame assuming that the pitch and roll angles are negligible. The virtual actuators that we consider are the references u_d, ψ_d and z_d, which have to vary slowly enough for the inner-loops to track them reasonably well. The relevant information for formation control that each vehicle obtains from its sensors, besides the variables that we consider to be already integrated in the inner-loops (attitude, surge speed, depth), can be summarized by:

$$y_{\text{LSV}} = \text{sonar data from the surrounding environment} \quad (3)$$

$$y_{\text{SSV}} = \boldsymbol{p}_{\text{SSV}} \quad \text{(global position from GPS)} \quad (4)$$

$$y_{\text{GCV}} = \begin{bmatrix} \boldsymbol{p}_{\text{LSV}} - \boldsymbol{p}_{\text{GCV}} \\ \boldsymbol{p}_{\text{SSV}} - \boldsymbol{p}_{\text{GCV}} \\ \boldsymbol{p}_{\text{CVL}} - \boldsymbol{p}_{\text{GCV}} \\ \boldsymbol{p}_{\text{CVR}} - \boldsymbol{p}_{\text{GCV}} \end{bmatrix} \quad \begin{matrix}\text{(relative positions of} \\ \text{all other vehicles)}\end{matrix} \quad (5)$$

$$y_{\text{CV*}} = \begin{bmatrix} \|\boldsymbol{p}_{\text{LSV}} - \boldsymbol{p}_{\text{CV*}}\| \\ \|\boldsymbol{p}_{\text{GCV}} - \boldsymbol{p}_{\text{CV*}}\| \end{bmatrix} \quad \begin{matrix}\text{(ranges from each} \\ \text{CV to LSV and GCV)}\end{matrix} \quad (6)$$

D. *Formation control objective*

The formation control objective can now be clearly stated: using the available actuation mechanisms, the information obtained from the sensors onboard each vehicle, and the available acoustic communication links, under the assumptions stated above, *devise a strategy that drives the position of each follower vehicle to a virtual target point, obtained by an appropriately-defined position offset with respect to the leader (LSV)*. The scheme should determine the algorithms run in each of the follower vehicles, as well as the variables to be transmitted between the vehicles via the acoustic communication channels. The solution should take into account that the pre-planned missions to be performed by the LSV have the typical characteristics of surveying missions, i.e. low, constant speed along smooth paths containing only straight line and arc segments.

III. PROPOSED SOLUTION

In this section, we start by showing how the formation control problem was tackled for a single follower vehicle, defining the algorithms that run in the follower and the information needed. We then look at the information to be broadcast between vehicles and auxiliary computations so that all of the followers in the MORPH formation obtain the necessary information.

A. *Navigation and control on a follower vehicle*

The general architecture for navigation and control running in the follower vehicles is illustrated in fig. 3. The blocks concerning guidance and control are shown in gray. In each follower vehicle, the information needed by these blocks to perform tracking is:

- the relative position between the leader and the follower vehicle itself in the horizontal plane, denoted by $\boldsymbol{p}-\boldsymbol{p}_{\text{leader}}$ (obtained from different sources depending on the vehicle, as will be seen later),
- the ocean current velocity vector $\boldsymbol{v}_c$,
- the depth of the leader, denoted by z_{leader}, and
- the *leader velocity information*: speed over ground, course angle, and course angle rate (turning rate) of the leader vehicle, denoted by v_{leader}, χ_{leader}, and $\dot{\chi}_{\text{leader}}$ respectively. This information should be broadcast from the leader to the follower vehicles, and if needed re-broadcast by the vehicles that receive it until it reaches all the followers.

Measurements of relative position (USBL) and range, as well as data broadcast between vehicles using acoustic communication channels, are inherently delayed, noisy, and obtained at rates too slow for control. The Time-Division Multiple Access (TDMA) interrogation scheme used to this effect, developed by the project partner CMRE, takes 5 seconds per cycle. The goal of the estimator blocks, shown in green, is to estimate the variables needed by the control and guidance blocks at 5 Hz. The functionality of these estimator blocks will now be described, followed by the guidance and control blocks.

1) Leader velocity estimator: The leader velocity estimator receives the leader velocity information (v_{leader}, χ_{leader}, $\dot{\chi}_{\text{leader}}$) every 5 seconds. To estimate these variables at 5 Hz, we use the assumption stated previously that the planned trajectory for the leader (LSV) is a constant-velocity smooth concatenation of straight line and arc segments. We then assume that v_{leader} and $\dot{\chi}_{\text{leader}}$ are constant, and integrate $\dot{\chi}_{\text{leader}}$ to obtain estimates of χ_{leader}. The continuous-time state space model is

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} 0 \\ \dot{\chi}_{\text{leader}} \\ 0 \end{bmatrix}, \quad \boldsymbol{y}(t) = \boldsymbol{x}(t-\Delta), \tag{7}$$

where

$$\boldsymbol{x} := \begin{bmatrix} v_{\text{leader}} \\ \chi_{\text{leader}} \\ \dot{\chi}_{\text{leader}} \end{bmatrix}. \tag{8}$$

Note that the model includes delayed outputs; this reflects the time it takes for the information to arrive at a follower vehicle after being broadcast by the leader, and can also take into account indirect transmissions (i.e. when the information is relayed through another vehicle, instead of being obtained directly from the leader). The estimator was implemented using a Kalman Filter based on a discretized version of the model in eqs. (7) and (8), where the measurements are obtained every 5 seconds (0.2 Hz). The delay Δ is explicitly taken into account in the update step. The estimator is then run at 5 Hz.

2) Relative position and ocean current estimator: To estimate the relative position of the leader and the ocean current velocity, each follower runs an estimator that uses relative-position measurements $\boldsymbol{p}_{\text{leader}} - \boldsymbol{p}$, the velocity vector of the leader vehicle $\dot{\boldsymbol{p}}_{\text{leader}}$ (obtained by converting to cartesian coordinates the estimates of its speed v_{leader} and course angle χ_{leader} output by the estimator in section III-A1) and the odometry of the follower vehicle itself (its velocity vector with respect to the water, $\boldsymbol{v}_{\text{w}}$). Since we assume the ocean current velocity to be constant, $\dot{\boldsymbol{v}}_c = 0$, the continuous-time design model used is

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{v}_{\text{w}} + \boldsymbol{v}_c - \dot{\boldsymbol{p}}_{\text{leader}} \\ \boldsymbol{0} \end{bmatrix}, \tag{9}$$

where

$$\boldsymbol{x} := \begin{bmatrix} \boldsymbol{p} - \boldsymbol{p}_{\text{leader}} \\ \boldsymbol{v}_c \end{bmatrix}. \tag{10}$$

Besides the velocity vector of the leader vehicle, we also assume that the velocity of the follower vehicle with respect to the water, $\boldsymbol{v}_{\text{w}}$, is known, and as such it is also treated as an input. Assuming no DVL measurements are available, this velocity is usually approximated as $R(\psi)\,[u \quad 0]^T$. However, even u may not be available as a direct measurement. For example, in the specific case of the MEDUSA vehicles, u cannot be measured directly by any of its sensors and is obtained from a scaling of the thrusters' RPMs, without considering any dynamics. The performance is still reasonable but this also imposes limitations, namely the need to avoid abrupt accelerations where the vehicle dynamics of the vehicle become increasingly important to estimate the surge speed. The model can be rewritten as

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t), \tag{11}$$

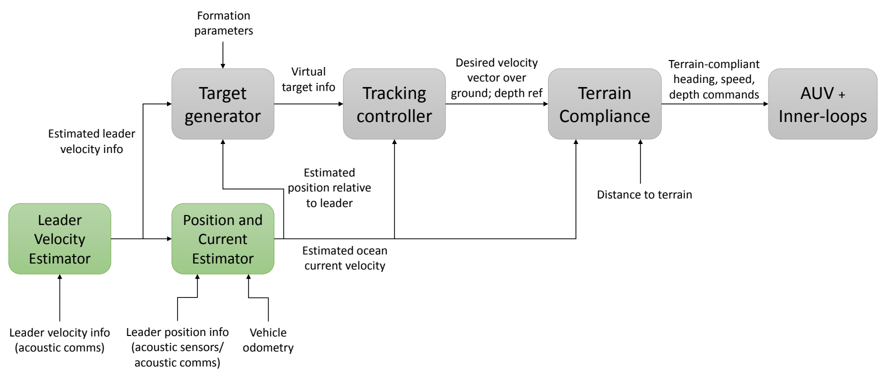

Fig. 3: Block diagram of general architecture running in each of the follower vehicles.

where

$$A := \begin{bmatrix} \mathbf{0} & I_2 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, B := \begin{bmatrix} I_2 & -I_2 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \boldsymbol{u} := \begin{bmatrix} R(\psi) \begin{bmatrix} u \\ 0 \end{bmatrix} \\ \dot{\boldsymbol{p}}_{\text{leader}} \end{bmatrix}. \quad (12)$$

As was done for the previous estimator, we again assume that the followers have delayed relative-position measurements with respect to the leader, that is,

$$\boldsymbol{y}_1(t) = C_1 \boldsymbol{x}(t - \Delta), \quad (13)$$

where

$$C_1 := [I_2 \quad \mathbf{0}]. \quad (14)$$

Observability of the system introduced in eqs. (11) to (14) can be verified. However, in case additional measurements from a DVL are available and reliable (not compromised by the terrain) then we can integrate in the filter direct measurements of velocity over water $\boldsymbol{v}_\text{w}$ (replacing $R(\psi)\,[u \quad 0]^T$), and measurements of velocity over ground $\dot{\boldsymbol{p}}$ with an extra 'virtual' measurement of the current, $\boldsymbol{y}_2(t) := \dot{\boldsymbol{p}} - \boldsymbol{v}_\text{w}$. This is added to the model with the measurement equation

$$\boldsymbol{y}_2(t) = C_2 \boldsymbol{x}(t), \quad (15)$$

where

$$C_2 : = [\mathbf{0} \quad I_2]. \quad (16)$$

This estimator runs in the follower vehicles at 5 Hz. The measurements $\boldsymbol{y}_1$ are obtained at 0.2 Hz and the measurements $\boldsymbol{y}_2$, if available, are obtained at 5 Hz. Finally, the depth of the leader z_{leader} is assumed to remain constant and is not integrated in the estimator.

3) Target generator: The target generator block uses the information about the motion of the leader and the formation parameters to generate the position and velocity vector of the virtual target on the horizontal plane, $\boldsymbol{p}_{\text{target}}$ and $\dot{\boldsymbol{p}}_{\text{target}}$ respectively, and its depth, z_{target}. We show here the most general case - the target generator for the CVs - which defines the target also in terms of the *formation roll* angle. For the remaining followers, the target generator is essentially the same with formation roll set to zero.

The position of the target to be tracked is defined by position offsets relative to the previous path of the leader vehicle on the horizontal plane. The along-path offset is denoted by

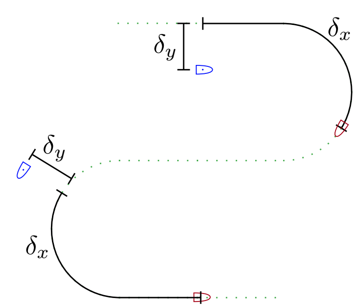

Fig. 4: Virtual target (blue) to be tracked by the follower as a function of the leader (red), its past path, and formation offsets δ_x, δ_y.

δ_x, while δ_y defines the nominal lateral offset perpendicular to the path. Finally, δ_z defines the nominal offset in depth. Figure 4 illustrates the position offsets on the horizontal plane. To include a formation roll angle ϕ_f around the path described by the leader (LSV), we simply rotate the vector (δ_y, δ_z) accordingly, yielding the new parameters δ'_y, δ'_z which are then used for the remainder of the target generator algorithm. Note that δ_x remains unchanged when a formation roll angle is applied. The position of the virtual target is defined in terms of the position of the leader and the formation parameters δ_x, δ'_y, δ'_z in two steps:

1) Integrate the velocity vector of the leader vehicle backwards along a distance of δ_x using the leader turning rate to get the projection of the target on the path,

$$\boldsymbol{p}'_{\text{target}} = \boldsymbol{p}_{\text{leader}} - \int_0^{\frac{\delta_x}{v_{\text{leader}}}} R(\chi_{\text{leader}} - \dot{\chi}_{\text{leader}} t) \begin{bmatrix} v_{\text{leader}} \\ 0 \end{bmatrix} dt \quad (17)$$

where $R(\cdot)$ is the rotation matrix as defined above;

2) add to this position the lateral offset perpendicular to the path at the point obtained in the previous step,

$$\boldsymbol{p}_{\text{target}} = \boldsymbol{p}'_{\text{target}} + R(\chi_1) \begin{bmatrix} 0 \\ \delta'_y \end{bmatrix}, \quad (18)$$

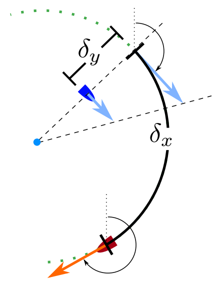

Fig. 5: Computation of the position and velocity of a virtual target (blue) during a turn.

where χ_1 is the direction tangent to the path at $\boldsymbol{p}'_{\text{target}}$ defined by

$$\chi_1 = \chi_{\text{target}} - \dot{\chi}_{\text{target}} \frac{\delta_x}{v_{\text{leader}}}. \tag{19}$$

Note that the *relative* position of the leader, $\boldsymbol{p} - \boldsymbol{p}_{\text{leader}}$, is assumed to be known, and not its global position $\boldsymbol{p}_{\text{leader}}$. Similarly, the output of the target generator should be a *relative* position of the target, $\boldsymbol{p} - \boldsymbol{p}_{\text{target}}$, instead of its global position $\boldsymbol{p}_{\text{leader}}$. We obtain this output by substituting the expression for $\boldsymbol{p}_{\text{target}}$ given in eqs. (17) to (19) into $\boldsymbol{p} - \boldsymbol{p}_{\text{target}}$, yielding an expression where $\boldsymbol{p} - \boldsymbol{p}_{\text{leader}}$ shows up instead of $\boldsymbol{p}_{\text{leader}}$.

Using a 'back-propagation' reasoning similar to what was done for the target position, we can compute the velocity vector of this target in the horizontal plane to obtain

$$\dot{\boldsymbol{p}}_{\text{target}} = R(\chi_1) \begin{bmatrix} v_{\text{leader}} \\ 0 \end{bmatrix} + R(\chi_1) S(\dot{\chi}_{\text{leader}}) \begin{bmatrix} 0 \\ \delta'_y \end{bmatrix}, \tag{20}$$

where $S(\cdot)$ is a skew-symmetric matrix related to the rotation rate, defined for a generic rotation rate r as

$$S(r) = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}. \tag{21}$$

Equation (20) can be interpreted in simple geometric terms. The first term is the velocity vector of the projection of the target on the leader's path, $\boldsymbol{p}'_{\text{target}}$. The second term scales this velocity vector during turns, so that a point on the outer side of the turn can move faster and one on the inner side can move slower. Figure 5 illustrates these facts.

Finally, the depth reference is simply set as the depth of the leader with the desired offset,

$$z_{\text{target}} = z_{\text{leader}} + \delta'_z. \tag{22}$$

It is important to note that during this derivation we assumed that the velocity and turning rate of the leader remained constant throughout the entire integration distance δ_x. This is true only when the follower is in the same path segment as the leader (e.g. same arc or same straight line). This strategy can be generalized to the case where the follower is also allowed to be in the segment behind the one of the leader using some logic on the leader turning rates received in consecutive communication cycles - see [5], [9] for details. Also, note that the integral in eq. (17) has a closed-form, which however is singular for $\dot{\chi}_{\text{leader}} = 0$, i.e. for straight line segments. Although this singularity is removable, the integral notation is more convenient since it can deal with straight segments without introducing piecewise-defined functions.

After being generated, the position and velocity of the virtual target on the horizontal plane $\boldsymbol{p}_{\text{target}}$ and $\dot{\boldsymbol{p}}_{\text{target}}$ and the target depth z_{target} are output by the target generator block.

4) Tracking controller: The tracking controller receives the information about the position and velocity of the virtual target and produces a desired velocity vector over ground which, if applied, would drive the vehicle to the target.

We define the error vector to be driven to zero as the position of the vehicle relative to the target output by the target generator, that is,

$$\boldsymbol{e}_h = \boldsymbol{p} - \boldsymbol{p}_{\text{target}} \tag{23}$$

with dynamics

$$\dot{\boldsymbol{e}}_h = \dot{\boldsymbol{p}} - \dot{\boldsymbol{p}}_{\text{target}}. \tag{24}$$

Assuming that $\dot{\boldsymbol{p}}$ follows a desired velocity over ground defined as

$$\dot{\boldsymbol{p}}_d = -\text{sat}(K\boldsymbol{e}_h) + \dot{\boldsymbol{p}}_{\text{target}} \tag{25}$$

where the first term is a *saturated position feedback* and the second is *feedforward on target velocity*, the resulting closed-loop dynamics are

$$\dot{\boldsymbol{e}}_h = -\text{sat}(K\boldsymbol{e}_h), \tag{26}$$

where $\text{sat}(\cdot)$ is the usual saturation function, whose saturation value (a parameter) is the maximum speed the vehicle should move at when approaching a stopped target from a distance, and K is a square matrix of size 2. If $K > 0$ these dynamics are asymptotically stable, and the vehicle will converge to the target. Additionally, if $\dot{\boldsymbol{p}}_{\text{target}}$ is not known precisely but estimated with some error $\boldsymbol{\epsilon}$, possibly due to some error on the estimate of $\dot{\boldsymbol{p}}_{\text{leader}}$, the closed-loop dynamics are

$$\dot{\boldsymbol{e}}_h = -\text{sat}(K\boldsymbol{e}_h) + \boldsymbol{\epsilon}, \tag{27}$$

which are locally input-to-state stable [10], ensuring that for $\|\boldsymbol{\epsilon}\|$ below some bound, dependent on the saturation value, the error will remain bounded [5], [9]. Given these desirable properties, the desired velocity vector over ground defined in eq. (25) is output by the tracking controller. As for the depth reference, no controller is needed in this outer layer; the reference z_d is simply passed on to the next block.

5) Terrain compliance: The terrain compliance block receives the desired velocity vector over ground $\dot{\boldsymbol{p}}_d$, the depth reference z_d, and the estimate of the ocean current from the estimator. Since some vehicles have extra information about the terrain, the purpose of this block is to integrate that information in the control architecture. In the particular case of MORPH, the CVs carry a mechanical scanning sonar sweeping an angular section of the $Y - Z$ plane of the vehicles that can provide the distance to the terrain. Specifically, the purpose of this block is to act as supervisor and, if needed, override the velocity and depth references produced by the tracking controller, so as to avoid any collisions with the terrain while still attempting to comply with the formation objectives. It should not have any influence on control when the vehicle is far from the terrain. Since these vehicles should be far from the terrain at all times, in their case this block does not change the references passed on from the tracking controller.

The basic functionality of the block was first developed by IST/ISR, and later adapted by the partners Atlas Elektronik and UdG, to accommodate for the different sensors onboard their vehicles. In simple terms, the strategy developed by IST can be described in a number of steps:

- Detect if the closest terrain is the seabed or a wall.
- If the closest terrain is the seabed, saturate the depth reference to ensure a minimum altitude above the terrain. This *minimum distance to terrain* is an adjustable parameter of the block, and also applies to the case of a wall.
- If the closest terrain is a wall, start by identifying its slope (steepness) using the sonar data. Then, using the direction (bearing relative to North) of the wall[1], we take the desired velocity vector $\dot{\boldsymbol{p}}_d$ and determine its component normal to the wall, v_{wall}. By convention, we define this velocity as positive towards the wall. We then saturate this so that the reaction to the terrain satisfies some requirements: 1) forces the vehicle away from the wall gradually when the distance to the terrain is below the specified *minimum distance to terrain*; 2) does not let the vehicle move towards the wall at very high speeds when the distance to the terrain is still above, but close to, the *minimum distance to terrain*; and 3) does not have any influence on the velocity vector prescribed by the tracking controller when the vehicle is very far from the terrain. In practice, this is implemented using the expression
$$(v_{\text{wall}})_{\text{saturated}} = \min\left(v_{\text{wall}},\ \frac{v_{\text{esc}}}{d_{\text{min}}}d - v_{\text{esc}}\right) \tag{28}$$
where v_{esc} is a parameter determining the *escape velocity* (velocity at which the vehicle should move away from the wall, for the extreme case where it is actually in contact with it), d_{min} is the *minimum distance to terrain*, and d is the distance to the wall detected by the sonar. The desired velocity vector over ground is then reconstructed using this saturated component normal to the wall, yielding the 'terrain-saturated' vector $\dot{\boldsymbol{p}}'_d$. It can be verified that the three requirements stated above are satisfied using this strategy. Also note that, assuming that the formation is moving parallel to the wall, which should be ensured by the LSV, this strategy does not influence the along-track component of formation control, so the vehicles reacting to terrain still avoid getting delayed with respect to the remaining vehicles in the formation.
- From $\dot{\boldsymbol{p}}'_d$, obtain the desired velocity vector *with respect to the water* by compensating for the ocean current, and then translate this into references in surge speed and heading to be passed on to the inner loops. Recall the simplified model from eq. (1)
$$\dot{\boldsymbol{p}} = R(\psi_d)\left[u_d \quad 0\right]^T + \boldsymbol{v}_c, \tag{29}$$
so the heading and surge speed references should be chosen so as to satisfy
$$R(\psi_d)\left[u_d \quad 0\right]^T = \dot{\boldsymbol{p}}'_d - \boldsymbol{v}_c. \tag{30}$$
This is achieved by setting u_d and ψ_d as
$$u_d = \left\|\dot{\boldsymbol{p}}'_d - \boldsymbol{v}_c\right\| \tag{31}$$
$$\psi_d = \angle(\dot{\boldsymbol{p}}'_d - \boldsymbol{v}_c). \tag{32}$$
The angle function $\angle(\cdot)$ is implemented with the usual 4-quadrant arc tangent function, atan2$(\cdot)$. This strategy is very simple but has some limitations, the most important of which is the singularity of ψ_d when the desired velocity vector is 0. In practice, this can be overcome by implementing a hysteresis on the norm of the desired velocity vector, freezing the value of ψ_d when the speed falls below some limit.

[1]This information is contained in χ_{leader} broadcast by the LSV to the followers, since we assume that in case there is a wall, the LSV plans the trajectory so as to move parallel to it.

At this point, the final references u_d, ψ_d and z_d are sent to the inner loops of the vehicle. Ideally, these references should vary slowly enough to be tracked by the inner loops, so the gain matrix K, the minimum distance to terrain d_{min} and the escape velocity v_{esc} should be adjusted accordingly.

As a final remark, note that compensating for the ocean current in the feedforward term in eq. (25) along with the velocity of the target would be a valid alternative, yielding directly a reference velocity vector with respect to the water. However, this would be valid only far from the terrain. This is because the influence of the terrain on the control should ideally be independent of the ocean current, so the reference velocity vector to be 'saturated' by the terrain should be the *desired velocity vector over ground.* Still, the considerations stated previously regarding the influence of the error ϵ affecting the estimate of $\dot{\boldsymbol{p}}_{\text{target}}$, as introduced in eq. (27), also apply when ϵ is seen as the error affecting $\dot{\boldsymbol{p}}_{\text{target}} - \boldsymbol{v}_c$.

B. Application to MORPH formation

Until now, we have looked at the architecture running in each follower assuming that they have access to the information needed to estimate the variables used for formation control. In this section, we look at how to apply this architecture in the MORPH formation, by defining what information should be broadcast among vehicles and adding any necessary auxiliary calculations to the architecture. We go through each vehicle role individually, look at the information it needs, and finally define how the vehicle can obtain this information from the available sensors and communication links.

1) LSV: The LSV is the leader of the formation and as such runs a path-following algorithm on a planned trajectory (see [2] for details). The only information the LSV needs to receive continuously (i.e. every communication cycle) is its position in the earth-fixed frame; how this information is obtained will become clear when we look at the GCV. In terms of out-going information, the LSV needs to broadcast information about its motion (v_{leader}, χ_{leader}, $\dot{\chi}_{\text{leader}}$), its depth z_d, and the prescribed *formation roll* for the CVs (ϕ_f) based on the identified wall slope.

2) GCV: For formation control, the GCV needs a) the relative position to the LSV, obtained using its USBL, and b) v_{leader}, χ_{leader}, $\dot{\chi}_{\text{leader}}$, and z_{leader} obtained directly from the LSV via the acoustic communication channel. Being the vehicle responsible for relaying information, the GCV also needs to broadcast some information to other vehicles:

- LSV's position in the earth-fixed frame, so that the LSV can perform path-following. Since the GCV carries a USBL that provides it with the relative positions of all vehicles in the formation, it can a) obtain the relative position measurements LSV-GCV and SSV-GCV, b) receive the position of the SSV in the earth-fixed frame (this information is broadcast by the SSV, as will be seen later), and c) using this, compute the position of the LSV in the earth-fixed frame and broadcast it. This information is also received by the SSV, which uses it to determine its position relative to LSV and perform tracking.

- v_{leader}, χ_{leader}, and $\dot{\chi}_{\text{leader}}$, needed by the SSV to follow the LSV. These variables are simply relayed (i.e. received from the LSV and re-broadcast). Note that the SSV does not need the *formation roll* ϕ_f (which applies only to the CVs) nor z_{leader} since it is a surface vehicle, so it cannot control depth.

- Additional data needed for the CVs to localize themselves using ranges: the direction in the horizontal plane of the line between the LSV and GCV, denoted by $\psi_{\text{M}} := \angle(\boldsymbol{p}_{\text{LSV}} - \boldsymbol{p}_{\text{GCV}})$ (obtained from the USBL); the depth of the GCV z_{GCV}; and a disambiguation bit for each CV. The reason the CVs need this information will become clear later in this section.

3) SSV: The SSV needs to receive v_{leader}, χ_{leader}, $\dot{\chi}_{\text{leader}}$, and LSV's position in the earth-fixed frame, so that it can compute its position relative to LSV and perform tracking. These variables are all broadcast by the GCV. To provide global navigation to the formation, the SSV also needs to broadcast its position in the earth-fixed frame (obtained from GPS), which is used by the GCV as stated above.

4) CVs: The CVs need to receive v_{leader}, χ_{leader}, $\dot{\chi}_{\text{leader}}$, z_{leader}, and ϕ_f for the target generator. Additionally, as stated before, they have no access to relative position measurements, and instead use range measurements to the LSV and GCV to obtain the relative position with respect to LSV. This is why they need the additional information broadcast by the GCV. The disambiguation bit mentioned above is simply an auxiliary binary variable needed to determine if each CV lies to the East or West of the line joining the LSV and GCV (again, see [5], [9] for details). For these auxiliary computations the CVs also need the distance between the GCV and LSV, denoted by $\|\boldsymbol{p}_{\text{LSV}} - \boldsymbol{p}_{\text{GCV}}\|$. This information is automatically available at the CVs as a by-product of the interrogation scheme.

The exchange of information among vehicles as described above is summarised in fig. 6. Note that an advantage of this scheme is scalability, since the number of CVs can be increased without much impact on the amount of information to be broadcast; only the number of disambiguation bits has to be increased (one per CV). Still, this is a very small amount of information, hence allowing the formation to be extended to more CVs if needed.

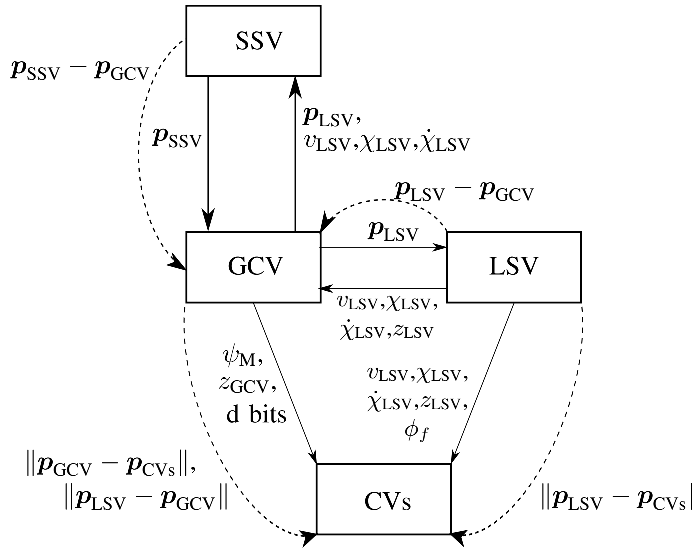

Fig. 6: Summary of the information exchanged among vehicles. Dashed lines indicate information obtained from the interrogation scheme using USBL and modems (ranges and relative position measurements); solid lines indicate information broadcast by the vehicles.

IV. Results

This section illustrates the performance of the systems described before with experimental results obtained in the course of MORPH project trials involving multiple partners, in the Azores, Portugal and in Sant Feliu de Guíxols, near Girona, Spain.

A. Azores trials, September 2014

The Azores trials in September 2014 were the first to include missions with five vehicles playing the roles defined before. All project partners were involved. The test site was Baía de Porto Pim, in the island of Faial, Azores, Portugal. At this time, the functionality to detect and follow walls was not yet implemented, so the missions were performed in open water areas. Also, the role of formation leader was played by the SSV, so that the path-following could be performed using GPS, without depending on information broadcast by other vehicles. This simplified the communications architecture and allowed the team to focus on testing and tuning the formation control functionality; the information exchange scheme presented above, with the LSV as the leader vehicle, was developed only after these trials. It should be noted that the formation control algorithm running in the CVs at the time used a different approach (see [7], [8]) and hence we focus the analysis of these results on the two upper segment followers (GCV and LSV in these trials) running the formation control strategy described in this paper.

Figure 7 shows the trajectories of the five vehicles in a selected part of a mission performed in the trials. The nominal mission path to be followed by the SSV (leader) consisted of several loops offset by 2 m from each other; we only show one loop for clarity. Its nominal speed was set to 0.3 m/s. The LSV and GCV were set to track it with $\delta_x = 5$ m and 17 m respectively, both with no lateral offset ($\delta_y = 0$ m). For these two vehicles, the trajectories shown include both their positions estimated onboard and the ones obtained by USBL. The CVs moved 5 m to the right and left of the center point between LSV and GCV. Figure 8 shows the estimated tracking error in the horizontal plane of the GCV and LSV, expressed in their body-fixed frames.

Visual analysis of the trajectories seems to indicate that the

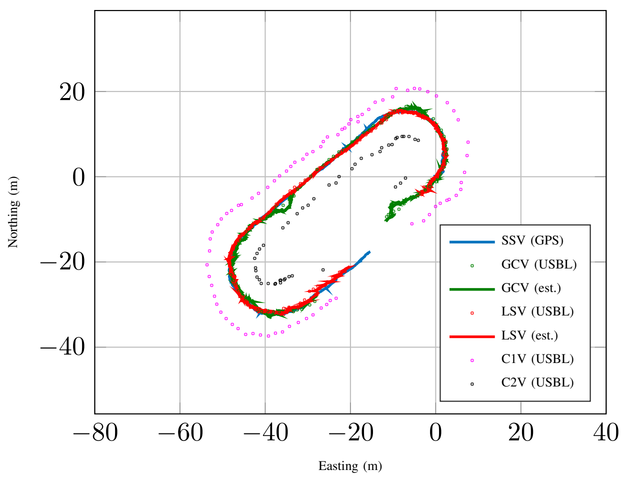

Fig. 7: Trajectories of all vehicles in one loop

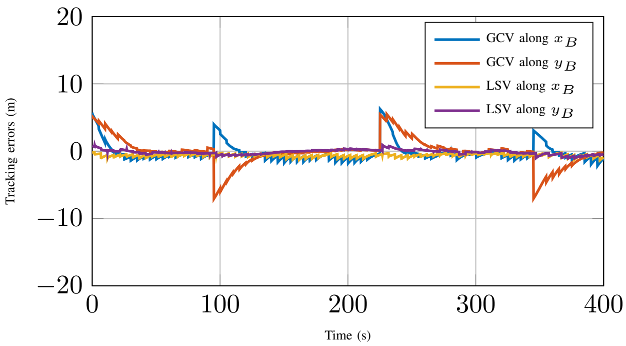

Fig. 8: Tracking error for upper segment vehicles.

followers tracked the expected trajectory, with small deviations from the path followed by the leader. The performance of the position estimates obtained onboard the vehicles also exhibited small differences from the positions measured with USBL, especially considering that the GCV and LSV did not use DVL for dead-reckoning in between USBL position fixes, resorting to the thruster model and attitude unit only. The fact that the thruster model used is static (i.e. does not take into account the acceleration dynamics) and imperfect causes the sawtooth shape seen in the tracking error plot; see [9] for a detailed explanation of this issue and how it can be attenuated. The peaks in tracking error plot are transients happening when the leader vehicle transitions from a straight segment to an arc or vice-versa, causing the position of the virtual target to jump. This was caused by the target generator assuming that the leader and the followers were in the same segment at all times. Although it did not have considerable implications in the trajectories of the follower vehicles, this issue was addressed by generalizing the target generator to allow a follower to be either in the same segment as the leader or in the previous one. Overall, the vehicles do not converge exactly to the virtual target position, but except for these peaks, they remain generally less than 1 m from the desired position - a very acceptable performance considering the mapping and surveying applications of the system envisioned in the project.

B. *Sant Feliu de Guíxols, June-July 2015*

Two MORPH project trials were held in Sant Feliu de Guíxols, Spain, in 2015. In the first trials, in April, the project partners focused on testing the performance of the formation control strategies as described in this paper, involving all the partners and vehicles. New trials were carried out in June and July, focused on testing extensively and ensuring robustness of the wall-following and terrain compliance functionalities, with a reduced number of vehicles. Vehicles from IST, ATLAS Elektronik and UdG were involved. The architecture used was the one described in this paper, but without the SSV vehicle role in order to simplify operations. Instead, the GCV was set to move at the surface and use GPS to provide global navigation to the formation.

We now show results of a mission from the June-July trials, highlighting how the terrain compliance strategy prevents collisions with the terrain while attempting to influence the formation control as little as possible. Figure 9 shows the initial segment of a mission with three vehicles (full mission not shown for clarity). The pre-planned mission path of the LSV is shown in solid black line, starting from the black square. Given the objectives of the trials, the path initially runs against a wall in the harbor, shown in green. The wall-following running in the LSV, developed by UdG, should detect this wall, identify its direction, and re-plan the mission path online so as to move parallel to it, keeping a distance of 10 m. The GCV formation configuration was $\delta_x = 5$ m (5 m behind the LSV). The CV starts with a formation configuration of $\delta_x = \delta_y = 5$ m (5 m behind and 5 m to the right of the LSV) for a formation roll angle of zero. As the formation approaches the wall, the LSV should also use the sonars to identify the slope of the wall and prescribe a formation roll angle to the CV, causing it to move towards the wall. The formation was configured so that, considering this prescribed formation roll angle for this particular wall (around 40 degrees), the nominal position of the virtual target for the CV would be closer to some underwater rocks along the wall, not visible in the figure, than the *minimum distance to terrain* (set to 3 m) allows. This was intended to force the terrain compliance module to act in order to keep the vehicle at a safe distance from the wall. Finally, as soon as the initial nominal mission no longer intersected the wall, the LSV should stop following the wall and return to this initially planned path. It should also reset the prescribed formation roll angle to zero.

The trajectories of the vehicles, also shown in fig. 9, again seem to show that the performance was reasonable. We draw attention to the trajectory of the CV and of its virtual target, shown in yellow and purple respectively. As the prescribed formation roll pushed the CV virtual target towards the wall, the vehicle followed it until it became too close to the rocks. The vehicle then kept a distance of around 3 m from the virtual target. This is more easily observed in fig. 10, showing the tracking error (difference between the position of the CV and the position of its virtual target) from the start of the mission until the end of the segment parallel to the wall. This tracking error is expressed in a frame with axes perpendicular (across) and parallel (along) to the wall. After initial transients, the tracking error decreases to a value close to zero. Then, at around $t = 150$ s, the CV detects the proximity to the underwater rocks and the terrain compliance prevents it from getting closer. The component of the tracking error on the across-wall direction increases to around 3 m. Another key observation is that the along-wall component of the tracking error remains very close to zero, highlighting the fact that the terrain compliance functionality attempts not to influence the control except for the across-wall component.

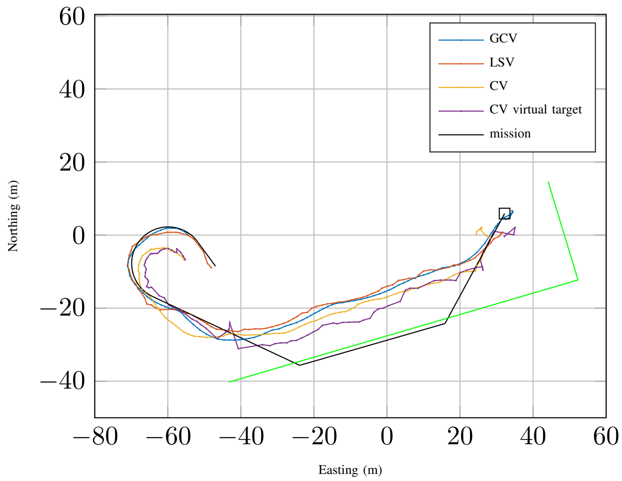

Fig. 9: Trajectories of all vehicles

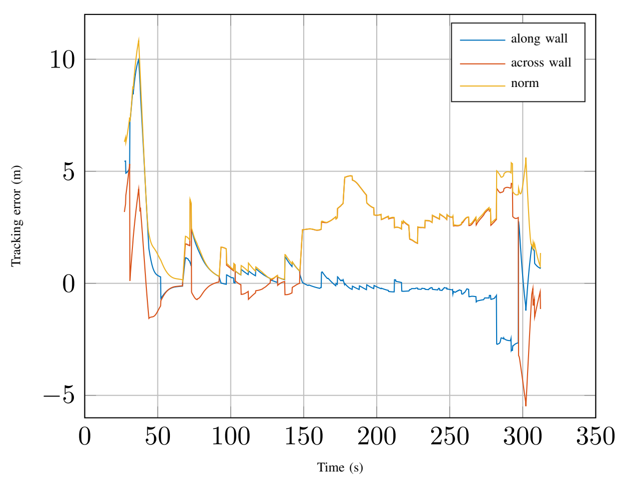

Fig. 10: Tracking error components along and across wall

V. Conclusions

In this paper, we described the formation control strategies developed by ISR/IST in the scope of the MORPH project. A general navigation and control architecture enabling a follower to track a leader vehicle with predefined offsets was presented. This architecture was then applied to the specific case of the MORPH formation (one leader, four followers) by defining what information to broadcast among the vehicles. A terrain compliance module was introduced as a supervisor block to make the vehicles maintain a safe distance from terrain while interfering as little as possible with the geometry of the formation. Although most of these systems were designed taking into account the specific limitations and requirements imposed by the practical applications of the system (i.e. scenarios of interest, speed and geometry of the missions, sensors available), the schemes presented can certainly be adapted to other applications that can benefit from a leader-follower(s) formation of autonomous marine vehicles. Results of trials at sea illustrated the performance of different features of the system implemented. The final project trials, to be held in the Azores in September 2015, will witness the demonstration of the entire MORPH system with a full vehicle formation in a scenario of interest for marine biologists, clearly illustrating the success of the project in achieving a working solution for a real application.

Acknowledgment

We are extremely grateful to everyone involved in the MORPH project for their effort and persistence in this exciting journey of developing, implementing, and sea testing a number of core systems for cooperative mission programming, communications, control, navigation, and mapping. We are especially indebted to all partner institutions outside IST for their commitment and the fantastic cooperation environment that they helped create. We trust that this collaborative effort will lead the MORPH team to a successful demonstration in the final trials of the project, to be held in September 2015 in the Azores.

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