Higher Order Statistics based Direction Of Arrival Estimation with Single Acoustic Vector Sensor in the Under-Determined Case

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Abstract— **Acoustic Vector Sensors (AVS) measure acoustic pressure as well as the acoustic particle velocity to estimate the acoustic intensity. The acoustic intensity, which is a vector quantity, represents the magnitude and direction of the active or propagating part of an acoustic field thus indicating the Direction of Arrival (DOA) of a received signal. This paper concerns with the feasibility of DOA estimation with a single acoustic vector sensor in the under-determined case where the number of sources are more than the number of sensors. It is proposed to use higher order statistics (cumulants) to increase the number of uniquely identifiable sources using a single vector sensor. The use of higher order statistics allows the increase in the dimension of the array steering matrix thereby resulting in increase of uniquely identifiable sources. The details of the proposed algorithm along with the number of sources which can be uniquely indentified are presented in this paper. It has been observed that a single AVS using fourth order statistics is capable of uniquely identifying 08 sources with not more than 04 sources in a single plane. The number of sources can be increased by working with further higher order statistics of the order of $2q$ (for $q \geq 2$). All the results of the proposed algorithm have been verified through suitable simulations.**

Keywords— Acoustic Vector Sensor, Under-determined DOA Estimation, Higher Order Statistics

I. INTRODUCTION

Acoustic emissions can provide an invaluable signature to detect, locate, and track targets/sources. A passive acoustic surveillance system allows us to monitor the environment acoustically without disturbing it or giving away information about one's own presence. The traditional solution for this purpose is to use a spatially distributed array of pressure sensors i.e. hydrophones or microphones. Many surveillance techniques for Direction of Arrival (DOA) estimation, retrieval of desired signal in the presence of noise and interference, and signal power estimation have evolved for the passive surveillance scenario [1],[2]. Acoustic emissions can also be characterized by using the particle velocity in addition to the pressure of the signal. The use of particle velocity which is a vector quantity along with the pressure would result in improved beamforming and DOA estimates with smaller arrays as compared to pressure sensors and at low signal-to-noise ratio. An acoustic vector sensor consists of three orthogonally oriented velocity sensors and a pressure hydrophone, all spatially collocated in a point-like geometry. By making additional use of the velocity information, systems built around acoustic vector sensors have numerous advantages over pressure sensor arrays and the same has been actively researched in the past [3]-[6].

A single vector sensor has an intrinsic azimuth-elevation directivity that is independent of signal frequency, signal bandwidth, and the source's location in the near field or far field. In contrast, the beam pattern and directivity obtainable from an array of spatially displaced pressure hydrophones or vector hydrophones would depend on the source frequency, distance from the array and inter-element spacing of the array. This appealing property of a single vector sensor makes it an ideal candidate for passive surveillance applications. However, a serious limitation of using single vector sensor is the limited number of sources which can be uniquely identified by a single vector sensor. It has been derived that only up to two arbitrarily oriented narrowband sources of same centre frequency can be uniquely identified by a single vector sensor [7] which make the processing of vector sensor data a over-determined one where the number of sources are less than the sensors (A single vector sensor comprises of four constituents sensors, 3 velocity sensor and 1 pressure sensor).

This paper concerns with the feasibility of DOA estimation in the under-determined case where the number of sources are lesser than the number of sensors. To provide a solution to the underdetermined case for non-Gaussian signals, we propose to use the concept of virtual arrays using higher order statistics similar to the algorithm proposed in [8] for linear array and circular array. The array steering matrix of a single vector sensor is different from conventional arrays and therefore, we evaluate the number of sources which can be uniquely identified using higher order statistics and confirm our observations through simulations. The use of higher order statistics also provide an additional advantage of immunity to Gaussian noise when compared to algorithms based on second order statistics.

Algorithms based on higher order statistics for vector sensors have been reported in [9] and [10] but not for the

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underdetermined case. An algorithm based on Khatri-Rao product for AVS in the underdetermined case has been proposed in [11]. The algorithm proposed in this paper is able to distinguish more sources using a single vector sensor than any of the algorithms reported in the past to the best of our knowledge.

The remainder of the paper is structured in the following manner. Section 2 describes the theoretical limit of uniquely identifiable sources for a passive array. Section 3 describes the proposed algorithm for identification of sources in the under-determined case with fourth order statistics. Section 4 indicates the limit on the number of sources which can be identified when they are located in the same plane. Simulation results are presented in Section 5. The proposed algorithm has been extended to even higher order statistics in Section 6. Comparison of our proposed algorithm with a recently reported algorithm based on the Khatri-Rao product has been described in Section 7. Concluding remarks are presented in Section 8.

II. UNIQUELY IDENTIFIABLE SOURCES USING PASSIVE SENSOR ARRAYS

A key problem of any passive surveillance system is to enumerate the number of narrowband sources with the same centre frequency which can be uniquely identified by the passive array. This figure is of both theoretical and practical importance and is referred to as the resolution capacity [12] of the array. This problem was addressed in [13] for a general passive array of M sensors receiving the following signal from D sources

$$\mathbf{U} = \mathbf{AS} + \mathbf{E} \tag{1}$$

where $\mathbf{A}$ is the array steering matrix for the D sources with dimensions $M \times D$, $\mathbf{S}$ is the signal matrix of $D \times N$ and $\mathbf{E}$ is the noise matrix of dimensions $M \times N$. The authors in [13] proved that the maximum number of sources which can be unambiguously identified by such an array is upper bounded by

$$a < \frac{M + \eta}{2} \tag{2}$$

where η is the rank of matrix $\mathbf{S}$.

However, the above problem did not consider the possibility of multi-dimensional signals and multi-dimensional parameter vector associated with each signal. The bound for the number of uniquely identifiable sources for a multi parameter source was reported in [14], where the following model was used for the received signal by M sensors for a single source

$$\mathbf{u} = \mathbf{a}(\boldsymbol{\theta}_1)\mathbf{s} + \mathbf{e} \tag{3}$$

where $\mathbf{s} \in \Re^q$ is the vector input signal corresponding to the source, $\mathbf{a}(\boldsymbol{\theta}_1) \in \Re^{m \times q}$ is the response matrix of the sensors to the signal, $\boldsymbol{\theta}_1 \in \Re^p$ is the p dimensional vector associated with the source and $\boldsymbol{\theta}_1 \in \Re^m$ is the additive noise.

For more than one source, the model becomes

$$\mathbf{U} = \mathbf{A}(\boldsymbol{\theta})\mathbf{S} + \mathbf{E} \tag{4}$$

where $\mathbf{S} = [\mathbf{s}_1^T, ..., \mathbf{s}_D^T]^T \in \Re^{Dq}$, $\boldsymbol{\theta} = [\boldsymbol{\theta}_1^T, ..., \boldsymbol{\theta}_D^T]^T \in \Re^{Dp}$ and $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\boldsymbol{\theta}_1), ..., \mathbf{a}(\boldsymbol{\theta}_D)] \in \Re^{M \times Dq}$

The autocorrelation of the zero-mean received signal in case of AWGN is given by

$$\begin{aligned} \mathbf{R_u} &= \mathrm{E}\{\mathbf{u}(n) * \mathbf{u}'(n)\} \\ &= \mathbf{A}(\boldsymbol{\theta})\mathbf{R_s}\mathbf{A}(\boldsymbol{\theta}) + \sigma^2\mathbf{I} \end{aligned} \tag{5}$$

For this model, the well posed solution based on the number of equations and number of variables has been proposed in [14] to be

$$a' \leq \frac{M\rho}{(p + \rho q)} \tag{6}$$

where ρ is the rank of the signal correlation matrix $\mathbf{A}(\boldsymbol{\theta})\mathbf{R_s}\mathbf{A}(\boldsymbol{\theta})$.

For the single acoustic vector sensor, M=4 and ρ =4 as the maximum number of rows in $\mathbf{A}(\boldsymbol{\theta})$ is 4, q=1 as the acoustic signal is one dimensional and p=2 as the signal is represented by two parameters (θ_l, ϕ_l). Hence the maximum number of sources which can be uniquely identified by a single vector sensor is 2 as

$$a_{AVS}' \leq \frac{4 \times 4}{(2 + 4 \times 1)} \leq 2.67 \tag{7}$$

The identification of just two sources with an acoustic vector sensor severely limits its practical utility. Hence, the current research has proposed to work with higher order statistics to increase the number of sources which can be identified using a single vector sensor.

III. PROPOSED ALGORITHM FOR SINGLE VECTOR SENSOR BASED ON HIGHER ORDER STATISTICS

We propose the algorithm based on Higher Order Statistics for a single vector sensor under the following assumptions:

A(1). The sources are statistically independent and non-Gaussian

A(2). The additive noise is Gaussian and i.i.d

The measurement model in a multi-source, single vector sensor environment with both the sensor and source considered point-like can be represented as:

$$\mathbf{u}(n) = \sum_{l=1}^{D} \mathbf{a}(\theta_l, \phi_l)\mathbf{s}_l(n) + \mathbf{e}(n) = \mathbf{As} + \mathbf{e}(n) \tag{8}$$

where $\mathbf{a}(\theta_l,\phi_l)$ is the steering vector corresponding to the l^{th} source at an azimuth angle of θ_l and elevation angle ϕ_l

$$\mathbf{a}(\theta_l, \phi_l) = [1, \cos\theta_l \cos\phi_l, \sin\theta_l \cos\phi_l, \sin\phi_l]^T \tag{9}$$

and, $\mathbf{A}$, $\mathbf{s}$ and $\mathbf{e}(n)$ correspond to the complete steering vector matrix of size $4 \times D$, the signal matrix of size $D \times N$ and the noise matrix of size $4 \times N$ respectively for the D sources and N snapshots

$$\mathbf{A} = [\mathbf{a}(\theta_1, \phi_1)\, \mathbf{a}(\theta_2, \phi_2) \ldots \mathbf{a}(\theta_D, \phi_D)] \tag{10}$$

$$\mathbf{s} = [\mathbf{s}_1(n), \mathbf{s}_2(n), \ldots \mathbf{s}_D(n)]^T \tag{11}$$

and, $\mathbf{u}(n)$ is the time series input at the four sensors

$$\mathbf{u}(n) = [\mathbf{p}(n), \mathbf{x}(n), \mathbf{y}(n), \mathbf{z}(n)]^T \tag{12}$$

The fourth order cumulant matrix $\mathbf{C}_u^4$ of the received signal can be written as

$$\begin{aligned} \mathbf{C}_u^4 &= \mathrm{E}\{(\mathbf{u}(n) \otimes \mathbf{u}'(n)) * (\mathbf{u}(n) \otimes \mathbf{u}'(n))'\} \\ &- \mathrm{E}\{(\mathbf{u}(n) \otimes \mathbf{u}'(n))\} * \mathrm{E}\{(\mathbf{u}(n) \otimes \mathbf{u}'(n))'\} \\ &- \mathrm{E}\{(\mathbf{u}(n) * \mathbf{u}'(n))\} \otimes \mathrm{E}\{(\mathbf{u}(n) * \mathbf{u}'(n))'\} \end{aligned} \tag{13}$$

where the dimension of the cumulant matrix is 16×16 . Since the fourth order cumulants of Gaussian noise are zero, the cumulant matrix can be alternatively represented as

$$\mathbf{C}_u^4 = (\mathbf{A} \otimes \mathbf{A}^*)\mathbf{C}_s^4(\mathbf{A} \otimes \mathbf{A}^*)^H \tag{14}$$

where $\mathbf{C}_s^4$ is the fourth order cumulant matrix of the desired signal vector $\mathbf{s}$ and is given by

$$\begin{aligned} \mathbf{C}_s^4 &= \mathrm{E}\{(\mathbf{s}(n) \otimes \mathbf{s}'(n)) * (\mathbf{s}(n) \otimes \mathbf{s}'(n))'\} \\ &- \mathrm{E}\{(\mathbf{s}(n) \otimes \mathbf{s}'(n))\} * \mathrm{E}\{(\mathbf{s}(n) \otimes \mathbf{s}'(n))'\} \\ &- \mathrm{E}\{(\mathbf{s}(n) * \mathbf{s}'(n))\} \otimes \mathrm{E}\{(\mathbf{s}(n) * \mathbf{s}'(n))'\} \end{aligned} \tag{15}$$

Since the sources are statistically independent, the cumulant matrix of the desired signal vector $\mathbf{s}$ would be a diagonal matrix and the cumulant matrix of the received signal can be written as

$$\mathbf{C}_u^4 = \sum_{l=1}^{D} (\mathbf{a}(\theta_l, \phi_l) \otimes \mathbf{a}^*(\theta_l, \phi_l))\kappa_4^l(\mathbf{a}(\theta_l, \phi_l) \otimes \mathbf{a}^*(\theta_l, \phi_l))^H \tag{16}$$

where κ_4^d is the fourth order cumulant of $\mathbf{s}_l$ for $l = 1,2...\mathrm{D}$.

In Eq.16 we can define a virtual array steering matrix $\mathbf{b}(\theta,\phi)$ of dimensions 16×1 as

$$\mathbf{b}(\theta, \phi) = \mathbf{a}(\theta, \phi) \otimes \mathbf{a}^*(\theta, \phi) \tag{17}$$

then Eq.15 can also be written as

$$\mathbf{C}_u^4 = \mathbf{BKB}^H \tag{18}$$

where

$$\mathbf{B} = [\mathbf{b}(\theta_1, \phi_1), \ldots, \mathbf{b}(\theta_D, \phi_D)] \tag{19}$$

$$\mathbf{K} = \mathrm{diag}\{\kappa_4^1, \kappa_4^2, \ldots, \kappa_4^{\mathrm{D}}\} \tag{20}$$

The cumulant matrix in Eq.18 maintains the same structure as the standard array processing problem and the spatial power spectrum for each (θ,ϕ) can be obtained using conventional techniques such as DAS or SCB. We propose the utilization of SCB with the cumulant matrix as indicated below for obtaining the spatial power spectrum and identifying peaks to estimate the DOA:-

$$\mathbf{P}_{SCB\_CUM}(\theta, \phi) = \frac{1}{\mathbf{b}^H(\theta, \phi)(\mathbf{C}_u^4)^{-1}\mathbf{b}(\theta, \phi)} \tag{21}$$

Since the dimension of the virtual array matrix has increased, we can detect more sources than originally possible. We use the upper bound indicated in Eq.6 to identify the maximum number of sources which can be uniquely identified by using the proposed algorithm based on higher order statistics. The maximum number of sources would depend on the rank of the matrix $\mathbf{b}^H(\theta,\phi)(\mathbf{C}_u^4)\mathbf{b}(\theta,\phi)$ **.** In case the sources are independent, $\mathbf{C}_u^4$ will be full rank with rank equal to the number of sources. Then the maximum number of sources would depend on the size and rank of the virtual array steering matrix $\mathbf{b}(\theta,\phi) = \mathbf{a}(\theta,\phi) \otimes \mathbf{a}^*(\theta,\phi)$.

The observation of the virtual array steering matrix of one source from the direction (θ_l,ϕ_l) is represented below at Eq.22 reveals that 6 elements of the vector are repeated (marked by #), thereby indicating that the virtual array matrix comprises of 10 distinct sensors. Since the number of distinct elements in $\mathbf{b}(\theta_l,\phi_l)$ is 10 the rank of the matrix $\mathbf{b}^H(\theta,\phi)(\mathbf{C}_u^4)\mathbf{b}(\theta,\phi)$ would also be upper bounded by 10.

$$\mathbf{b}(\theta_l, \phi_l) = \mathbf{a}(\theta_l, \phi_l) \otimes \mathbf{a}^*(\theta_l, \phi_l)$$
$$= \begin{bmatrix} 1 \\ \cos\theta_l \cos\phi_l \\ \sin\theta_l \cos\phi_l \\ \sin\phi_l \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \cos\theta_l \cos\phi_l \\ \sin\theta_l \cos\phi_l \\ \sin\phi_l \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ \cos\theta_l \cos\phi_l \\ \sin\theta_l \cos\phi_l \\ \sin\phi_l \\ \cos\theta_l \cos\phi_l{}^{\#} \\ \cos^2\theta_l \cos^2\phi_l \\ \cos\theta_l \sin\theta_l \cos^2\phi_l \\ \cos\theta_l \cos\phi_l \sin\phi_l \\ \sin\theta_l \cos\phi_l{}^{\#} \\ \cos\theta_l \sin\theta_l \cos^2\phi_l{}^{\#} \\ \sin^2\theta_l \cos^2\phi_l \\ \sin\theta_l \cos\phi_l \sin\phi_l \\ \sin\phi_l{}^{\#} \\ \cos\theta_l \cos\phi_l \sin\phi_l{}^{\#} \\ \sin^2\theta_l \cos^2\phi_l{}^{\#} \\ \sin^2\phi_l \end{bmatrix} \tag{22}$$

Substituting the value of M=10, ρ =10, p=2, q=1 in Eq.6 we get the maximum number of uniquely identifiable sources as,

$$a'_{4AVS} \le \frac{10\times10}{(2+10)} \le 8.33 \tag{23}$$

where a'_{4AVS} represents the maximum number of sources identifiable by a single AVS using fourth order statistics.

Hence, it can be concluded that 8 sources can be localized with a single AVS(comprising of four constituent sensors) in ideal conditions using fourth order statistics. However, there is a limitation of resolvability of only four targets collocated in a single plane. The reason for the same is explained in the next section.

IV. IDENTIFICATION OF SOURCES IN A SINGLE PLANE

Close observation of the virtual array steering matrix $\mathbf{b}(\theta,\phi)$ further reveals that only four unique sources can be identified in a single azimuth or in an elevation plane. This is because the number of distinct elements in the virtual array further reduces if we make the elevation or azimuth constant. For example, if we work in a single elevation plane and denote $\sin\phi_l = a$, $\cos\phi_l = b$, then the array steering matrix would be as shown in Eq.24. It can be seen that apart from the six elements which were repeated (marked by $^{\#}$), there are elements which are scalar scaled version of other elements when constants are introduced (marked by $^{\$}$). a and a^2 being constants in the search criterion are scaled version of 1. $ab\cos\phi_l$ is a scaled version of $b\cos\phi_l$. Similarly, $ab\sin\phi_l$ is a scaled version of $b\sin\phi_l$. Hence the distinct elements that remain are only six and substituting the same in Eq. 6, results in the maximum number of sources with either the elevation or azimuth constant as $6\times6/(2+6)\le4.5$.

$$\mathbf{b}(\theta_l,\phi_l) = \begin{bmatrix} 1 \\ b\cos\theta_l \\ b\sin\theta_l \\ a^{\$} \\ b\cos\theta_l{}^{\#} \\ b^2\cos^2\theta_l \\ b^2\cos\theta_l\sin\theta_l \\ ab\cos\theta_l{}^{\$} \\ b\sin\theta_l{}^{\#} \\ b^2\cos\theta_l\sin\theta_l{}^{\#} \\ b^2\sin^2\theta_l \\ ab\sin\theta_l{}^{\$} \\ a^{\#} \\ ab\cos\theta_l{}^{\#} \\ b^2\sin^2\theta_l{}^{\#} \\ a^{2\$} \end{bmatrix} \tag{24}$$

The upper limit of number of sources which can be identified in a single plane can also be verified by using a vector sensor with only three elements comprising of a pressure sensor and two velocity sensors in a single plane (azimuth/elevation only). The array steering matrix for such a sensor would be $\mathbf{a}(\theta_l) = [1, \cos\theta_l, \sin\theta_l]^T$ and the virtual array steering matrix using fourth order cumulants would be as given as

$$\begin{aligned} \mathbf{b}(\theta_l) &= \mathbf{a}(\theta_l)\otimes\mathbf{a}^*(\theta_l) \\ &= \begin{bmatrix} 1 \\ \cos\theta_l \\ \sin\theta_l \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \cos\theta_l \\ \sin\theta_l \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ \cos\theta_l \\ \sin\theta_l \\ \cos\theta_l{}^{\#} \\ \cos^2\theta_l \\ \cos\theta_l\sin\theta_l \\ \sin\theta_l{}^{\#} \\ \cos\theta_l\sin\theta_l{}^{\#} \\ \sin^2\theta_l \end{bmatrix} \end{aligned} \tag{25}$$

It can be seen that there are six distinct elements in the virtual array steering matrix and hence the maximum number of sources which can be found using fourth order statistics for this sensor would also be limited to four.

V. SIMULATION RESULTS

The findings were verified with simulations wherein eight sources were uniquely identified by the algorithm. For the simulation the source matrix was formed as given by Eq.16. Signals with equal fourth order cumulant were simulated from locations (Azimuth, Elevation) of (40°,10°), (80°,20°), (120°,30°), (160°,40°), (200°,50°), (240°,60°), (280°,70°), (320°,80°). Since the theoretical effect of Gaussian noise on the cumulant matrix is nil, the matrix in Eq.16 would tend to be singular in nature with difficulty in achieving the inverse matrix. To avoid this problem, a diagonal matrix has been added to the cumulant matrix which can also be considered as noise. For the simulation result indicated in Fig. 1, noise matrix of SNR 30 dB was added. The spatial spectrum obtained using Eq. 21 is plotted at Fig. 1. It can be seen that the eight sources can be distinguished using a single AVS.

It is further observed that eight is the theoretical limit and the actual number of sources would depend on the angular separation between the individual sources and the SNR. With an increase in the noise level, the performance for the same eight sources deteriorates as shown through simulation results shown in Fig. 2-3. In Fig. 2, we can observe that the same eight sources are barely discernable when the SNR is 15dB. When the SNR is 0 dB, the sources are not discernable as seen in Fig. 3.

The resolvability also depends on the angular separation between the individual sources. In Fig. 4, it can be seen that four sources at (40°,10°), (80°,20°), (120°,30°), (160°,40°) are not discernable at 0 dB SNR. However, if we increase the spacing between the sources, the same are resolved at 0 dB SNR as shown in Fig. 5 where the four sources are located at (60°,12°), (120°,24°), (180°,36°), (240°,48°). Even five sources located at similar angular separation can be identified as shown in Fig. 6. The sources are located at (60°,12°), (120°,24°) , (180°,36°), (240°,48°), (300°,60°).

The limitation of 4 sources in a single plane has also been verified through simulations. Four equi-powered sources were simulated with the same elevation of 10° and with varying azimuth of 60°, 120°, 180° and 240°. The noise was added with 30 dB SNR and all the sources were identified as indicated in the power spectrum displayed in Fig. 7. The four peaks can be clearly discerned.

However, when an additional source was added at the same elevation at an azimuth of 300°, the ability to distinguish the sources was lost even at the high SNR of 30 dB. The spatial power estimate obtained when five sources were present is displayed in Fig. 8. It can be seen that the five sources are not discernable.

Similar results are observed when the azimuth is kept constant and search is undertaken only in the elevation plane. Four sources can be clearly distinguished as seen in Fig. 9. The sources were at a constant azimuth of 60° and elevation of 10°, 20°, 30° and 40°.

However when one source is added at 50° elevation, the algorithm fails to detects beyond the fourth target as shown in Fig. 10. Five sources could not be localized and incorrect estimates of four targets would be provided in this case.

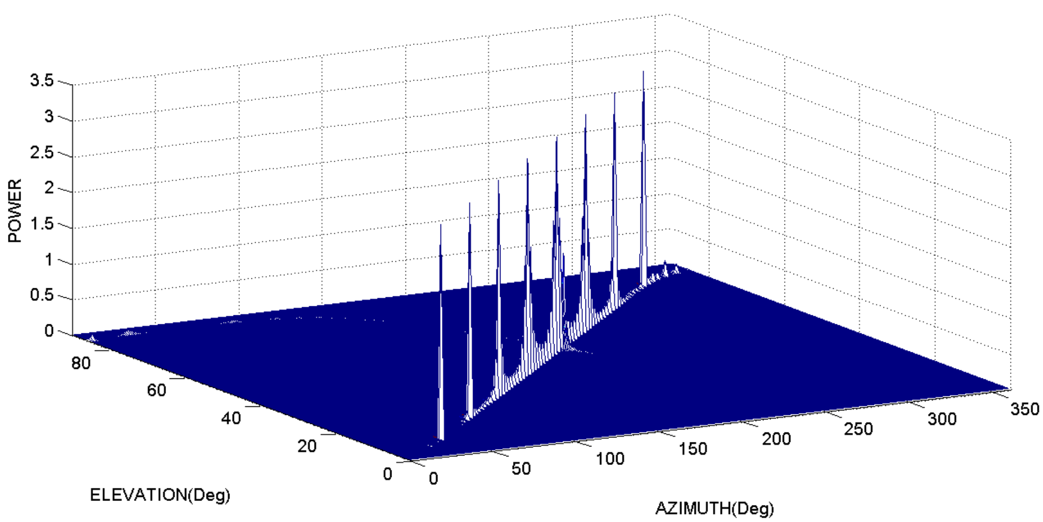

Fig. 1. Localization of Eight Sources by Single AVS at 30dB SNR

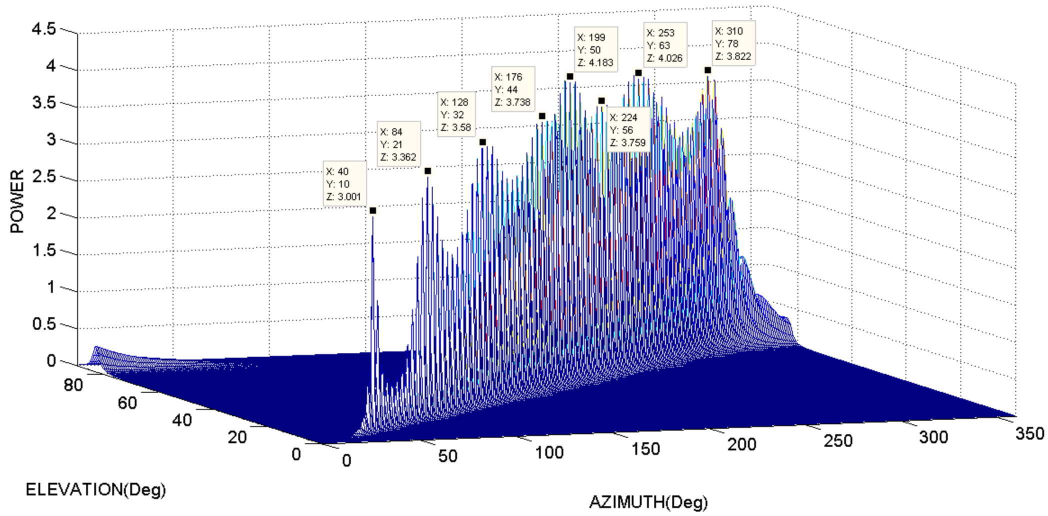

Fig. 2. Localization of Eight Sources by Single AVS at 15 dB SNR

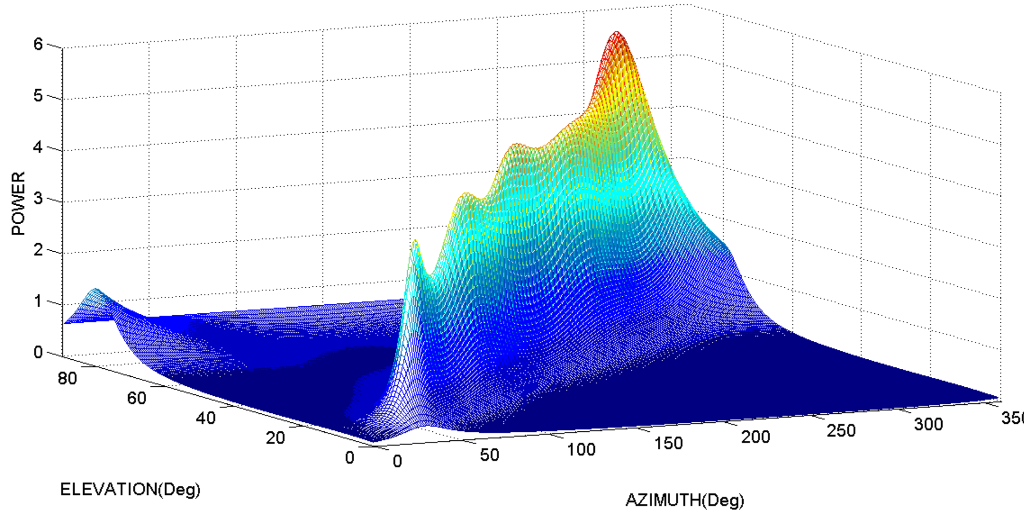

Fig. 3. Localization of Eight Sources by Single AVS at 0 dB SNR

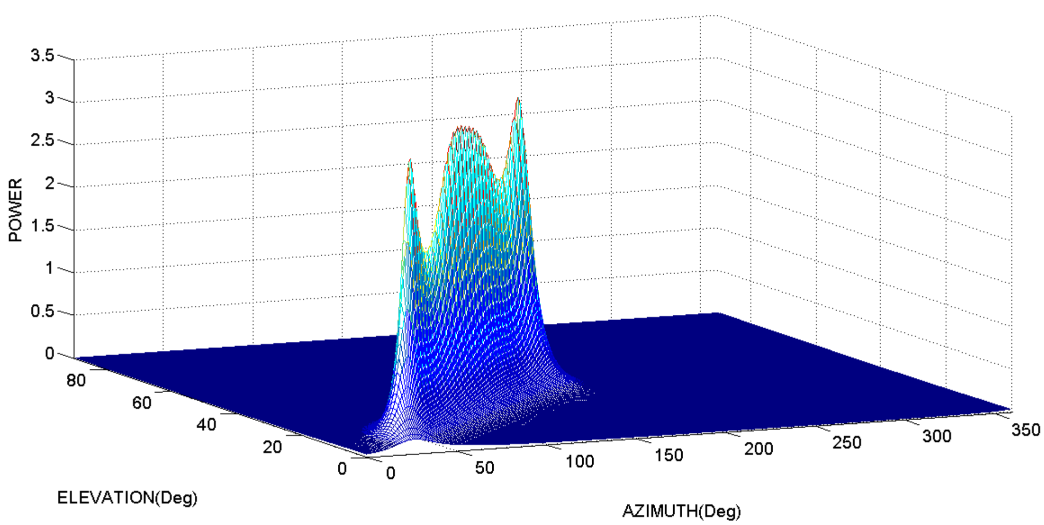

Fig. 4. Localization of Four Sources by Single AVS at 0 dB SNR

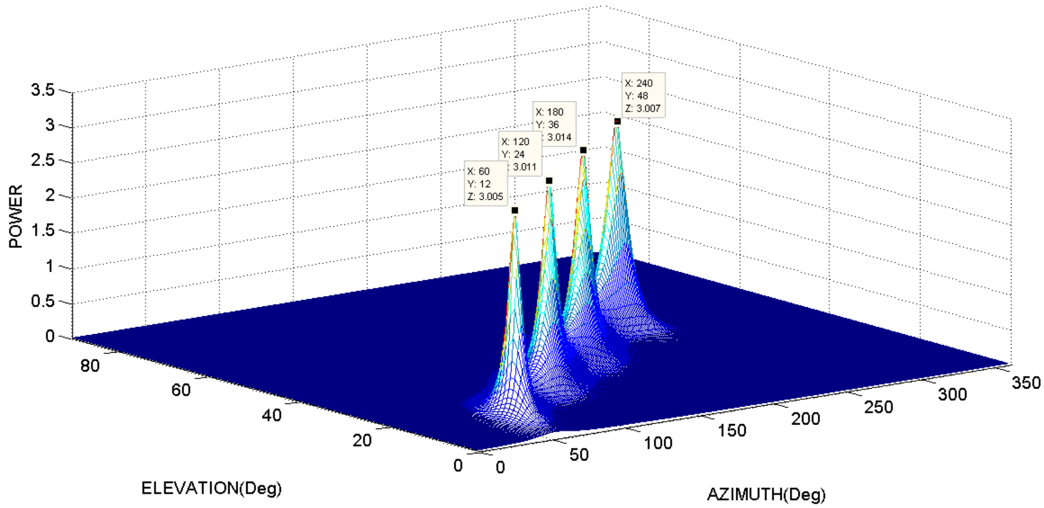

Fig. 5. Localization of Four Widely Separated Sources by Single AVS at 0 dB SNR

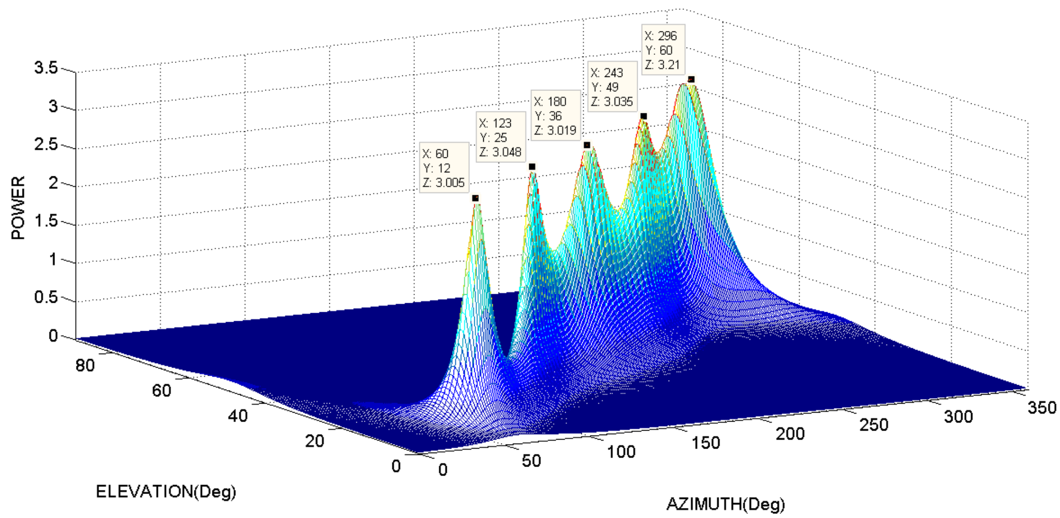

Fig. 6. Localization of Five Widely Separated Sources by Single AVS at 0 dB SNR

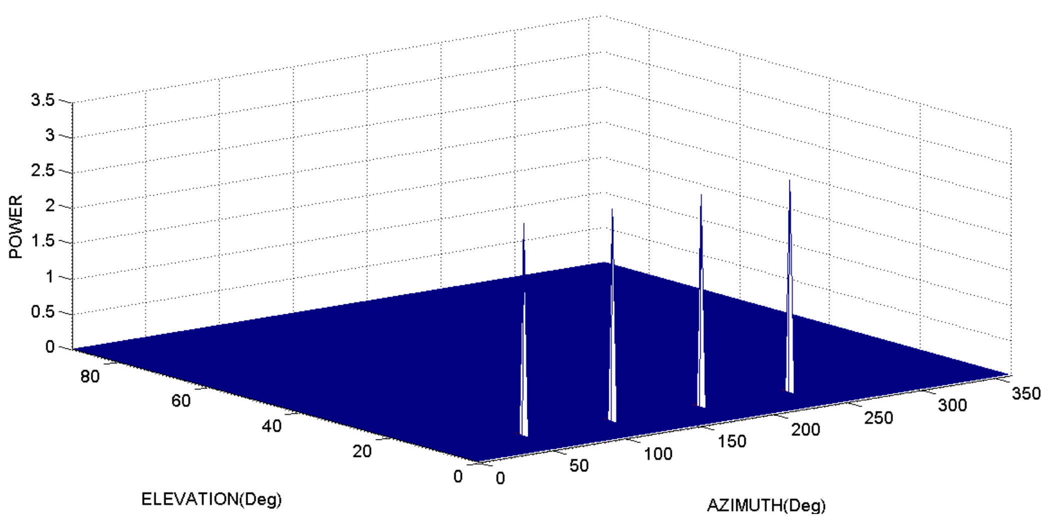

Fig. 7. Localization of Four sources at a constant Azimuth Plane at 30dB SNR

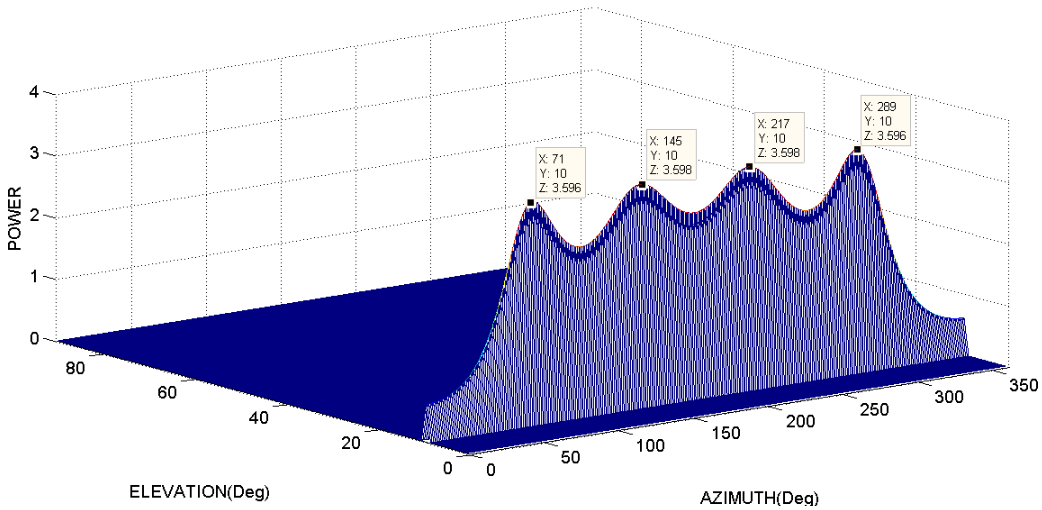

Fig. 8. Inability to Localize Five sources at a constant Azimuth Plane at 30dB SNR

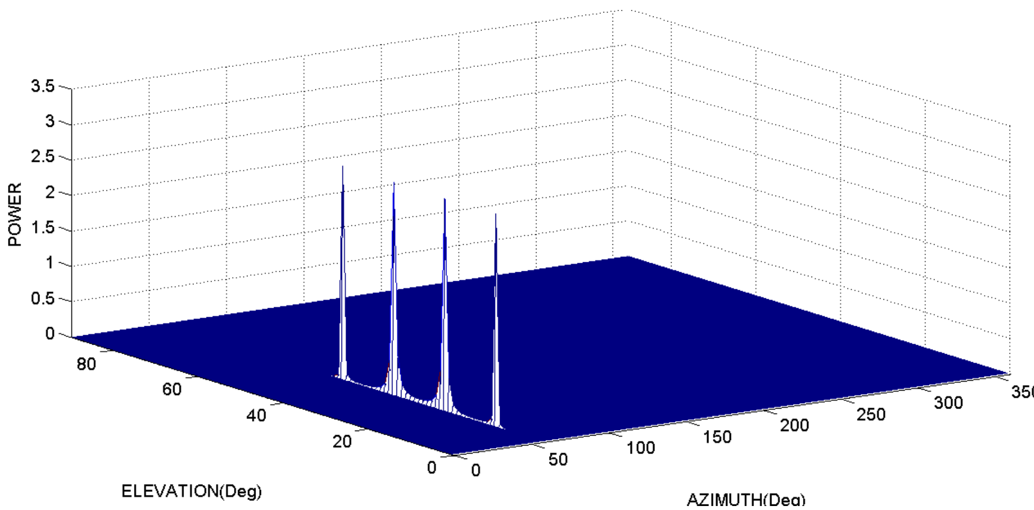

Fig. 9. Localization of Four sources at a constant Elevation Plane at 30dB SNR

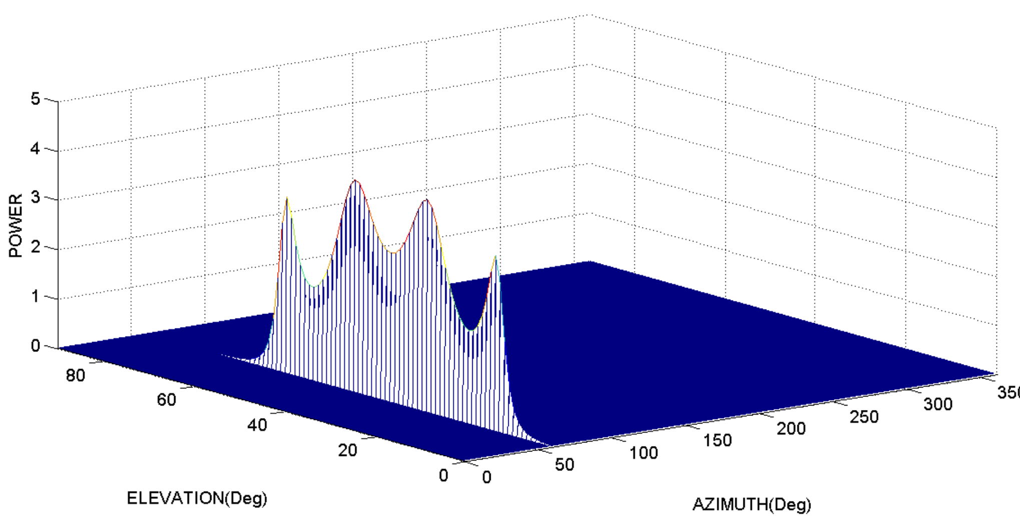

Fig. 10. Inability to Localize Five sources at a constant Elevation Plane at 30dB SNR

VI. NUMBER OF SOURCES WITH 2QTH ORDER ALGORITHM

Following the same principle as indicated in the earlier section for fourth order statistics, we can use even higher order statistics of the order of 2q (q≥2) to increase size of the virtual array [15] and thereby increase the number of uniquely identifiable sources. The spatial spectrum of the 2qth order would be

$$\mathbf{P}_{SCB\_CUM\_2q}(\theta,\phi) = \frac{1}{(\mathbf{a}(\theta,\phi)^{\otimes q})^H (\mathbf{C}_u^{2q})^{-1} (\mathbf{a}(\theta,\phi)^{\otimes q})} \quad (26)$$

where $\mathbf{a}(\theta,\phi)^{\otimes q} = \mathbf{a}(\theta,\phi) \otimes \mathbf{a}(\theta,\phi) \otimes \ldots\ q$ times.

The virtual array steering matrix for using the $2q^{\text{th}}(q \geq 2)$ order statistics would be $\mathbf{a}(\theta,\phi)^{\otimes q} = \mathbf{a}(\theta,\phi) \otimes \mathbf{a}(\theta,\phi) \otimes \ldots\ q$ times. This way, with increase in the order, the maximum number of sources which can be uniquely identified in ideal conditions can be increased. The increase in the number of sources will depend on the number of unique elements in the resultant virtual array matrix. As was observed in the kronecker product for the fourth order matrix, repetitions in the elements within the array steering matrix would occur for the higher orders also. The number of unique elements and the corresponding number of maximum number of sources that can be identified for the higher orders have been evaluated and listed in Table I. It is further reiterated that the values given in Table I are theoretical limits which can be achieved in ideal conditions. In practice, the DOA estimation will depend on the SNR and the angular separation between the sources.

TABLE I. MAXIMUM NUMBER OF SOURCES UNIQUELY IDENTIFIABLE SOURCES WITH HIGHER ORDER STATISTICS

ORDER	NO OF UNIQUE ELEMENTS IN THE STEERING VECTOR	MAXIMUM NUMBER OF IDENTIFIABLE SOURCES
4	10	8
6	20	18
8	35	33
10	56	54
12	86	84

VII. COMPARISON WITH ALGORITHM BASED ON KHATRI-RAO PRODUCT

Recently an algorithm for DOA estimation using AVS in the underdetermined case has been proposed based on the Khatri-Rao(KR) product [16]. The algorithm uses the following property of the 'vec' operator

$$\text{vec}(\mathbf{AXB}) = (\mathbf{B}^T \otimes \mathbf{A})\text{vec}(\mathbf{X}) \quad (27)$$

to process the received signal covariance matrix as indicated below:

$$\text{vec}(\mathbf{R_u}) = \text{vec}(\mathbf{A}(\boldsymbol{\theta})\mathbf{R_s}\mathbf{A}(\boldsymbol{\theta})) + \text{vec}(\mathbf{E}) \quad (28)$$

$$= (\mathbf{A}(\boldsymbol{\theta})\ \Theta\ \mathbf{A}(\boldsymbol{\theta}))\mathbf{r_s} + \text{vec}(\mathbf{E})$$

where Θ is the Khatri-Rao(KR) product and $(\mathbf{A}(\boldsymbol{\theta})\,\Theta\,\mathbf{A}(\boldsymbol{\theta}))$ = $[a(\boldsymbol{\theta}_1)\otimes a(\boldsymbol{\theta}_1),\ldots a(\boldsymbol{\theta}_D)\otimes a(\boldsymbol{\theta}_D)] \in \Re^{16\times D}$. The algorithm then concatenates the signal covariance matrix of several batches of a quasi-stationary signal and performs the MUSIC algorithm for DOA estimation. While the steering matrix of this algorithm is of similar dimensions as proposed in our algorithm, the processing is considerably different. While the algorithms based on KR product has achieved DOA estimation of up to six speech sources, our algorithm is capable of estimating eight sources with fourth order statistics and more with higher order statistics. The KR approach also has a limitation of four sources in a single plane as is the case for our algorithm. In contrast to the KR approach, our algorithms is based on higher order statistics which are impervious to all types Gaussian noise and hence is likely to give improved performance and increased number of detection of sources as compared with the KR-AVS approach.

VIII. CONCLUSION

A vector sensor allows the location of a target to be determined in a more accurate and efficient manner when compared to a traditional omni-directional hydrophone. However, Existing literature indicates that only two arbitrarily oriented sources of the same frequency can be uniquely identified by a single vector sensor. This would severely restrict the utility of a practical system involving a single vector sensor. We prove that eight sources with no more than four sources in a single azimuth / elevation plane can be uniquely localized when processing AVS data with fourth order statistics. The number of sources can be increased by working with further higher order statistics of the order of $2q$(for $q \geq 2$) . All the results of the proposed algorithm have been verified through suitable simulations. The proposed algorithm is able to distinguish more narrow band sources using a single vector sensor than any other algorithm reported in the past to the best of our knowledge. Increasing the number of sources using a single vector sensor significantly improves the practical utility of such sensors for passive surveillance applications.

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