Steered Nonlinear Field Computation and Measurement

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***Abstract*— One order parabolic approximation KZK equation is considered to be reliable inside ±15 degree relative to the KZK centerline, so KZK equation is unsuitable for studying steered beams in its classical configuration of having beam axis aligned with transducer central axis. An equivalent parametric array model was proposed based on the collimation characteristic of primary field, so the problem of steered beams calculation using KZK equation is converted to source condition calculation of equivalent parametric array. In this paper, a time domain solution of KZK equation for rectangular source is used to calculate steered nonlinear beams, the natural directivity beam calculated under this model is compared with classical KZK approach, and the influence of the distance between the center of parametric array and equivalent parametric array in calculating steered nonlinear field is analyzed at the same time. Finally, comparison is made between the calculated and measured steered nonlinear field which shows that the equivalent parametric array model is an effective theory model in steered nonlinear beams computation, and this research gives a helpful direction to the future study.**

Keywords—steered nonlinear beams; KZK equation; equivalent parametric array; time domain solution; channel tank measurement

I. INTRODUCTION

Finite-amplitude sound waves are used in many applications, such as underwater acoustic engineering, biomedicine, air acoustics and so on [1-3]. One typical example in underwater acoustic engineering is parametric sub-bottom profiling, when the parametric sonar emitting intensity primary waves with frequencies f_1 and f_2 (f_1<f_2 and f_0=(f_1+ f_2)/2 is central frequency), nonlinear interaction of sound with the water is created, and a part of energy is shifted from the primary waves to secondary waves with different frequencies such as 2 f_1, 2 f_2, f_1+ f_2, f_d=|f_1- f_2| and so on, where f_d is used to detect the structure and characteristics of sediment [4]. The application of nonlinear acoustic is based on the researches of nonlinear field. Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation is a popular model for calculating nonlinear sound field which can be solved in the time-domain or frequency domain [5]. Because KZK equation is a one order parabolic approximation equation, the nonlinear sound field calculated by KZK equation is reliable inside ±15 degree and outside of which it will suffer from a lack of accuracy, so the KZK equation is unsuitable for calculating the steered nonlinear field especially the beams outside this narrow field angle. Natural directivity finite-amplitude sound beams whose beam axis aligned with transducer central axis have been studied in more detail, but the steered nonlinear field studies are not as numerous.

The primary field of parametric array has a collimation characteristic, and based on this characteristic, an equivalent parametric array model is proposed to solve the disadvantage of KZK equation in calculating steered nonlinear field, so the steered nonlinear sound field can be calculated using the KZK equation's original algorithm structure, the only difference is the equivalent parametric source condition should be recalculated in the parametric array coordinate. The problem of steered beams calculation using KZK equation is converted to source condition calculation of equivalent parametric array. A coordinate transformation relationship between the coordinates of equivalent parametric array and the actual parametric array is given to realize the expression of steered nonlinear sound field in the parametric array coordinate.

The steered nonlinear sound field theoretical numerical computation and measurement of rectangular parametric sources are researched in this paper, the nonlinear sound field numerical computation is done in time-domain, and the measurement is done in a channel tank experiment.

II. STEERED NONLINEAR ACOUSTIC FIELD NUMERICAL COMPUTATION

A. Theory Equation and Geometry Model

The KZK equation is a model that is commonly used to calculate the paraxial region of a directive nonlinear acoustic field. Most of the parametric arrays used in underwater acoustic engineering especially in the field of parametric sub-bottom profiling are rectangular apertures, so we take a plane rectangular piston parametric transducer as subject investigated, and consider the nonlinear propagation occurs in water, the one order parabolic approximation KZK equation defined in the Cartesian coordinates is [6]

$$\frac{\partial^2 p}{\partial z \partial t'} = \frac{c_0}{2}\nabla_{\perp}^2 p + \frac{\delta}{2c_0^3}\frac{\partial^3 p}{\partial t'^3} + \frac{\beta}{2\rho_0 c_0^3}\frac{\partial^2 p^2}{\partial t'^2} \qquad (1)$$

where p is pressure, c_0 is small-signal sound velocity, ρ_0 is medium density, □ is the diffusivity of sound, β is the coefficient of nonlinearity, $\nabla^2_{\perp}$ is Cartesian coordinate. The terms on the right side of (1) account for diffraction, absorption and nonlinearity in turn.

Following the method of Lee and Hamilton [7], equation (1) is transformed into non-dimensional form on the Cartesian grid by coordinate transformation to improve the computational efficiency in the far field, and the resulting parabolic equation is integrated analytically once with respect to time. The dimensionless form of KZK equation can be written as

$$\frac{\partial P}{\partial \sigma} = \frac{R}{\pi(1+\sigma)^2}\int_{-\infty}^{\tau}\left(\frac{\partial^2 P}{\partial u_x^2}+\frac{\partial^2 P}{\partial u_y^2}\right)d\tau' + A\frac{\partial^2 P}{\partial \tau^2} + \frac{NP}{1+\sigma}\frac{\partial P}{\partial \tau} \quad (2)$$

where A is sound absorption coefficient, α_0 is attenuation coefficient, N is nonlinear coefficient, $R = b/a$ is aspect ratio, the parametric source dimension is $2a\times 2b$ (where a>b). Based on (2), a time-domain numerical algorithm for solving the KZK equation can be used to compute the nonlinear acoustic field P. This equation is solved sequentially at each range step in sequence of diffraction term, absorption term and nonlinear term [8]. Because one order parabolic approximation is applied in solving KZK equation, the nonlinear acoustic field can be accurately computed inside ±15 degree relative to natural directivity sound beam axis (or transducer central axis). The approximation fitting degree based on Muir square root expansion is descripted in Fig.1.

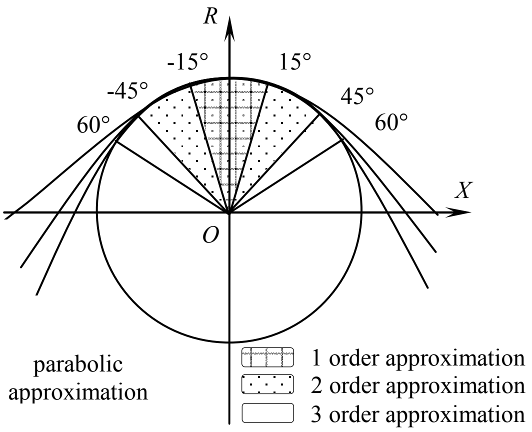


Fig.1 Parabolic approximation fitting degree

The primary field distribution of rectangular aperture parametric source is illustrated in Fig.2. The primary wave is assumed to be a collimated plane wave with a cross-sectional area equal to that of the source and propagating from the source face to a distance L_1=$4b^2/\lambda_0$, between L_1 and L_2 (where L_2=$4a^2/\lambda_0$) the primary sound spreads cylindrically, and beyond L_2 spherical spreading occurs [8].

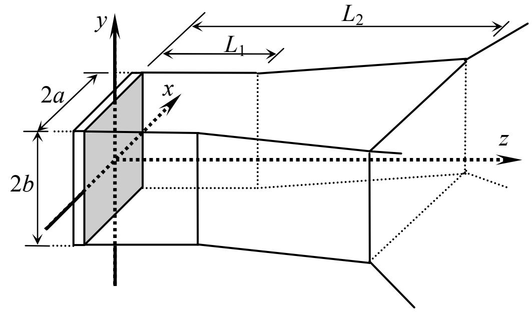
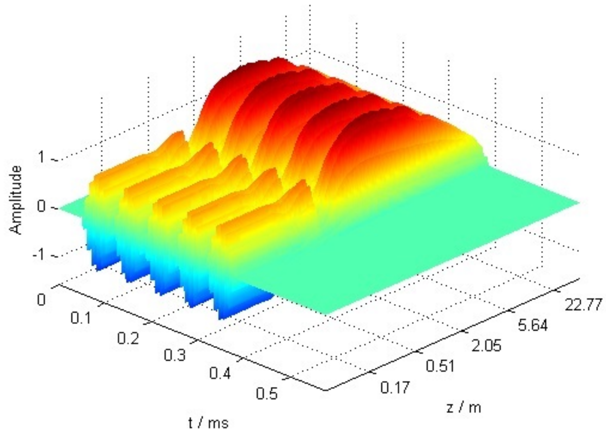
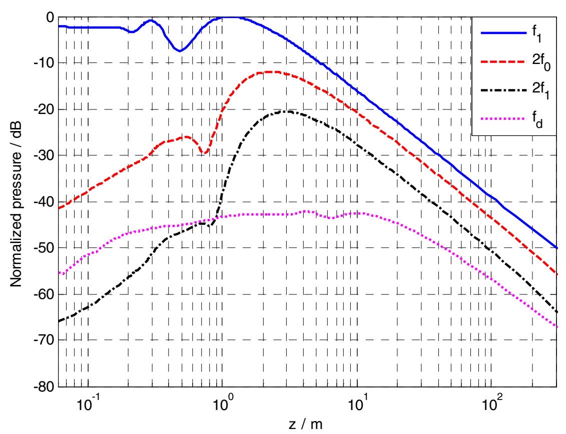


Fig.2 Geometrical model of primary field of rectangular aperture parametric source

Nonlinear sound waves on the transducer central axis changed with range z are given in Fig.3, the source signal is amplitude modulated (AM) signal whose carrier frequency is center frequency and the base band signal frequency is half of the difference frequency. We can see in Fig.3 that the waveform and the amplitude of the radiated AM signal nearby the radiation surface (z=0) change small.



(a) acoustic wave



(b) axis pressure

Fig.3. Nonlinear acoustic wave changed in the z direction

Based on the nearfield characteristic of primary wave, the nonlinear acoustic field can be considered as a field generated by a virtual source in the collimated zone vertical to the central axis of parametric array and has the same aperture of the parametric array. Similarly, the steered nonlinear field also can be considered as a field generated by a virtual source vertical to the axis of the steered beam, and the aperture of this virtual source is the profile of the collimation zone, so the steered nonlinear field are also can be computed by the original algorithm structure of KZK equation. In this paper, the virtual source is called equivalent parametric array. The geometric model of equivalent parametric array to generate the steered nonlinear field is given in Fig.4. Consider the parametric array in a Cartesian coordinate system (x, y, z) which is centered at the center of the radiate surface. For simplicity of discussion we take the beams steered in (x, z) plane for example to analysis steered nonlinear field. At the same time, the equivalent parametric array is considered in a dynamic coordinate system (x_E, y_E, z_E) relative to (x, y, z), the distance between the two origins is d_E, and the same field point F is descripted as (x, z) and (x_E, z_E) in the two coordinate systems respectively.

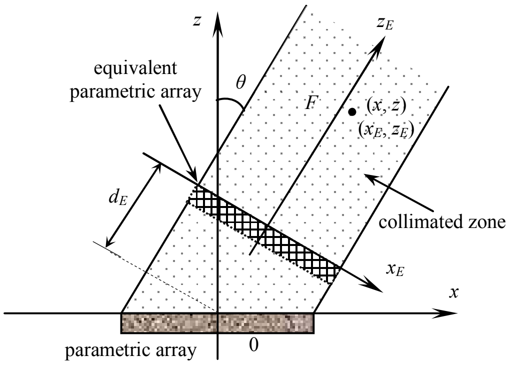


Fig.4. Geometric model of steered field generation

According to the geometrical relationship between the two coordinate systems, the coordinate transformation can be given by

$$\begin{cases} x = x_E \cos\theta + z_E \sin\theta + d_E \sin\theta \\ z = -x_E \sin\theta + z_E \cos\theta + d_E \cos\theta \end{cases} \quad (3)$$

where θ is steered angle.

B. Source Condition

When equivalent parametric array is proposed, steered nonlinear acoustic field can be computed using KZK equation's original algorithm structure, and the main job is source condition calculation. We can also find that the energy shift from primary waves to secondary waves is very small in the zone near the source surface, so the wave propagation can be considered as linear process, and the source condition can be approximatively calculated using linear model. On the assumption that the nonlinear source is surrounded by a plane rigid baffle, and the radiate surface source is divided into rectangular elements as shown in Fig.5, the sound pressure amplitude of field point F_r can be descripted as [9]

$$p_r = \frac{j\rho c}{\lambda} \sum_{n=1}^{N} u_n \int_{\Delta s} \frac{e^{-(\alpha + jk)r}}{r} ds \quad (4)$$

where ρ is density of water, c is sound velocity, λ is sound wave length, N is the number of elements, u_n is the complex surface velocity for element n, Δs is the area of a single element, α is attenuation coefficient, k is the wave number and r is the distance between element and the field point.

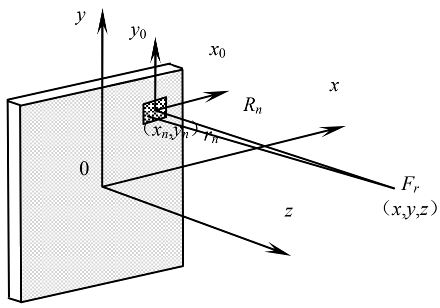
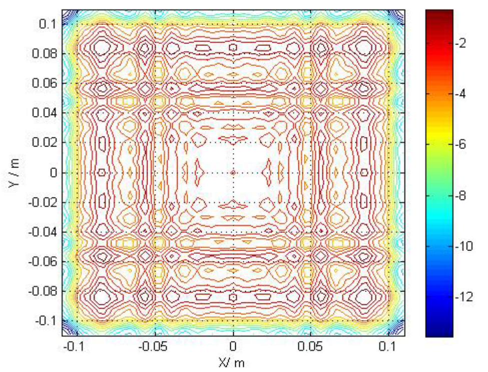


Fig.5. Linear acoustic field computation model

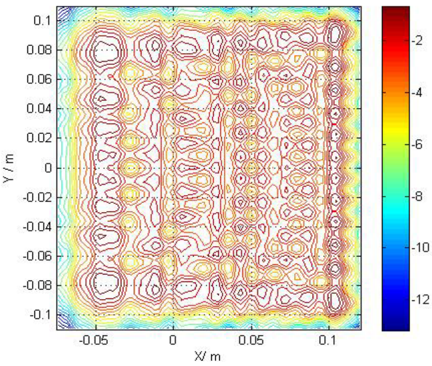
Base on the geometric model in Fig.4, when the steered angle is θ we can derive the coordinate value in (5), which is the field points on the equivalent parametric array radiate surface descripted in (x, z) coordinate, this is an important relationship in steered nonlinear acoustic field source condition calculation.

$$\begin{cases} x = d_E \sin(\theta) + x_E \cos^2(\theta) \\ z = d_E \cos(\theta) + x_E \cos(\theta)\sin(\theta) \end{cases} \quad (5)$$

where $x_E \in [-a, a]$. Combined (4) and (5), the steered nonlinear acoustic field source condition can be calculated (see Fig.6).



(a) 0 degree steered

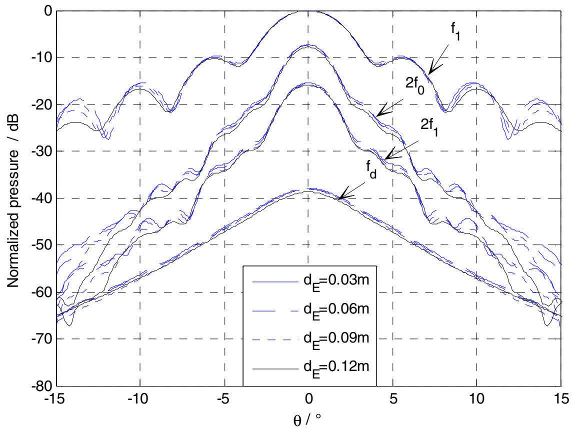


(b) 15 degree steered

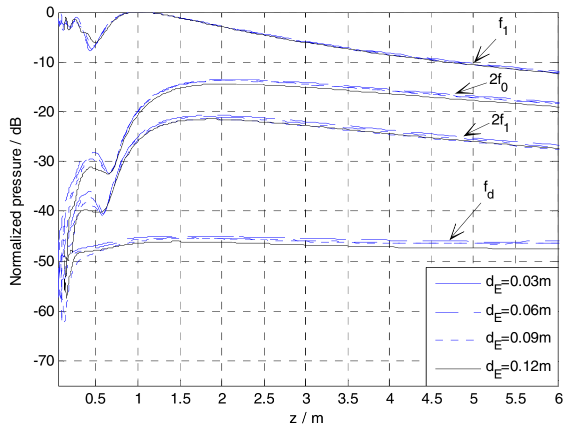
Fig.6. Profiles of steered nonlinear acoustic field

C. *Field Computation and Analysis*

Despite the primary frequency near field is collimated and the algorithm structure is fixed, but the source condition is different when we take different values of d_E. Assume the dimension of a parametric array is 0.22m×0.22m, and d_E = 0.03, 0.06, 0.09 and 0.12m, the directivity and axis pressure characteristics are illustrated in Fig. 7.



(a) directivity (z=12m)



(b) axis pressure

Fig.7 Nonlinear sound field characteristic with different d_E

With the value of d_E increased, the computation deviation is also increased gradually, for d_E=0.12m, a difference of about 1dB has been reached for the pressure of difference frequency f_d, but for the other values of d_E the differences are all less than 1dB or even smaller. The cause of this phenomenon is more energy in the sound field is shifted from the primary waves (f_1 and f_2) to secondary waves (nf_1, nf_2, nf_0, nf_d, where n=1, 2, 3···) with d_E increased, and the linear model for sound propagation is also gradually mismatch in calculating the source condition.

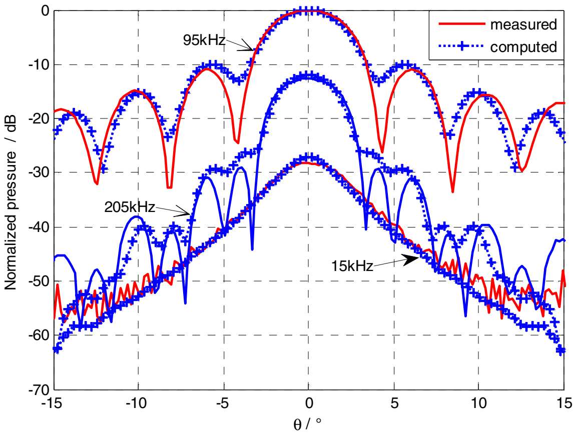
Even the steered angle is up to 60 degree, we can also use d_E less than 0.12m for this parametric array, so if we use the equivalent virtual array near the parametric source adequately, we can calculate the steered nonlinear acoustic field very well.

III. EXPERIMENT RESULTS

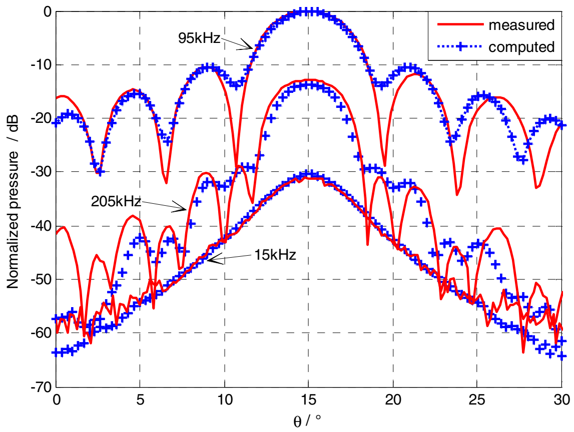
Measurement experiment is done in a channel tank, size of the tank is 45m × 6m × 5m. We used SES2000 standard parametric system as the transmit module of experimental system which could generate 4kHz, 5kHz, 6kHz, 8kHz, 10kHz, 12kHz and 15kHz difference frequency sound wave, and the steered angle can be set manually in a range of ±16 degree in 0.5 degree step. The hydrophone is RESON TC4038 (10~800kHz), data acquisition unit is TOPVIEW2000 whose maximum sampling frequency is 25MHz. A sound profiler HY 1200 is used to measure the sound profile in the channel tank to offer a real sound velocity for nonlinear acoustic field numerical computation. The parametric array and the measurement scene are shown in Fig.8.

Fig.8 Photograph of experimental system and measurement process

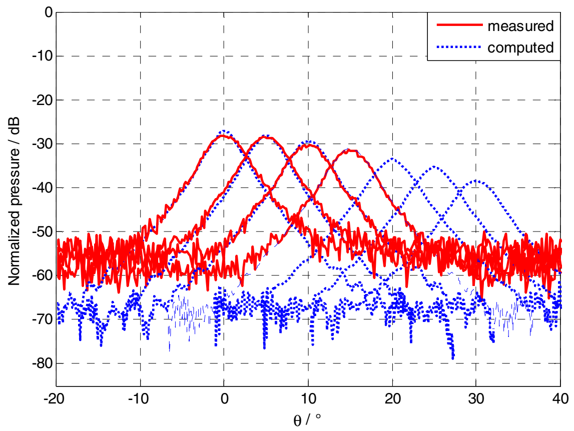
The aperture of SES2000 standard parametric array is about 0.22m×0.22m, the measured sound velocity at depth of 1.5m is 1474m/s, and the research is focused on the steered nonlinear acoustic field corresponding to 15kHz difference frequency. Simulated and measured results of directivity for the steered nonlinear acoustic field are given in Fig.9, in order to clearly illustrate the steered nonlinear field computational algorithm, the measured and computed beam patterns of primary frequency (95kHz), sum frequency (205kHz) and difference frequency (15kHz) when steered angle is 0 degree (Fig.9 (a)) and 15 degree (Fig.9 (b)) are compared, the results are in good agreement in the main lobes, but there is a certain mismatch in the side lobes which is mainly introduced by the measure errors and one order parabolic approximation, and this is inevitable. The steered difference frequency beam patterns in range of 0 degree to 15 degree in 5 degree step are also compared in detail, we can see in Fig.9 (c) that the level of computed side lobes are all lower than the measured lobes, this is because the signals in these angles are drowned in the background noises. All the comparison indicates that the theoretical model is correct and valid.



(a) 0 degree



(b) 15 degree



(c) steered difference frequency beam patterns

Fig.9 Measured and computed steered nonlinear beam directivity

The measured and computed axial pressure characteristics are also compared in Fig.10, and the two results are in good agreements, the correctness and validity of the theoretical model and computation method are verified again.

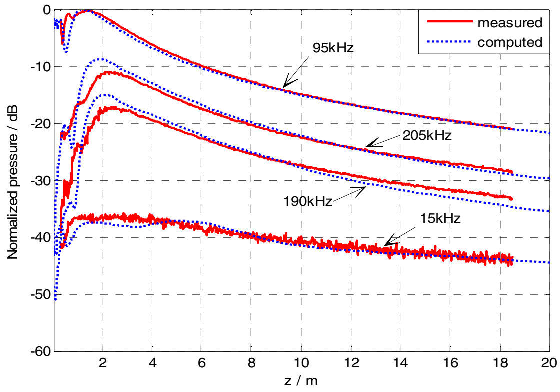


Fig.10 Measured and computed axial pressure

IV. Conclusion

Based on the collimation characteristic of primary frequency field of parametric array, an equivalent parametric array model is proposed to realize the steered nonlinear acoustic field computation using one order parabolic approximation KZK equation, and the problem of steered nonlinear field computation is no longer the problem of algorithm structure mismatch, but the source condition computation. By coordinate transformation, we realized the source condition calculation. Measurement and computation comparison verified the correctness and validity of this model and computation method.

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