Fatigue Load and Life Estimation of Composite Turbine Blades under Random Ocean Current

Takuya Suzuki
Structural Strength Department, Research Laboratory
IHI Corporation
Yokohama, Kanagawa, Japan
takuya_suzuki@ihi.co.jp

Hassan Mahfuz and Marco Canino
Department of Ocean and Mechanical Engineering
Florida Atlantic University
Boca Raton, Florida, USA
hmahfuz@fau.edu, mcanino@fau.edu

Abstract—**In the present study, the fatigue load and the fatigue life of the composite ocean current turbine (OCT) blade were estimated under random ocean current. Since the OCT blade is submerged into the water that has relatively high fluid density compared to that of air, the conventional design technique for wind turbine blades, i.e. blade element momentum (BEM) theory was modified. The modifications in terms of the section modulus and mass density of the core, buoyancy, and the added mass were applied to a set of BEM codes developed by National Renewable Energy Laboratory (NREL). On the other hand, a statistical approach was implemented to characterize randomly fluctuating ocean current. The representative model for the average velocity as well as for the power spectral density was developed through the statistical analysis, and the time history of the current magnitude was simulated. Using the simulated current history and the fatigue analysis procedure, the fatigue life of the OCT blade was estimated. The estimated current history and the fatigue life were compared with those of collected data, and a good agreement was observed.**

Keywords—Ocean current turbine; Gulf Stream; Composite blade; Fatigue life estimation; Blade element momentum theory

I. INTRODUCTION

In order to meet the growing energy demand, attempts have been made to explore the marine hydrokinetic energy. Among them, extracting energy from ocean current is relatively a new field, and it is getting more attention as a base power source owing to the steady nature of ocean current. For example, the Gulf Stream at the Atlantic Ocean [1] is one of the most abundant source of marine hydrokinetic energy. The Gulf Stream is 100 km wide in average, and it has high energy density near the coast of south Florida. It was estimated that capturing 1/1,000th of the available energy from the Gulf Stream meets 35 % of energy needs in Florida [2]. The marine hydrokinetic energy can be extracted by deploying the ocean current turbine (OCT) in the sea, where the flow of ocean current is converted into electricity by turbine's rotation. The concept of OCT has been demonstrated by Southeast National Marine Renewable Energy Center (SNMREC) at Florida Atlantic University [3] using the Gulf Stream.

However, developing the OCT has complex factors in terms of the design of its blade. One of them is the complexity of determining hydrodynamic loads on the blade. Since the OCT has many analogies to wind turbines, the conventional design technique, i.e. blade element momentum (BEM) theory [4] can be applied. However, the most significant difference is the fluid that surrounds the OCT, where the water has about 800 times higher density compared to that of air. In such situation, BEM theory needs to be modified to include the difference caused by the oceanic environment [5,6]. An example of modification is the effect of added mass. Since the fluid around the blade is also accelerated due to the motion of the blade, the resultant force should be considered. In addition, the blade is subjected to buoyancy, where canceling the gravity by buoyancy is out of the conventional BEM theory. Therefore, these modifications are necessary to accurately calculate the hydrodynamic loads. Previously, BEM theory was applied to calculate the hydrodynamic loads of the tidal turbine blade including the effect of wave, and a reasonable agreement was observed between experiments and simulations [6].

Another issue is the difficulty in determining the fatigue load due to randomly fluctuating ocean current. Since the characteristics of ocean current is highly random in nature, the statistical approaches are preferred rather than applying the deterministic techniques. Despite plenty of studies regarding the statistical nature of ocean current [7], few of them were focused on the engineering point of view such as design load of the offshore structures or marine hydrokinetic devices [8]. Therefore, the characterization of ocean current is necessary as the source of identifying the fatigue load.

In order to overcome above issues, the present study modified the BEM codes to obtain realistic hydrodynamic loads in oceanic environment, and proposed a method to determine the fatigue life based on the statistical approach. The modifications were implemented to a set of BEM codes developed by National Renewable Energy Laboratory (NREL) [9] in terms of the section modulus and mass density of the core, buoyancy, and the added mass. Moreover, the time history of the current velocity collected at the Gulf Stream near Florida was analyzed by statistical approaches, and the representative model of the ocean current was developed. The model was used to numerically simulate the velocity history, and it was used for the fatigue life estimation. The fatigue life was calculated through the rainflow counting of the fluctuating velocity, the calculation of the equivalent mean and alternate stresses, and Palmgren-Miner's rule. The calculated fatigue life

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was compared with the one from collected data, and the validity of the proposed method was investigated.

II. Design of Ocean Current Turbine Blade

Fig. 1 shows the composite OCT blade designed by SNMREC. The Wortmann FX77-series airfoils were used for each cross section of the blade with changing their chord length and twist angle so that it can extract maximum power from the Gulf Stream. The turbine has a diameter of 3 m with blade length of 1.22 m, and it was designed to rotate with 50 rpm under the rated operation. The blade consists of a composite skin, and it is filled with foam as a core material. Since it is required for the skin to have high modulus and strength, carbon/epoxy composite was used. The stacking sequence of the laminate is $[45/\text{-}45/0_2]_s$ with the total thickness of 4 mm. On the other hand, the core material should have light weight to achieve neutral buoyancy as well as to resist hydrostatic pressure. For this purpose, the epoxy based syntactic foam with SF46 micro-balloons [10] was used. The material properties of the carbon/epoxy composite and the syntactic foam are shown in Table 1 [10,11].

III. Theory and Method

A. Statistical Analysis of Ocean Current

Since the characteristic of ocean current is highly random in nature, statistical analysis is often applied to analyze the ocean current data. One of the most useful property is probability density function (PDF). The PDF shows the probability of occurrence of a random variable; for example,

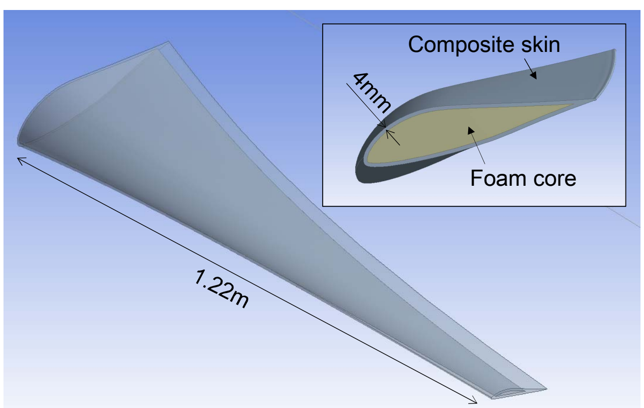


Fig. 1. Ocean current turbine blade.

TABLE I. Material properties of carbon/epoxy composite and syntactic foam.

	E_1 [GPa]	E_2 [GPa]	E_3 [GPa]	G_{12} [GPa]	G_{23} [GPa]	G_{13} [GPa]
Carbon/Epoxy	147	10.3	10.3	7.0	3.7	7.0
	ν_{12}	ν_{23}	ν_{13}			
	0.27	0.54	0.27			
Syntactic Foam	E [GPa]	G [GPa]	ν			
	2.5	0.96	0.3			

current velocity or current direction. At the Gulf Stream, it was investigated that the PDF of the current velocity can be expressed by Gaussian distribution [1]. The Gaussian distribution is described by Eq. (1) [12];

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[\frac{-(x-\overline{X})^2}{2\sigma^2}\right] \tag{1}$$

where, $f(x)$ is the probability density of variable x, σ^2 is the variance, and $\overline{X}$ is the mean of x. In the present study, we modeled the PDF of current magnitude by the Gaussian distribution. It should be noted that the direction of ocean current also have an effect on the statistical properties, however, we assumed the change in current direction is relatively small, and ignored it.

Another statistical approach is power spectral density (PSD). While PDF gives mean and variance of the variable, PSD gives a contribution of frequency to its amplitude. The signal of the variable is divided into a finite number of waves with different frequencies through the spectral analysis. After decomposing into several waves, the PSD of each wave is expressed by Eq. (2) [4];

$$\begin{aligned} S(f) &= \frac{1}{T}X(f,T)X^*(f,T) \\ where \quad X(f,T) &= \int_0^T x(t)e^{-i2\pi ft}dt \end{aligned} \tag{2}$$

where, $S(f)$ is the PSD of frequency f, $X(f,T)$ is the Fourier transform of the variable $x(t)$ over the time period T, and $X^*(f,T)$ is the complex conjugate of $X(f, T)$. For the calculation of $X(f,T)$, Fast Fourier Transform was performed by MatLab built-in function.

B. Blade Element Momentum Theory

BEM theory is the combination of momentum theory and blade element theory, which was originally developed for the design of wind turbine blade. In momentum theory, the rate of change of momentum due to a pressure difference across the rotor plane is considered, and the induced velocities in the axial and tangential directions are calculated. The expressions of the differential contribution to the thrust and torque are described by Eqs. (3) and (4) [4];

$$dT = \rho U^2 4a(1-a)\pi r dr \tag{3}$$

$$dQ = 4a'(1-a)\rho U\pi r^3\Omega dr \tag{4}$$

where, ρ is the density of the fluid, U is the far-field velocity, a is the axial induction factor, a' is the angular induction factor, Ω is the angular velocity, r is the radius of the blade element, and dr is its length. In blade element theory, the blade is divided into small blade elements, and each blade element acts independently as a two-dimensional airfoils to calculate the aerodynamic loads. The aerodynamic loads of the blade element are calculated by Eqs. (5) and (6) [4];

$$dF_N = \frac{1}{2}\rho U_{rel}^2 (C_l \cos\varphi + C_d \sin\varphi) c dr \qquad (5)$$

$$dF_T = \frac{1}{2}\rho U_{rel}^2 (C_l \sin\varphi - C_d \cos\varphi) c dr \qquad (6)$$

where, dF_N is the normal force to the rotational plane, F_T is the tangential force, U_{rel} is the velocity of relative flow into the airfoil, C_l and C_d are the lift and drag coefficients of the airfoil, φ is the sum of section pitch angle and angle of attack, and c is the chord length. Combining Eqs. (3) to (6) and iterating for the axial and angular induction factors, the aerodynamic loads are calculated at each airfoils.

The hydrodynamic loads on the OCT blade were calculated by modifying the input of conventional BEM codes; i.e. PreComp, BModes, FAST and AeroDyn developed by NREL [9]. Fig. 2 shows the flow diagram of conventional BEM procedure and following stress analysis for fatigue life estimation. The hatched box are the modifications to apply for the OCT blades. Firstly, the increase in section modulus as well as mass density due to the core was considered. Since PreComp is designed to calculate the section properties of hollow turbine blades, those properties of the core were calculated separately by commercial CAD software. 2D properties of the core such as the flap- and edge-wise, the torsion-wise stiffness, axial stiffness and the mass density were calculated, and they were added to the properties of the skin. This procedure was repeated for each hydrofoil section.

Secondly, the effect of added mass was considered in the simulation. Under the water, the fluid around the blade is accelerated simultaneously by the motion of the blade. This is not significant for wind turbine blades due to a low density of the air, therefore, the conventional BEM codes does not take into account the added mass. For the OCT blade, the added mass of the hydrofoil was considered by multiplying the added mass coefficient to the cross-section area of the hydrofoil by Eq. (7) [5];

$$m_a = \rho C_A S \qquad (7)$$

where, m_a is the added mass, ρ is the density of water, C_A is the added mass coefficient, and S is the area of the hydrofoil. According to DNV standard [13], C_A=0.5 was used since it gives reasonable estimation for the hydrofoil. At the end, the input file of BModes was overwritten by adding the core properties and the added mass.

Finally, the buoyancy of the blade was considered [5]. Since FAST only accepts a constant gravitational acceleration, the average net gravitational acceleration was given to FAST.

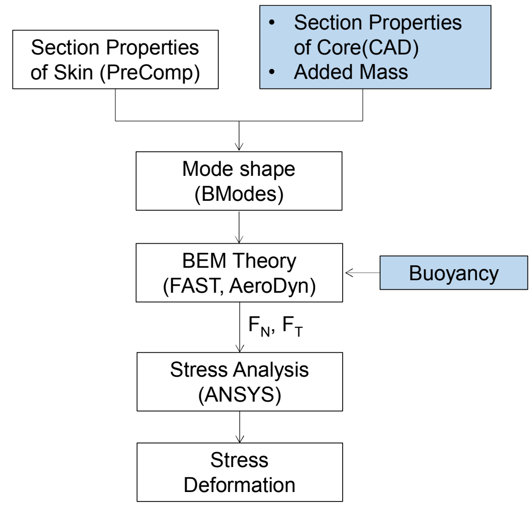


Fig. 2. Flow diagram of the modified BEM code.

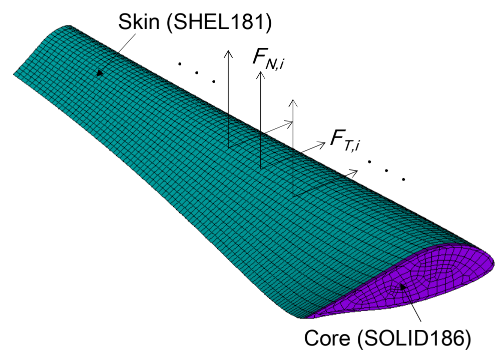


Fig. 3. Finite element model of the OCT blade.

The net gravitational acceleration at each hydrofoil was calculated using Eq. (8);

$$g_a = \frac{m_b - m_w}{m_b} g \qquad (8)$$

where, g_a is the net gravitational acceleration, m_b is the mass density of the hydrofoil, m_w is the mass density of the displaced water and g is the gravitational acceleration. The calculated net gravitational acceleration was averaged over the blade length, and it was given to FAST as a constant value. The average value was found to be negligibly small, therefore, the blade has neutral buoyancy.

C. Finite Element Analysis of OCT blade

The static analysis was performed to determine the stress range of skin and core due to fatigue load. Fig. 3 shows the finite element model of the OCT blade. The commercial software ANSYS 15.0 was used for the calculation. The skin was modeled by 4-node shell elements (SHELL181), and the core was modeled by 20-node structural solid elements (SOLID186). The number of elements in the model was 22407, and the number of nodes was 64031. The skin was modeled as orthotropic material, and the core was modeled as isotropic material. The material properties shown in Table 1 were used for the analysis. At each cross section of hydrofoils, the hydrodynamic loads were applied at the nearest node of the quarter chord as shown in Fig. 3. The hydrodynamic loads have two components: F_N which is the component normal to the rotational plane, and F_T which is the component toward the tangential direction of the rotation. On the other hand, all the degrees of freedom were constrained at the root section.

D. Fatigue Analysis

The first step of the fatigue analysis is to count the number of cycles of fatigue load. Among the several cycle counting methods, the rainflow counting [14] was used in the present study. It is known that the number of cycles counted by rainflow counting is related to that of repeated hysteresis loop of stress-strain curve. Although it was originally applied to metals, the use of it for composite materials has been observed in literature [4].

The second step is to convert the mean and alternate stresses into equivalent values at a particular stress ratio using the concept of constant life diagram (CLD). In the present study, the Goodman diagram was used as the CLD, which is expressed as Eq. (9) [4];

$$\sigma_a = \sigma_w \left(1 - \frac{\sigma_m}{\sigma_T} \right) \tag{9}$$

where, σ_m and σ_a are the mean and alternate stresses, σ_t is the tensile strength, and σ_w is the fatigue strength at R=-1.0. Along the Goodman diagram, the mean and alternate stresses can be converted into the equivalent values as shown in Fig. 4. R=0.1 is usually chosen as the equivalent value since most of fatigue tests have been performed at this stress ratio. This conversion is often applied when the experiments at different stress ratios

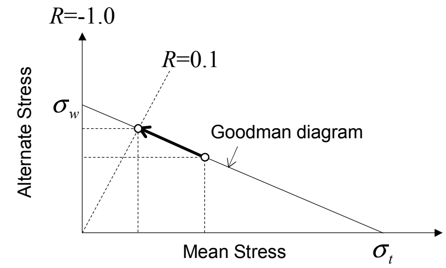


Fig. 4. Goodman diagram and the equivalent mean and alternate stresses.

are difficult due to time or cost issue. Several CLD models have been proposed so far [15], however, the Goodman diagram was used in the present study.

The last step is to calculate cumulative damage over the examined period. To estimate the fatigue life under the variable-amplitude load, Palmgren-Miner's rule [16] was used. Palmgren-Miner's rule is expressed as;

$$D = \sum_{i=1}^{k} D_i = \sum_{i=1}^{k} \frac{n_i}{N_i} \tag{10}$$

where, D is the cumulative damage, D_i is the damage at a particular stress range, n_i is the number of cycles that was applied by the stress range, and N_i is the fatigue life. The rainflow counting provides n_i from the loading spectrum, while N_i is obtained from the S-N diagram of the material. The fatigue failure is expected to occur when D reaches unity.

E. Procedure for Fatigue Life Estimation of OCT blades

The procedure to estimate the fatigue life of the OCT blade is proposed by combining the above techniques. Fig. 5 shows the schematic of the proposed procedure. Firstly, the statistical approaches were applied to collected data of ocean current. We

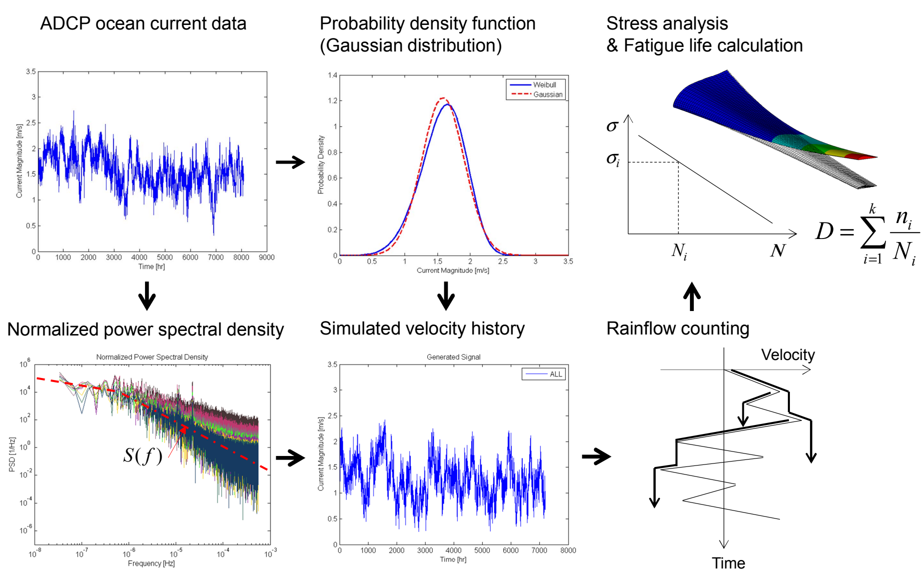


Fig. 5. The procedure for fatigue life estimation of the OCT blade.

focused on the magnitude of ocean current velocities, and calculated the PDF and PSD of them. The PDF of current magnitude was modeled by the Gaussian distribution, and the average velocity was derived. Since the average velocity decreases with increasing depth, i.e. velocity shear, the average velocity can be expressed as a function of depth as $\overline{u}(d)$. The PSD was calculated using normalized velocity so that the representative PSD model at the Gulf Stream can be proposed. In the present study, the normalized PSD was modeled by Eq. (11);

$$S(f) = Cf^{n} \tag{11}$$

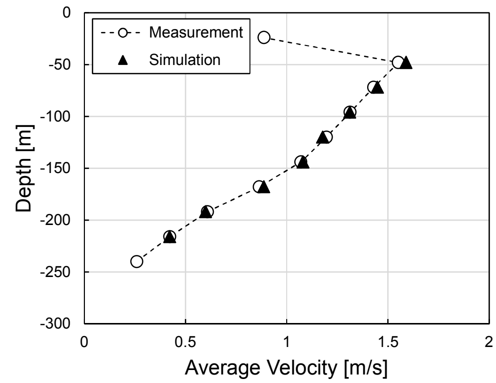
where, C and n are the constants obtained from the normalized PSD. Although this is a very simple model compared to the one used for wind turbine's design [4], this model gives us a preliminary estimation of fatigue life. Equation (11) has been investigated by Ref [17] to model the PSD of ocean current velocity. In the present study, we applied several slope 'n' with respect to the frequency region so that it can express the normalized PSD accurately.

Secondly, having determined the average velocity and the normalized PSD model, the time history of the current magnitude was simulated for the fatigue analysis. In accordance with the wind turbines' procedure, Shinozuka method [4] was used;

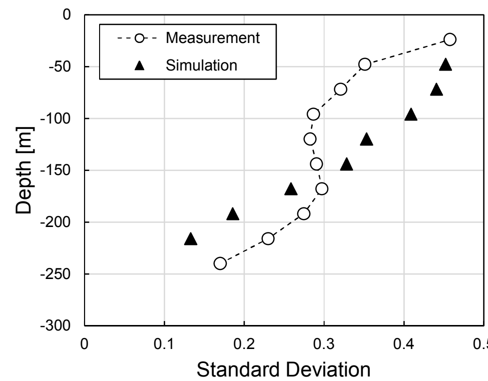
$$\begin{aligned} u &= \overline{u} + \widetilde{u} \\ \widetilde{u} &= \overline{u}\left[\sqrt{2\Delta f}\sum_{i=1}^{N}\sqrt{S(f_i)}\sin(2\pi f_i t + \phi_i)\right] \end{aligned} \tag{12}$$

where, u is the total velocity, $\overline{u}$ is the average velocity, $\widetilde{u}$ is the random component of the velocity, Δf is the frequency increment, $S(f_i)$ is the normalized PSD at frequency f_i, and ϕ_i is randomly generated phase. Since the normalized PDF has been developed, the velocity can be simulated at any depth.

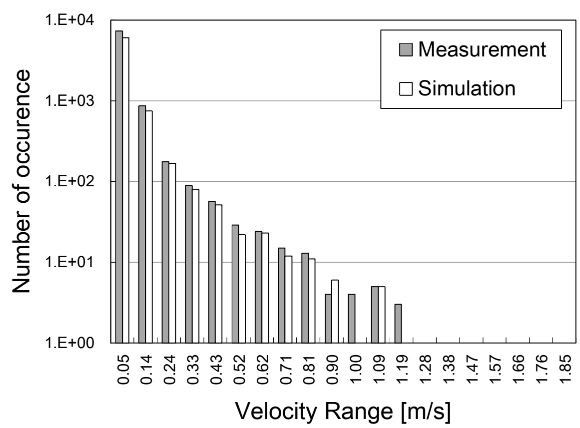
Finally, the simulated current magnitude was analyzed by ranflow counting, and corresponding mean and alternate velocities were calculated. Since the stress analysis gives us the relationship between applied velocity and resulting stress, the histogram of alternate velocity can be converted to that of alternate stress. Using the stress conversion and Palmgren-Miner's rule, the resulting damage was calculated. By summing the damage over the different alternate stress, the cumulative damage was obtained.



(a) Average velocity.



(b) Standard deviation.



(c) Velocity range counted by rainflow counting at 144m.

Fig. 6. Comparisons of the collected data and simulated results.

IV. RESULTS AND DISCUSSIONS

A. Comparison of collected and simulated velocity

The average velocity model and the normalized PSD model was calibrated using the ocean current data that was collected at the Gulf Stream near Fort Lauderdale in Florida [18] by Acoustic Doppler Current Profiler (ADCP). These data were recorded from 5/25/2013 to 4/28/2014 with a sampling period of 15 minutes. Since ADCP is able to collect the current velocity from the depth that the device was deployed to the sea surface, the velocity distribution over the depth was also available. In the present study, the current magnitude of 120 m, 180 m and 240 m at measurement site "B2" were analyzed, and the average velocity and the normalized PSD models were developed.

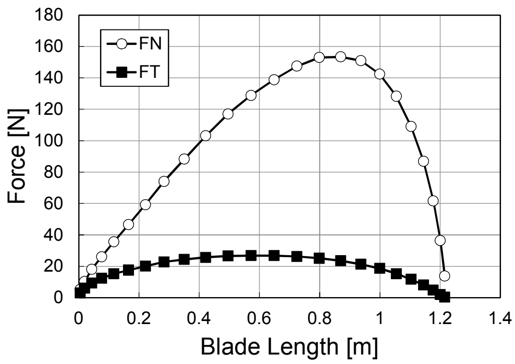


Fig. 7. Load distribution under the velocity of 1.5 m/s.

According to the procedures described in the previous section, the time history of the current magnitude was simulated. The time history was simulated using the information of measurement site "B5" so that the proposed models were validated. Fig. 6 shows comparisons of average velocity, standard deviation and velocity range counted by rainflow counting. Although the difference in standard deviation is not negligible, the proposed model gives reasonable estimation for each characteristics of ocean current.

B. Calculation of Fatigue Load using Modified BEM codes

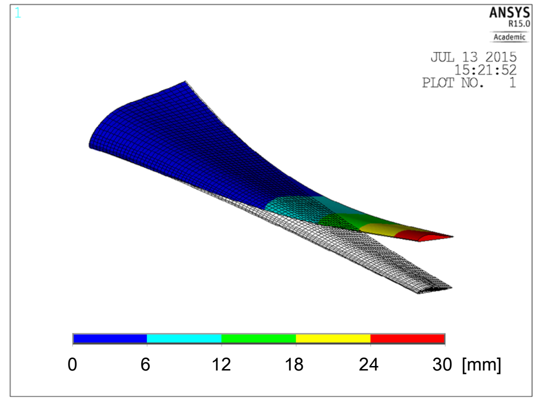
The hydrodynamic loads were calculated using the modified BEM codes. Fig. 7 shows the example of the hydrodynamic loads, where F_N is the out-of-plane component and F_T is the tangential component. These loads were calculated under the velocity of 1.5 m/s and the rotational speed of 50 rpm. The hydrodynamic loads under other velocities were calculated by the same method.

C. Stress analysis

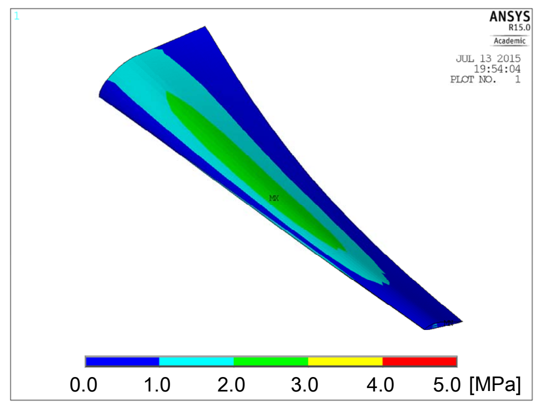
Fig. 8(a) shows the deformation of the blade under the velocity of 2.7 m/s. The deformation calculated from finite element analysis was compared with the one from BEM codes, and a good agreement was observed. Therefore, it was validated that the section moduli of the core were appropriately included in BEM codes. Fig. 8(b) shows von Mises stress distribution of the core. The stress distribution showed that the stress due to the flap-wise bending was dominant. The calculated stress value will be used for the following fatigue analysis.

D. Fatigue Life Estimation

The equivalent mean and alternate stresses were calculated along the Goodman diagram. Here, the stress ratio of R=0.1 was used since the following experimental S-N diagram was obtained at this stress ratio. In addition, the mean and alternate shear stress as well as S-N diagram of shear fatigue strength were used for the fatigue life calculation since the composite/foam sandwich panel tends to fail by transverse shear stress under bending [19,20]. To supplement a lack of experimental study, the shear strength and S-N diagram were generated by reference [20] and [21]. The fatigue life was estimated for the collected and simulated time history of the velocity magnitude at 48 m and 144 m of site "B5". Table 2 shows a comparison of them, where a reasonable agreement was observed in the fatigue life. Therefore, it was concluded that the proposed procedure can be used to estimate the fatigue life of the OCT blade at the Gulf Stream. It should be noted that the calculated fatigue life was very small since we focused on the velocity fluctuation per sampling frequency of ADCP, i.e. 15 minutes. In the actual situation, there are more frequent velocity changes during the sampling interval due to turbulence of ocean current. These factors may shorten the fatigue life of the OCT blade, therefore, they should be included into the fatigue design.



(a) Deformation.



(b) Von Mises stress distribution of the core.

Fig. 8. The results of stress analysis under the velocity of 2.7 m/s.

TABLE II. COMPARISON OF THE ESTIMATED FATIGUE LIFE AT SITE "B5".

Data	Depth	Cumulative Damage [/year]	Fatigue Life [year]
Collected	48m	0.0508	19.7
Simulated		0.0505	19.8
Collected	144m	0.0151	66.3
Simulated		0.0189	52.8

V. SUMMARY

In the present study, the fatigue load and the fatigue life of the OCT blade were estimated under random ocean current at the Gulf Stream. In order to accurately determine the hydrodynamic loads, a set of BEM codes were modified by including the effect of core properties, buoyancy, and added mass. These modifications were applied to the conventional BEM codes, and the normal and tangential load of the OCT blade were extracted. In addition, a statistical approach was applied to analyze the random behavior of ocean current fluctuation. The representative models of the average velocity and power spectral density were proposed, and they were used to simulate the time history of velocity magnitude. The time history of velocity magnitude was simulated at 144 m of site "B5", and a good agreement was observed in the average velocity, standard deviation and velocity range counted by rainflow counting. The fatigue life was also estimated at 48 m and 144 m depth by combining stress analysis and Palmgren-Miner's rule. The estimated fatigue life was close to the one calculated from collected data, therefore, the validity of the proposed method was confirmed. It should be noted that the velocity fluctuations in ADCP's sampling interval also contribute to fatigue cycles, therefore, the fatigue design of OCT blades needs to include those fluctuations.

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