Underwater Acoustic Sensor Localization Using a Broadband Sound Source in Uniform Linear Motion

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Abstract*-Acoustic sensors deployed on the sea floor can be localized using a broadband sound source travelling along a linear trajectory at a constant velocity and a constant depth below the sea surface. The absolute positions (*X*- and *Y*-coordinates) of the sensors are estimated in two steps, assuming that the *XY*-plane coincides with the (flat) sea surface. First, a local Cartesian *x-y* coordinate system is set up in such a way that, when the sensors and the moving source are projected onto the local *xy*-plane, its origin coincides with the projection of one of the sensors (called the reference sensor) and the *x*-axis is parallel to the projection of the source's linear trajectory which intersects the positive *y*-axis. The projection of the source trajectory onto the *xy*-plane is described by three motion parameters: the source speed together with the time and horizontal range at which the source is at the closest point of approach (CPA) to the reference sensor. The relative positions (*x*- and *y*-coordinates) of all other sensors, along with the three motion parameters, are estimated by measuring the temporal variation of the differential time-of-arrival (DTOA) of the signal emitted by the moving source at each pair of sensors and then minimizing the sum of the squared deviations of the noisy DTOA estimates from their predicted values over a sufficiently long period of time for all pairs of sensors. This relative position estimation assumes *a priori* knowledge of the source depth, the sensor depths and the side (either left or right) on which the source transits past the reference sensor. In the second step, the relative position estimate of each sensor is converted into an absolute position estimate by rotating the *x*- and *y*-axes by an angle equal to the source bearing at CPA, followed by a translational displacement determined by the absolute position of the reference sensor. The source bearing at CPA can be estimated if either the direction of travel of the source or the absolute position of another sensor is known. It can also be derived together with the absolute position of the reference sensor if the absolute position of the moving source is known as a function of time (e.g. from a GPS receiver attached to the source). The proposed sensor localization method is applied to real acoustic data recorded in a shallow water experiment where a small vessel travelled at a constant speed (at zero depth) past a wide-aperture linear horizontal array of eight hydrophones mounted 1 m above the sea bed. Assuming that the absolute positions of two of the sensors are known, the effectiveness of the method is verified by comparing the estimated absolute positions of the other six sensors with their nominal values.

I. INTRODUCTION

Distributed sensor networks have recently attracted much attention for both defense and civilian applications such as battlefield surveillance, passive detection and localization of underwater acoustic sources, early warning for natural disasters, seismic imaging of undersea oilfields, and environmental monitoring [1-5]. In a distributed sensor network, low-cost sensors are spatially distributed in the area of interest and it is necessary to know their locations in order to correctly interpret the sensor network data, e.g., localizing an intrusion, tracking a moving target or monitoring the physical conditions over the area. For land-based sensor networks, a global positioning system (GPS) receiver can be attached to each sensor to provide a location estimate for the sensor. Other sensor localization techniques, which employ radio-frequency (RF) or acoustic calibration sources, measure the times-of-arrival of the emitted signals at the sensors, the differential times-of-arrival (DTOA) of the emitted signals at different pairs of sensors, or the received signal strengths at the sensors [6-9]. With these techniques, fixed sources are deployed in the neighbourhood of the sensors, or alternatively, a mobile source moves around within the network of sensors. Some of the techniques do not require a *priori* knowledge of the source positions which are estimated simultaneously with the sensor positions provided there are some reference sensors whose locations are known. However, for underwater sensor networks, the high attenuation of RF waves in water renders GPS and other RF-based localization techniques impractical, so acoustic-based localization techniques need to be used.

Recently, a DTOA-based sensor localization method, which requires fixed sources at known positions, was used to locate the sensor elements of a wide-aperture hydrophone array deployed in Sydney Harbour [10]. The estimated sensor positions were then used to locate other underwater acoustic sources. The close agreement between the estimated and actual source positions verified the accuracy of the estimated sensor positions. This sensor localization method has also been evaluated using real data recorded in a land-based field experiment where six widely separated microphones and multiple acoustic transient signal sources were fixed on the ground and the sources were activated one at a time [11]. The localization accuracy of the method was quantified in terms of average radial bias error and average circular error probable.

In this paper, a new DTOA-based method is described for localization of underwater acoustic sensors. Instead of using fixed transient sources at known positions, this method uses a moving source that emits a continuous broadband random acoustic signal as it travels in a straight line at a constant speed and a constant depth (below the sea surface). Section II formulates the problem and describes a method for estimating the relative positions of sensors deployed on the sea floor. Section III describes how the relative sensor positions can be transformed into absolute positions. Three cases are considered, each corresponding to a different assumption on

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the additional source or sensor information available. In Section IV, the proposed sensor localization method is applied to real acoustic data recorded in a shallow water experiment where a small vessel travelled at a constant speed past a wide-aperture linear horizontal array of eight hydrophones located 1 m above the sea floor. Assuming that the absolute positions of two of the sensors are known, the absolute positions of the other six sensors are estimated and the results compared with the nominal values. Section V gives the conclusions.

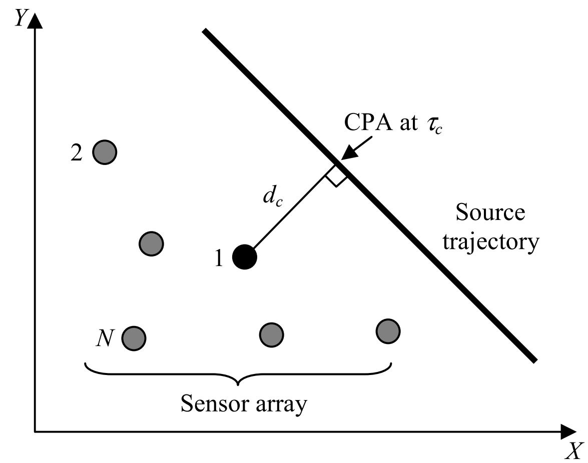

Figure 1. Plan view of an array of *N* acoustic sensors deployed on the sea floor and the trajectory of an underwater moving sound source.

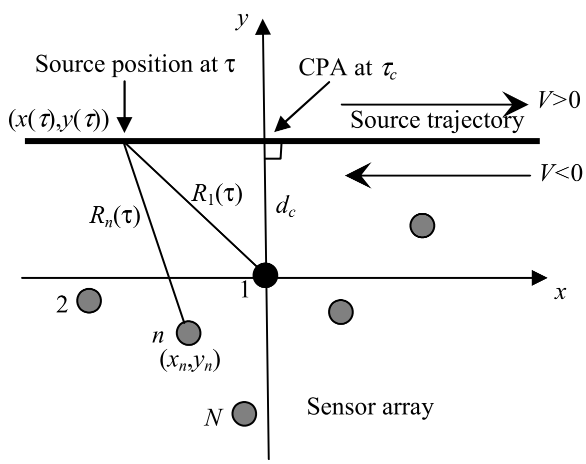

Figure 2. Projections of the sensor array and source trajectory in local *x-y* coordinate system.

II. Relative Position Estimation

Figure 1 shows the plan view of an array of *N* acoustic sensors deployed just above the sea floor and the trajectory of an underwater moving sound source. The *XY*-plane coincides with the (flat) sea surface. The absolute (or global) position of sensor *n* is given by (X_n, Y_n) and the depth of the sensor is d_n (below the sea surface). The source emits a continuous broadband random acoustic signal as it moves along its linear trajectory at a constant (subsonic) speed *V* and a constant depth d_s. All the *N* sensors are located on one side of the source trajectory. At time τ_c, the source is at the closest point of approach (CPA) to sensor 1 and its horizontal range from sensor 1 is d_c.

A local *x-y* coordinate system is set up on the *XY*-plane in such a way that, when the sensors and the moving source are projected onto the *XY*-plane (or *xy*-plane), the origin of the local *x-y* coordinate system coincides with the projection of sensor 1 and the *x*-axis is parallel to the projection of the source's linear trajectory which intersects the *positive y*-axis – see Fig. 2. This point of intersection corresponds to the CPA of the source to sensor 1 and its *y*-coordinate is equal to the CPA horizontal range d_c. As shown in Fig. 2, if the source travels past sensor 1 on the right hand side (represented by an arrow pointing to the right), its direction of travel is in the positive *x* direction and the source velocity *V* is positive; if the source travels past sensor 1 on the left hand side (represented by an arrow pointing to the left), its direction of travel is in the negative *x* direction and the source velocity *V* is negative. The relative (or local) position of the source at time *t* is given by

$$x(t) = (t - \tau_c)V \tag{1}$$

$$y(t) = d_c \tag{2}$$

where $d_c > 0$ and $-c < V < c$ with c being the speed of sound in water. (It is assumed that the water is an isospeed sound propagation medium.)

Suppose the relative position of sensor n is given by (x_n, y_n) for $1 \le n \le N$ with $x_1 = y_1 = 0$. Since the source speed is much less than the speed of sound in water, the displacement of the source during the propagation of the emitted signal from the source to the sensors (commonly referred to as the *retardation effect*) can be ignored. In other words, the source position is assumed to remain unchanged until the signal emitted by the source at that position arrives at the sensors. Under this 'start-stop' assumption, the DTOA (or time delay) of the emitted signal at sensors *m* and *n* ($1 \le m, n \le N$ and $m \ne n$) at time *t*, denoted as $\beta_{nm}(t)$, is given by

$$\beta_{nm}(t) = [R_n(t) - R_m(t)]/c \tag{3}$$

where $R_n(t)$ is the radial distance to the source from sensor *n* ($1 \le n \le N$) at time *t*, which can be expressed as

$$R_n(t) = \{[x(t) - x_n(t)]^2 + [y(t) - y_n(t)]^2 + (d_s - d_n)^2\}^{1/2} . \tag{4}$$

Substituting (1) and (2) into (4) gives

$$R_1(t) = [V^2(t - \tau_c)^2 + d_c^2 + (d_s - d_1)^2]^{1/2} \tag{5}$$

for $n = 1$, and

$$\begin{aligned} R_n(t) = [&V^2(t - \tau_c)^2 + d_c^2 + (d_s - d_n)^2 \\ &+ x_n^2 + y_n^2 - 2d_c y_n - 2V(t - \tau_c)x_n]^{1/2} \end{aligned} \tag{6}$$

for $2 \le n \le N$.

Equations (3), (5) and (6) provide an expression for $\beta_{nm}(t)$ that is an explicit function of time *t*, the source velocity *V*, the time τ_c and horizontal range d_c at which the source is at the CPA to sensor 1, and the relative coordinates of sensors *n* and *m*: (x_n, y_n) and (x_m, y_m) for $2 \le m, n \le N$ and $m \ne n$, the relative coordinates of sensor *n* for $2 \le n \le N$ and $m = 1$, or the relative coordinates of sensor *m* for $2 \le m \le N$ and $n = 1$. Define the source motion parameter vector $\mathbf{p} = [v, \tau_c, d_c]^T$ where the superscript *T* denotes vector transpose, the relative sensor position vector $\mathbf{x} = [x_2, y_2, \cdots, x_N, y_N]^T$ and the time delay vector $\mathbf{f}(t)$ which consists of the time delays $\beta_{mn}(t)$ for different pairs of sensors ($1 \le m, n \le N$ and $m \ne n$) at time *t*. The problem is to estimate $\mathbf{x}$ from a time series of time delay vector estimates $\{\hat{\mathbf{f}}(t_1), \hat{\mathbf{f}}(t_2), \ldots, \hat{\mathbf{f}}(t_M)\}$, where $\hat{\mathbf{f}}(t_m)$ denotes the estimate of $\mathbf{f}(t)$ at time t_m for $1 \le m \le M$ and *M* is the number of estimates, without a *priori* knowledge of the actual value of $\mathbf{p}$. It is assumed that the observation time for $\mathbf{f}(t)$ is sufficiently long so that it encompasses both inbound (closing in range) and outbound (opening in range) legs of the source transit. Also, the source depth d_s, the sensor depths d_n for $1 \le n \le N$, and the sign of *V* (i.e., the side on which the source transits past sensor 1) are known.

This problem is solved by jointly estimating the relative sensor position vector $\mathbf{x}$ and the source motion parameter vector $\mathbf{p}$. Define the composite parameter vector $\boldsymbol{\lambda} = [\mathbf{x}^T, \mathbf{p}^T]^T$. The (nonlinear) LS estimate of $\boldsymbol{\lambda}$ is the vector $\hat{\boldsymbol{\lambda}}$ that minimizes the following cost function:

$$P(\boldsymbol{\lambda}) = \sum_{m=1}^{M} \| \hat{\mathbf{f}}_m - \mathbf{f}_m(\boldsymbol{\lambda}) \|^2 \tag{7}$$

where $\hat{\mathbf{f}}_m \equiv \hat{\mathbf{f}}(t_m)$ is the estimated time delay vector at time t_m, $\mathbf{f}_m(\boldsymbol{\lambda}) \equiv \mathbf{f}(t_m)$ is the predicted time delay vector at time t_m, which is a function of $\boldsymbol{\lambda}$, and $\|\cdot\|$ denotes l_2 norm. The cost function $P(\boldsymbol{\lambda})$ is minimized using the Levenberg-Marquardt optimization algorithm [12]. Denote the *k*th estimate of $\boldsymbol{\lambda}$ as $\hat{\boldsymbol{\lambda}}_k$. The *k*+1th estimate is given by:

$$\begin{aligned}\hat{\boldsymbol{\lambda}}_{k+1} = \hat{\boldsymbol{\lambda}}_k + (\textstyle\sum_{m=1}^{M} \mathbf{G}_m^T \mathbf{G}_m + \mu \mathbf{I})^{-1} \\ \times \textstyle\sum_{m=1}^{M} \mathbf{G}_m^T [\hat{\mathbf{f}}_m - \mathbf{f}_m(\hat{\boldsymbol{\lambda}}_k)]\end{aligned} \tag{8}$$

where μ is a convergence factor, $\mathbf{I}$ is an $(2N+1) \times (2N+1)$ identity matrix, and $\mathbf{G}_m$ is the $(N-1) \times (2N+1)$ Jacobian matrix evaluated at $\hat{\boldsymbol{\lambda}}_k$, whose transpose is given by

$$\mathbf{G}_m^T = \nabla_{\lambda} \mathbf{f}_m^T(\hat{\boldsymbol{\lambda}}_k) \tag{9}$$

where $\nabla_{\lambda} = \left[\frac{\partial}{\partial V}, \frac{\partial}{\partial \tau_c}, \frac{\partial}{\partial d_c}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial y_2}, \cdots, \frac{\partial}{\partial x_N}, \frac{\partial}{\partial y_N} \right]^T$ is the gradient operator. The Jacobian matrix $\mathbf{G}_m$ can be derived using (3), (5) and (6). The iteration (8) is repeated until $|\hat{\boldsymbol{\lambda}}_{k+1} - \hat{\boldsymbol{\lambda}}_k|$ is less than the specified tolerance. Convergence of (8) to the LS solution $\hat{\boldsymbol{\lambda}}$ does not critically depend on the choice of the initial estimate of $\boldsymbol{\lambda}$ provided that the correct sign is used for the initial estimate of *V*. It has been shown elsewhere that using a wrong sign for the initial estimate of *V* results in estimates of the *mirror* positions (about the *y*-axis) rather than the actual positions of the sensors [13].

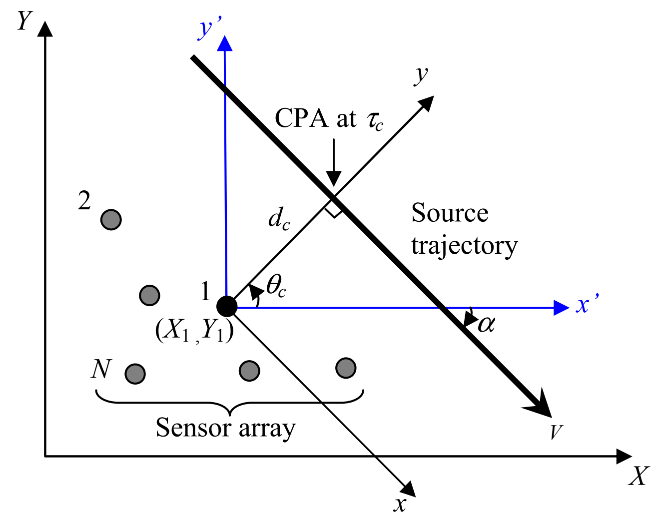

Figure 3. Transformation from relative coordinates (*x*,*y*) to absolute coordinates (*X*,*Y*). In this figure, α<0 and the source travelling in the direction of the positive *x*-axis passes sensor 1 on the right hand side so that *V*>0.

III. Coordinate Transformation

Denote the LS estimate of the relative sensor position vector as $\hat{\mathbf{x}} = [\hat{x}_2, \hat{y}_2, \cdots, \hat{x}_N, \hat{y}_N]^T$ and the LS estimate of the source motion parameter vector as $\hat{\mathbf{p}} = [\hat{V}, \hat{\tau}_c, \hat{d}_c]^T$. Given the absolute position (X_1, Y_1) of sensor 1 (the reference sensor) and the bearing θ_c of the source at CPA, the relative position estimates for sensors 2 to *N* can be transformed into absolute position estimates by a rotation of the local *x*- and *y*-axes by θ_c, followed by a translation of the local origin by (X_1, Y_1). This coordinate transformation is depicted in Fig. 3 where θ_c is the source bearing at CPA and α is the direction of travel of the source, both measured relative to the *X*-axis. The source bearing at CPA can be estimated if either the direction of travel of the source or the absolute position of another sensor is known. It can also be derived together with the absolute position of the reference sensor if the absolute position of the moving source is known as a function of time.

A. (X_1, Y_1) and α are known

Denote **u** and **v** as the unit vectors in the direction of the positive *x*-axis and in the direction of travel of the source respectively. They can be expressed in the *X*-*Y* coordinate system as $\mathbf{u} = [\sin\theta_c, -\cos\theta_c]^T$ and $\mathbf{v} = [\cos\alpha, \sin\alpha]^T$. In Fig. 3, $\alpha < 0$, $V > 0$ and $\mathbf{v} = \mathbf{u}$. But in general, **v** is related to **u** by $\mathbf{v} = \mathrm{sign}(V)\mathbf{u}$, where $\mathrm{sign}(V)$ equals 1 for $V > 0$ and -1 for $V < 0$. Equating the two expressions for **v** gives the relationship between α and θ_c:

$$\sin\theta_c = \mathrm{sign}(V)\cos\alpha \tag{10}$$

$$\cos\theta_c = -\mathrm{sign}(V)\sin\alpha \tag{11}$$

Equations (10) and (11) can be used to compute θ_c for a given sign of V and α. From Fig. 3, the absolute position estimate for sensor n ($2 \le n \le N$) is given by

$$\begin{bmatrix} \hat{X}_n \\ \hat{Y}_n \end{bmatrix} = \begin{bmatrix} \sin\theta_c & \cos\theta_c \\ -\cos\theta_c & \sin\theta_c \end{bmatrix} \begin{bmatrix} \hat{x}_n \\ \hat{y}_n \end{bmatrix} + \begin{bmatrix} X_1 \\ Y_1 \end{bmatrix}. \tag{12}$$

B. (X_1, Y_1) and (X_k, Y_k) are known

If the absolute position of another sensor, say, sensor k $(2 \le k \le N)$ rather than the direction of travel α of the source is known, then θ_c can be estimated using the method described below. The estimated value of θ_c is then substituted into (12) to compute the absolute position estimates for all other sensors ($2 \le n \le N$ and $n \ne k$).

According to (12), the absolute position (X_k, Y_k) for sensor k can be written as

$$\begin{bmatrix} X_k \\ Y_k \end{bmatrix} \cong \begin{bmatrix} \hat{X}_k \\ \hat{Y}_k \end{bmatrix} = \begin{bmatrix} \sin\theta_c & \cos\theta_c \\ -\cos\theta_c & \sin\theta_c \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{y}_k \end{bmatrix} + \begin{bmatrix} X_1 \\ Y_1 \end{bmatrix}. \tag{13}$$

Rearranging the above equation gives

$$\begin{bmatrix} \cos\theta_c \\ \sin\theta_c \end{bmatrix} \cong \begin{bmatrix} \hat{y}_k & \hat{x}_k \\ -\hat{x}_k & \hat{y}_k \end{bmatrix}^{-1} \begin{bmatrix} X_k - X_1 \\ Y_k - Y_1 \end{bmatrix}. \tag{14}$$

An estimate of θ_c can be obtained from (14) as

$$\hat{\theta}_c = \tan^{-1}\left[\frac{\hat{x}_k(X_k - X_1) + \hat{y}_k(Y_k - Y_1)}{\hat{y}_k(X_k - X_1) - \hat{x}_k(Y_k - Y_1)}\right]. \tag{15}$$

It can be shown that $\hat{\theta}_c$ is also a LS estimate of θ_c, which minimizes the squared l_2 norm of the error vector defined as the difference between the actual and estimated absolute position vectors of sensor k as given by the left hand and right hand sides of (13) respectively.

C. $(X(t), Y(t))$ is known as function of t

If both the direction of travel α of the source and the absolute position (X_1, Y_1) of the reference sensor are unknown, but instead the absolute position $(X(t), Y(t))$ of the moving source is known as a function of time, then the absolute position estimates for all sensors can be estimated by first estimating θ_c together with (X_1, Y_1) and then substituting their estimated values into (12). To estimate θ_c requires an estimate of α which can be computed by fitting a straight line to the absolute positions of the moving source. Using the estimated value for α in (10) and (11) provides an estimate of θ_c. The absolute position (X_1, Y_1) of sensor 1 is estimated using the following procedure:

1. Interpolate the available source position data at $\hat{\tau}_c$ to obtain $(X(\hat{\tau}_c), Y(\hat{\tau}_c))$ which represents an estimate of the source's absolute position at CPA.
2. Compute an estimate of the source's relative position at CPA: $(0, \hat{d}_c)$ in local *x* and *y* coordinates.
3. Transform the relative position $(0, \hat{d}_c)$ to absolute position $(\hat{X}_c, \hat{Y}_c)$ using (12) with θ_c replaced by its estimated value $\hat{\theta}_c$:

$$\begin{bmatrix} \hat{X}_c \\ \hat{Y}_c \end{bmatrix} \cong \begin{bmatrix} \sin\hat{\theta}_c & \cos\hat{\theta}_c \\ -\cos\hat{\theta}_c & \sin\hat{\theta}_c \end{bmatrix} \begin{bmatrix} 0 \\ \hat{d}_c \end{bmatrix} + \begin{bmatrix} X_1 \\ Y_1 \end{bmatrix}.$$

4. Equate $(\hat{X}_c, \hat{Y}_c)$ and $(X(\hat{\tau}_c), Y(\hat{\tau}_c))$ to obtain an estimate of the absolute position (X_1, Y_1) of sensor 1:

$$\hat{X}_1 = X(\hat{\tau}_c) - \hat{d}_c \cos\hat{\theta}_c \tag{16}$$

$$\hat{Y}_1 = Y(\hat{\tau}_c) - \hat{d}_c \sin\hat{\theta}_c \tag{17}$$

IV. Experimental Results

In a shallow water experiment (water depth ≈ 20 m), eight hydrophones were located at a nominal height of 1 m above the sea floor. The sensor configuration was (almost) linear with an inter-sensor spacing of about 14 m. A small vessel (acoustic source) travelled in a straight line at a nominal (constant) speed of 9 knots (4.63 m/s) past the sea-bottom-mounted wide-aperture linear hydrophone array on the right hand side at a horizontal distance of less than 90 m from the centre of the array. In the return trip, the vessel travelled past the array again, only this time on the left hand side, at a similar horizontal distance. The output of each hydrophone was sampled at a frequency of 250 kHz. The sensor data recorded during the two vessel transits were processed to evaluate the accuracy of the proposed sensor localization method.

To reduce the processing time, the data recorded from each hydrophone were down-sampled to a frequency of 4 kHz. The resulting data from each sensor were processed in non-overlapped blocks, each containing 2048 samples. Each data block from one sensor was cross-correlated with the corresponding data block from another sensor. The phase transform prefiltering technique was used to suppress ambiguous peaks which would otherwise have appeared in the cross-correlation function due to the strong harmonic components of the source signal. The generalized cross-correlation processing (with phase transform prefiltering) was implemented in the frequency domain using the fast Fourier transform, with a rectangular spectral window from 50 to 1500 Hz. The peak of the generalized cross-correlation function was refined using a 3-point quadratic interpolation and the position along the lag axis of the refined peak gives the time delay estimate. In this paper, the generalized cross-correlation processing was performed for six different pairs of sensors: (1,2), (3,2), (4,2), (5,6), (7,6) and (8,6). The speed of sound propagation in water was 1520 m/s.

Figure 4 shows, as circles, the estimated time delays for the six sensor pairs as functions of time for the first transit of the vessel. The LS estimate of $\boldsymbol{\lambda}$ corresponds to the vector $\hat{\boldsymbol{\lambda}} = [\hat{V}, \hat{\tau}_c, \hat{d}_c, \hat{x}_2, \hat{y}_2, \ldots, \hat{x}_8, \hat{y}_8]^T$ that provides a LS fit of the time delay model (3) to the temporal sequences of time delay estimates for the six sensor pairs. An initial estimate of $\boldsymbol{\lambda}$ was obtained as follows. The initial estimates of the x and y coordinates for each of the seven sensors (2 to 8) were simply set to zero. The initial estimates of the source speed and CPA horizontal range were set to 1 m/s and 1 km respectively. An initial estimate of the CPA time τ_c was obtained by finding the time when the received signal energy at sensor 1 attained its maximum. The solid curves in Fig. 4 represent the LS fit of the time delay model to the temporal sequences of time delay estimates for the six sensor pairs. (The nominal and estimated values of the speed and CPA horizontal range of the vessel are shown on the top of Fig. 5.)

To enable the transformation of the relative sensor position estimates into absolute position estimates, additional information to the absolute position of sensor 1 (the reference sensor) was required. By assuming that the absolute position of sensor 8 was also known, the source bearing θ_c at CPA was estimated by substituting the nominal absolute positions of these two sensors (1 and 8) and the relative position estimate for sensor 8 into (15). The estimated value of θ_c, denoted as $\hat{\theta}_c$, was then substituted into (12) to convert the relative position estimates for the other six sensors into absolute position estimates. Figure 5 shows the nominal absolute positions for all eight sensors and the estimated absolute positions for sensors 2 to 7. Also included in Fig. 5 is the linear trajectory of the vessel predicted using the four estimated motion parameters: $\hat{V}, \hat{\tau}_c, \hat{d}_c$ and $\hat{\theta}_c$ which are shown on the top of the figure. Next, the radial errors for sensors 2 to 7 were calculated and the results shown as blue-filled circles in Fig. 6. (Radial error is defined here as the separation distance between the estimated and nominal positions of a sensor.) The maximum radial error is about 0.49 m for the first vessel transit.

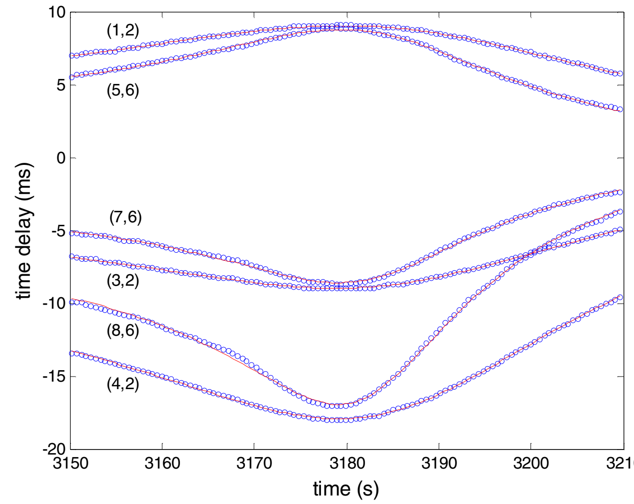

Figure 4. Time delays for the six sensor pairs as functions of time for the first vessel transit. Estimated values (blue circles). LS fit (red lines).

Parameter	Nominal	Estimated
V (kn)	9	8.7
τ_c (s)	unknown	3185.6
d_c (m)	132	136.6
$\hat{\theta}_c$ (deg)	unknown	17.3

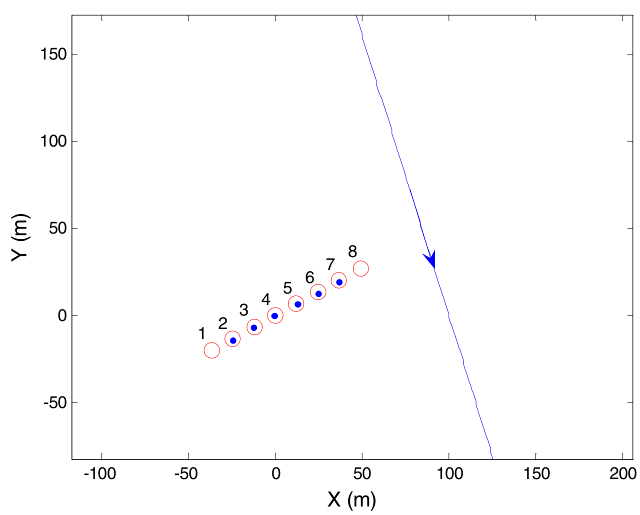

Figure 5. Estimated sensor positions (blue dots) and predicted source trajectory (blue line) for the first vessel transit. Centres of red circles denote nominal sensor positions.

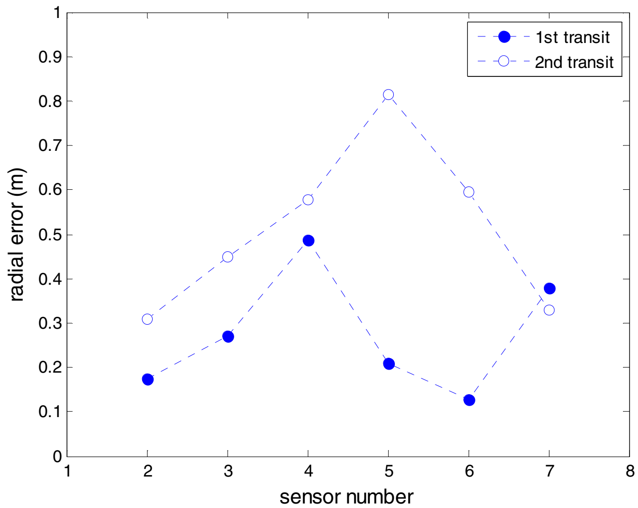

Figure 6. Radial error versus sensor number for both vessel transits.

The hydrophone data recorded during the second vessel transit were processed in the same way except that the initial estimate of the source speed was now set to -1 m/s because the vessel travelled past the array on the left hand side. Figure 7 shows the estimated time delays for all six sensor pairs as functions of time and the LS fit. Figure 8 shows the nominal absolute positions for all eight sensors and the estimated absolute positions for sensors 2 to 7, together with the linear trajectory of the vessel predicted using the four estimated motion parameters: $\hat{V}, \hat{\tau}_c, \hat{d}_c$ and $\hat{\theta}_c$ as shown on the top of the figure. The radial errors for sensors 2 to 7 were shown as unfilled circles in Fig. 6. The maximum radial error is about 0.82 m for the second vessel transit.

V. CONCLUSIONS

A time-delay based method has been described for localization of underwater acoustic sensors using a moving source that emits a continuous broadband acoustic signal as it travels in a straight line at a constant speed and a constant depth below the sea surface. The sensor localization method consists of two main steps: relative position estimation and coordinate transformation (axis rotation followed by a translational displacement). The method has been tested using real acoustic data recorded in a shallow water experiment where a small vessel transited past a sea-bottom-mounted wide-aperture linear horizontal array of eight hydrophones on two occasions. The sensor localization results are satisfactory. Assuming that the absolute positions of the (first and eighth) sensors at the two ends of the array are known, the maximum radial errors in the position estimates of the other six sensors are approximately 0.49 m for the first transit and 0.82 m for the second transit of the vessel. Additionally, the method is able to estimate both speed and CPA horizontal range of the vessel to within 4 % of their nominal values for each of the two transits.

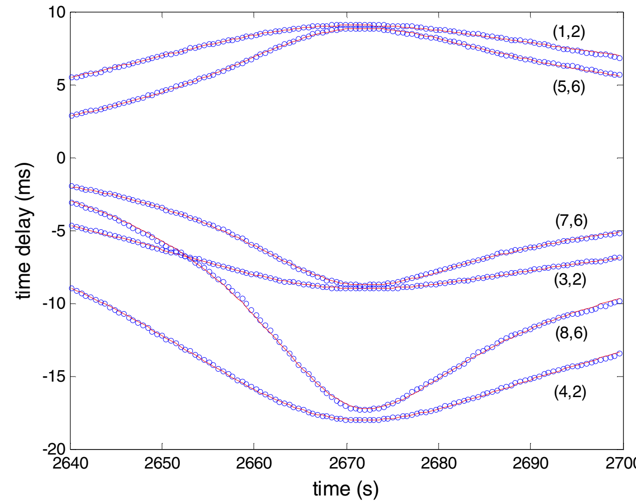

Figure 7. Time delays for the six sensor pairs as functions of time for the second vessel transit. Estimated values (blue circles). LS fit (red lines).

Parameter	Nominal	Estimated
V (kn)	-9	-8.7
τ_c (s)	unknown	2665.6
d_c (m)	138	132.6
$\hat{\theta}_c$ (deg)	unknown	16.4

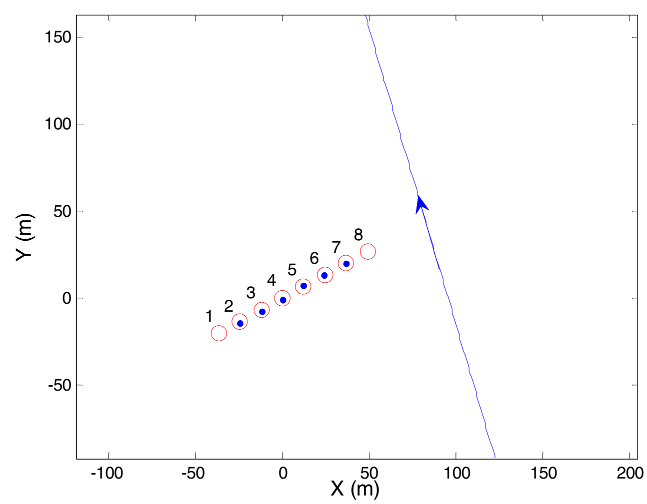

Figure 8. Estimated sensor positions (blue dots) and predicted source trajectory (blue line) for the second vessel transit. Centres of red circles denote nominal sensor positions.

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