InSAR Kalman Filter Phase Unwrapping Algorithm Based on Topographic Factors

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Abstract—**Phase unwrapping is the key step in Digital Elevation Model extraction and the measurement of surface deformation of Interferometric Synthetic Aperture Radar (InSAR). When in steep terrain or larger slope, the unwrapping result is bad and causes error transmission using the existing Kalman Filter phase unwrapping algorithm. Considering this situation, this paper presents an improved Kalman Filter phase unwrapping algorithm based on topographic factors for InSAR. It can be implemented through the introduction of the input control variable associated with topographic factors to the state-space model of Kalman Filter. Owing to the fact that the interference fringes directly reflect the change of the terrain and local fringe frequency is closely related with the local terrain slope, the local fringe frequency estimation can be used as the input control variable. In the local frequency estimation, using two-dimensional Chirp-Z transform, better estimate of the results may be quickly get. In this paper, using simulated data and real InSAR data to do the experiment, it can gain more reliable result compared with the conventional Kalman filter phase unwrapping algorithm. It is verified that the proposed algorithm can effectively deal with the situation of steep terrain and larger slope.**

Keywords-InSAR; phase unwrapping; kalman filter; topographic factors; local fringe frequency estimation; Chirp-Z transform

I. INTRODUCTION

InSAR is a great potential technology among the Earth observation techniques. It has a wide application in terrain mapping, seismic deformation, volcanic activity, glacial drift, ground subsidence, landslides, the measurement of ocean current velocity. Phase unwrapping is the crucial procedure in InSAR data processing. In the ideal case, phase unwrapping is easier to achieve, through extracting the partial derivatives of phase in vertical and horizontal direction, and then integrating along these two directions, the true phase can be restored. In fact, there are inevitably the top and bottom of the displacement caused by undulating terrain, radar shadow and errors generated in the original radar signal processing in InSAR data in spaceborne or airborne. This can arouse that phase data is not continuous, giving rise to local phase error [1]. Therefore, phase unwrapping has been a difficult and hot issue for research scholars at home and abroad in InSAR area.

Currently, the traditional phase unwrapping algorithms are broadly divided into four categories. The first is based on the path following method [2-4], typically including Goldstein's branch cut method. The second is based on the minimum norm algorithm [5-7], representatively comprising minimum Lp norm algorithm. The third is based on the optimal estimation method, such as network flow model [8], Kalman filter model [9-12]. The fourth is based on feature extraction algorithm, for example, region segmentation method [13]. Using Kalman filter model in Reference [9-10], phase unwrapping problem is transformed to state estimation issue, doing a direct effect on the real and imaginary part of complex interferogram, and to estimate true phase through the establishment of the dynamic equation and observation equation of phase, achieving unwrapping and noise eliminating simultaneously.

The conventional Kalman filter phase unwrapping algorithm [12] can get reliable and stable unwrapping result in the condition that terrain is flat. However, unwrapping result is bad and it causes errors transmission when in steep terrain or larger slope. This paper presents a Kalman filter phase unwrapping algorithm based on topographic factors. Then the feasibility and effectiveness of this improved method are tested using simulated data and real InSAR data to experiment.

II. KALMAN FILTER MODEL CONSIDERING TOPOGRAPHIC FACTORS

A. Observation Model

Consisting of inphase and quadrature components of the complex interferogram as two noisy observations of the true interferometric phase [9]. In order to writing conveniently, substituting again the n,m dependence by a 1-D k dependence, we let

$$y(k)=\begin{bmatrix}\mathrm{Re}\left\{\dfrac{z(k)}{a(k)}\right\}\\ \mathrm{Im}\left\{\dfrac{z(k)}{a(k)}\right\}\end{bmatrix}\triangleq\begin{bmatrix}\cos(\varphi(k))\\ \sin(\varphi(k))\end{bmatrix}+\begin{bmatrix}v_1(k)\\ v_2(k)\end{bmatrix} \qquad (1)$$
$$=h(x(k))+v(k)$$

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Where, $z(k)$ is complex interferogram, $a(k)$ is observed interferogram amplitude, $\varphi(k)$ is the true unambiguous phase, $h(\cdot)$ is nonlinearized mapping between $y(k)$ and state vector $\varphi(k)$ .Since there is no difference between true phase and wrapped phase in the complex domain. Thus, any filter directly operating on the complex data, rather than on the phase calculated from the complex value, will not suffer from the phase wrapping effect. The noise processes $v_1(k)$ and $v_2(k)$ are assumed to be zero-mean white Gaussian (the Gaussianess assumption is not correct in the strict sense, but practically not very crucial) noise with known covariance depending on the coherence γ, i.e.

$$\begin{aligned} E\{v(k)\} &= 0 \\ E\{v(k)v(j)^T\} &= \underset{i=1,2}{\text{diag}}\left[\frac{1}{2|\gamma|^2}-\frac{1}{2}\right]\cdot\delta(k,j) \\ &= R(k)\cdot\delta(k,j) \end{aligned} \tag{2}$$

$\delta(k,j)$ is Kronecker function, and

$$\delta(k,j)=\begin{cases}1, & if \quad k=j\\ 0, & else\end{cases}$$

B. State-Space Model based on Topographic Factors

When the interference phase is in the discrete-time case, we consider the terrain factors impact on phase unwrapping. The state space model of existing Kalman filter phase unwrapping is modified as the improved state space model in follows:

$$x(k+1) = Ax(k) + B\eta + w(k) \tag{3}$$

$$E\{w(k)\}=0 \ ; \ E\{w(k)w(j)\}=Q(k)\delta(k,j) \tag{4}$$

Where, $x(k)$ is the real phase at the point k, A is system matrix of given state-space model, B is the input control matrix, η is the input control variable related with topographic factors, $w(k)$ is state noise, $Q(k)$ is state noise covariance matrix, indicating the impact that caused by the uncertainty of state and model.

The improved state-space model is considered to introduce the input control variable with topographic factors from 3 aspects.

1) The size of spectrum offset is closely related to the slope angle of terrain where the pixel is, so the topography can be taken into account by solving the spectrum offset [14].

2) If the DEM of the measured area gained in advance by other means (such as optics or The United States SRTM space missions) is known, it can be used to regard topography [15].

3) The terrain slope is considered through linear phase model obtained from the interferometric phase [15].

The linear phase model can be got through the local frequency in range and azimuth direction, and spectrum offset and local fringe frequency of the interferogram have the relationship of 2π multiples, so the problem can be transformed into solving local frequency of interferogram in range and azimuth direction so as to take into account the local topography. And interferogram fringes really reflect ups and downs of the terrain, local fringe frequency f and local terrain slope α have the following relationship [16]:

$$f=\frac{cB_\perp}{\lambda R\tan(\theta-\alpha)} \tag{5}$$

Where, c is the propagation velocity of radar wave. $B_\perp$ is the projection that baseline is vertical to the line of sight, namely effective baseline length. λ is the wave length. R is the slant range. θ is the incidence angle.

C. Local Frequency Estimation Algorithm Associated FFT and Chirp-Z Transform

The local fringe frequency estimation of the interferogram can be achieved by spectral analysis. Although the result that got by using two-dimensional Fourier transform is the maximum likelihood [17], in order to accelerate computing speed, using two-dimensional zero-padding FFT in the implementation process. The calculation of the zero-padding FFT is excessive, so here two-dimensional FFT and Chirp-Z Transform (CZT) joint algorithm is applied to estimate the local frequency of interferogram fringe. The key of this algorithm is the selection of spectral range, the appropriate spectral range ensuring the accuracy of frequency estimation. It is embodied in a wavelet processing thought of local amplification, greatly improving spectral resolution on the basis of a limited increase in computational cost, and thereby enhanced estimation accuracy [18]. The basic idea of this algorithm is as follows:

For interferogram $z(x,y)$ in $M\times N$, the local fringe frequency f_x and f_y at the point (x,y) can be estimated by maximizing the likelihood function[17] in (6) as follow.

$$J(f_x,f_y)=\left|\sum_{x=M-(W_x-1)/2}^{M+(W_x-1)/2}\ \sum_{y=N-(W_y-1)/2}^{N+(W_y-1)/2} z(x,y)\exp(-j\cdot 2\pi(f_x x+f_y y))\right| \tag{6}$$

Where, W_x and W_y are the size of estimated window in x and y direction, respectively. The window size can be adjusted according to actual terrain. In the flatter terrain areas, the resulting interferogram fringes more sparse, it can use a larger window. But in the steeper terrain areas, generated fringes more intense, it may make use of a smaller window.

First of all, it uses two-dimensional discrete Fourier transform method to do "coarse search", that is a rough estimate of the maximum value of two-dimensional complex phase spectrum within the search window; then, as long as the local frequency estimation values of "coarse search" fall into the main lobe of two-dimensional spectrum, it can use two-dimensional Chirp-Z Transform to conduct "fine search" on this basis.

The specific approach of joint algorithm is that it is firstly determined the main lobe position of the spectrum (i.e. spectral peak) by conducting a two-dimensional FFT analysis with 32 × 32 points, then it has spectrum refinement by using Chirp-Z Transform with 128 × 128 points within the main lobe, so that the accuracy of obtained frequency estimate is not only more higher, but also the computation of joint algorithm is significantly less than directly using FFT method.

III. THE IMPLEMENTATION PROCESS OF KALMAN FILTER PHASE UNWRAPPING ALGORITHM BASED ON TOPOGRAPHIC FACTORS

The basic framework of two-dimensional Kalman filter phase unwrapping algorithm is shown in Figure 1. First, it is started from the two SAR coregistrated images, calculating the interferogram, coherence map and measured interferometric phase. Next, it need to figure out the necessary parameters for the Kalman filter. The observation noise variance used in the observation model of Kalman filter can be calculated from interferometric intensity and coherence. The local phase frequency estimation and error variance (i.e., the state noise variance) can be computed from the measured interferometric phase. Using these parameters, a very strong and effective state-space model will be established. Two-dimensional Kalman filter optimally combines the information from local frequency estimation with interferometric phase observations.

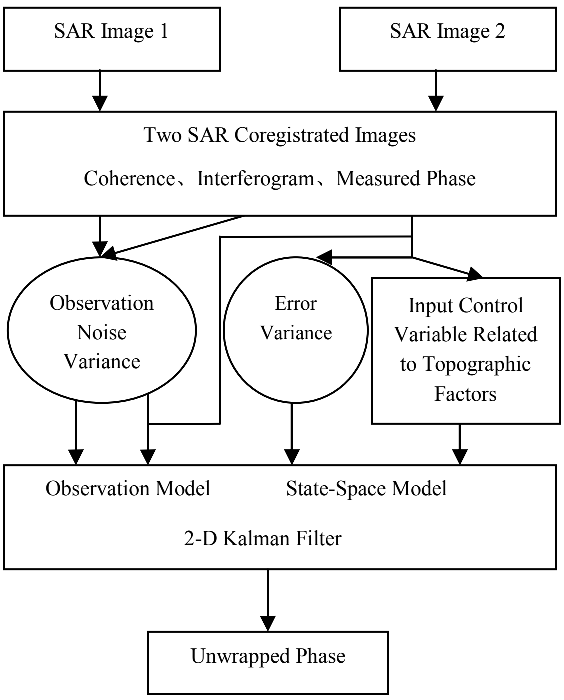

Fig. 1 The framework of Kalman filter phase unwrapping

Established for the above state-space model, if the statistical properties of the noise is known, it can follow extended two-dimensional linear Kalman filter for state estimation [19], the specific calculation process is as follows:

The first step, according to (7) and (8), it calculates predictive value of state vector $\hat{x}_{k+1,k}$ and the corresponding covariance matrix $P_{k+1,k}$:

$$\hat{x}_{k+1,k} = A\hat{x}_{k,k} + B\eta_{k,k} \tag{7}$$

$$P_{k+1,k} = AP_{k,k}A^T + Q_{k,k} \tag{8}$$

Where, $\eta_{k,k}$ is the local frequency estimation, $Q_{k,k}$ is the state noise covariance matrix. Here, the system matrix of state-space model A and input control matrix B are both taken as unit matrix. $\hat{x}_{1,1}$ and $P_{1,1}$ are the initial estimated phase and the corresponding covariance matrix respectively, selecting based on experience value.

The second step, following the predictive value $\hat{x}_{k+1,k}$ and covariance matrix $P_{k+1,k}$ from the first step, it computes state estimations $\hat{x}_{k+1,k+1}$ and the corresponding covariance matrix $P_{k+1,k+1}$ from (9) and (10):

$$\hat{x}_{k+1,k+1} = \hat{x}_{k+1,k} + J_{k+1}r_{k+1,k+1} \tag{9}$$

$$P_{k+1,k+1} = (I - J_{k+1}C_{k+1,k})P_{k+1,k} \tag{10}$$

Where, J_{k+1} is the filter gain matrix, $r_{k+1,k+1}$ is the residual, $C_{k+1,k}$ is linearized observation matrix. And

$$J_{k+1} = P_{k+1,k}C_{k+1,k}^T(C_{k+1,k}P_{k+1,k}C_{k+1,k}^T + R_{k+1,k+1})^{-1} \tag{11}$$

$$r_{k+1,k+1} = y_{k+1,k+1} - C_{k+1,k}\hat{x}_{k+1,k} \tag{12}$$

$$C_{k+1,k} = \frac{d}{dx}h(x)|_{\hat{x}_{k+1,k}} = [-\sin(\hat{x}_{k+1,k}),\cos(\hat{x}_{k+1,k})]^T \tag{13}$$

Here, $R_{k+1,k+1}$ is the observation noise covariance matrix, given by (2).

In fact, when it does phase unwrapping to interferogram, only the expected formula is to be corrected in the Kalman filter algorithm duing to any pre-estimation is calculated in the two adjacent regions. Using m as the column index and n as the row index, then it get:

$$\begin{aligned}\hat{x}_{k+1,k}(m,n) &= \frac{1}{2}\left[\hat{x}_{k,k}(m-1,n)+\hat{x}_{k,k}(m,n-1)\right]+\frac{1}{2}\left[\eta_m(m-1,n)+\eta_n(m,n-1)\right]\\ P_{k+1,k}(m,n) &= \frac{1}{4}\left[P_{k,k}(m-1,n)+P_{k,k}(m,n-1)\right]+M_{wm}Q(m-1,n)M_{wm}^T+M_{wn}Q(m,n-1)M_{wn}^T\end{aligned} \tag{14}$$

Where, $M_{wm} = [0.5 \quad 0]$, $M_{wn} = [0 \quad 0.5]$.

$\eta_m(m-1,n)$ and $\eta_n(m,n-1)$ are local frequency estimation in column and row direction respectively. $Q(m,n)$ is the state noise covariance matrix, estimated from the normalized power spectral density of the interferogram in a rectangular window centered around the pixel of interest [9].

Based on the above two steps, the final unwrapping phase $\hat{x}_{k+1,k+1}$ can be gained from the calculation of the second step by predictive value $\hat{x}_{k+1,k}$ and its variance matrix $P_{k+1,k}$.

IV. Analysis of Experimental Results

A. Method Validation Using Simulated Data

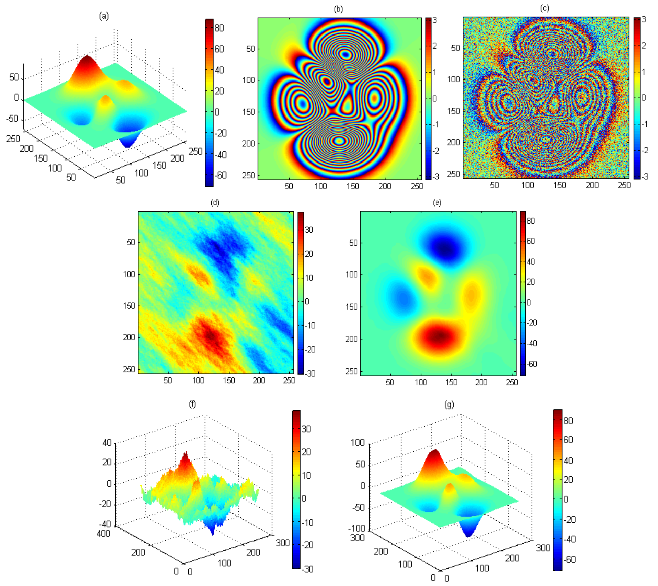

Fig. 2 Simulated phase images. (a) Simulated phase surface. (b) Wrapping phase without noise. (c) Noisy wrapped phase. (d)(f)The 2-D and 3-D map of original method's unwrapping result. (e)(g)The 2-D and 3-D map of improved method's unwrapping result.

In order to test the effectiveness of this improved method, this paper first uses the simulated data for validation. In a simulated interferometric phase (Figure 2 (a), the max. phase value is about 89 rad, the min. is about -72 rad) which the size is 256×256 and terrain is relatively steep, it is added 0.6 as coherence factor, corresponding to Gaussian phase noise where the standard deviation is 1.2178 rad in 1 look, shown in Figure 2 (c) below. Respectively, using Kalman filter method (called original method) and improved Kalman filter method to unwrap, results are displayed in Figure 2 (d) ~ (g).

Seen from Figure 2, unwrapping result of improved method (the max. phase value is about 90 rad, the min. is about -72 rad) is closer to the true phase, compared with estimated phase of original method (the max. value is about 39 rad, the min. is about -31 rad).

Figure 3 is the curve of unwrapping accuracy calculated by Kalman filter phase unwrapping method and improved method using Monte Carlo method (one for each coherence coefficient 50 times) under the different coherence coefficients. The horizontal axis is for the coherence coefficient, and the vertical axis is for the RMS phase error between unwrapped phase and true phase. Table 1 is the accuracy comparison of improved Kalman filter phase unwrapping method, original method and least-squares phase unwrapping method. From Figure 3 and Table 1 shown that, the accuracy of this improved method is not only significantly higher than the original method but also precision improves faster with the coherence coefficient increases.

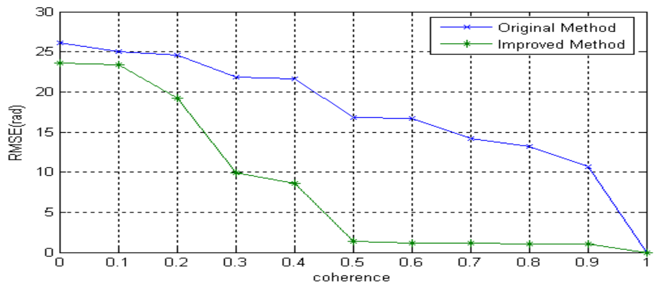

Fig. 3 The curve of unwrapping accuracy

Table1 The accuracy comparison of phase unwrapping methods

Coherence	RMSE of phase unwrapping (rad)		
	Improved method	*Original method*	*Least-squares method*
0.9	1.0523	10.6877	10.8061
0.8	1.0528	13.2436	13.9672
0.7	1.1325	14.1633	15.0702
0.6	1.1390	16.6724	16.8958
0.5	1.3786	16.7621	18.3765

B. Method Validation Using Real Data

Adopting two ERS1 satellite SAR images acquired in the town of schefferville in the Canada province of Quebec in January 1994 to do interferometric processing, the 150×150 area with terrain fluctuating and detail richer is selected from flattened interferogram (Figure 4 (a)), and coherence map is in Figure 4 (b) below. Using original method and this improved method to unwrap respectively, the results are shown in Figure 4 (c) ~ (f).

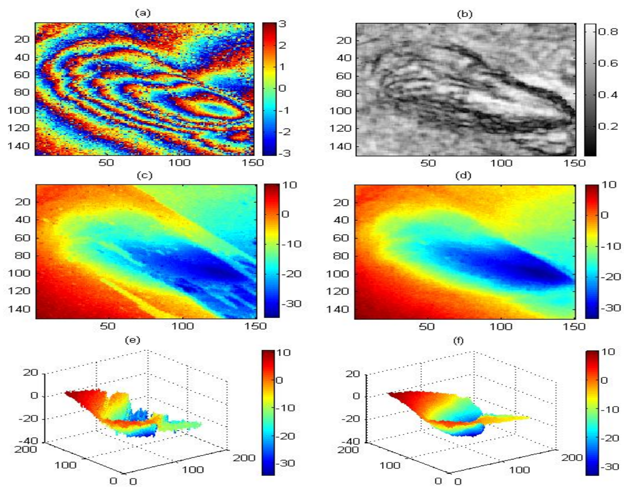

Fig. 4 Real data phase images (a) Interferometric phase image (b) Coherence image (c)(e)The 2-D and 3-D map of original method's unwrapping result (d)(f)The 2-D and 3-D map of improved method's unwrapping result

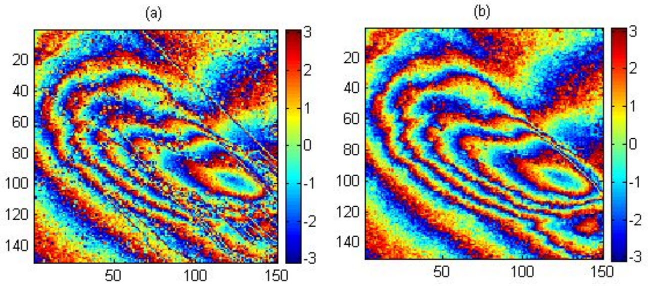

Fig. 5 Rewrapped phase images of unwrapping result. (a) Kalman Filter unwrapping method (b)Improved method

As can be seen from Figure 4, unwrapping result (Figure 4 (c) and (e)) of original method has clearly error transmission, losing part of the edge information; however, the result (Figure 4 (d) and (f)) of this improved method is smooth, almost no error propagation. From the rewrapped result of unwrapping (Figure 5), improved method is better than original method in filter effect, and the noise is almost completely removed.

The performance of improved method is evaluated from two aspects of the number of discontinuities and ε value [20]. The fewer the number of discontinuities, indicating the better the performance of anti-phase distortion; the smaller the ε value, the higher the unwrapping quality. Table 2 shows the number of discontinuities and ε value of original method and improved method. Figure 6 gives these two discontinuity maps of two methods. As can be seen, it is clear that the number of discontinuities and ε value of improved method are both less than that of original method. It is demonstrated that the anti-phase distortion performance of this method is better and the unwrapping quality is relatively higher.

Table 2 The number of discontinuities and ε value in 2 unwrapping methods

Phase unwrapping method	**The number of discontinuities**	**ε value**
Original method	818	0.88548
Improved method	52	0.81521

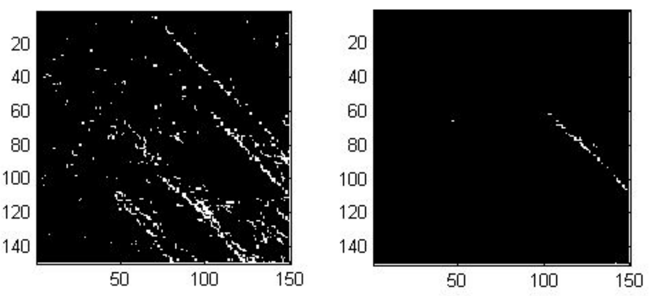

Fig. 6 The discontinuity maps (a) Kalman Filter unwrapping method (b) Improved Kalman Filter unwrapping method

V. CONCLUSION

Considering the problem that the existing Kalman filter phase unwrapping algorithm has a greater error propagation in the steep terrain or larger slope, through introducing input control variable related to topographic factors in state space model of Kalman filter, this paper presents a Kalman filter phase unwrapping algorithm based on topographic factors. And using two-dimensional Chirp-Z transform in local frequency estimation, computing speed and accuracy are improved. Unwrapping results of simulated data and real data verify that this algorithm is feasible and effective when in the steep terrain or larger slope. However, there is still model error in Kalman filter model, how to take into account model error, making the results closer to real phase, as well as these aspects that performance comparison of this algorithm with other unwrapping algorithms is quantitatively analyzed from efficiency and accuracy of unwrapping are still required further research.

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