Frequency-Domain Channel Estimation for Nonlinear Multicarrier Underwater Communication Systems

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***Abstract*—Underwater acoustic (UWA) communications have drawn attention to many researchers in recent years. The major obstacles to reliable UWA communications include limited bandwidth, long multipath delay and large Doppler shifts of the UWA channel. Due to the hostile UWA environment, implementation of a high-speed UWA communication system often requires advanced modulation and signal processing techniques. The multicarrier modulation in the form of orthogonal frequency-division multiplexing (OFDM) has many attractive features which make it a potential candidate for UWA communications. However, the OFDM signal has a high peak-to-average-power ratio (PAPR) and hence is highly susceptible to nonlinearities in the communication link. For combating channel nonlinearities, knowledge of the nonlinear channel is essential. In this paper, we propose a novel method for the estimation of nonlinear channels in OFDM UWA communications. Compared to conventional methods, the proposed method requires a significantly smaller amount of data to achieve the channel estimation. This makes the proposed method suitable for time-varying UWA channels. The effectiveness of the proposed method is demonstrated by applying it to estimate the nonlinear channel of a simulated OFDM UWA communication system.**

I. Introduction

Reliable and high-speed UWA communication is challenging due to the limited bandwidth, severe multipath fading, large Doppler shifts, and extended delay spread of the UWA channel [1]–[5]. Many early UWA communication systems adopted incoherent modulation techniques due to their simplicity and reliability [6]–[8]. In the past two decades, tremendous efforts have been made to the employment of coherent modulation techniques in UWA communications [9]–[11]. Recently, the potential application of the multicarrier modulation [12] to UWA communications has attracted much attention [13]–[17]. The multicarrier modulation in the form of OFDM has the advantages of robustness against intersymbol interference (ISI) and multipath fading, high spectral efficiency, low sensitivity to timing synchronization errors, and simple frequency-domain equalization [12]. These attractive features make it an potential candidate for implementing UWA communication systems. On the other hand, the OFDM requires precise frequency synchronization, which suggests that it is vulnerable under Doppler shifts. Moreover, a perhaps more important implementation challenge that faces the OFDM is its high PAPR. The high PAPR signal may easily saturate the power amplifier and generate intermodulation distortion in the transmitted signal. The undesirable effect is often avoided by backing off the operation point of the power amplifier to a linear region. This strategy, however, causes the OFDM system to suffer from poor power efficiency [18]. To improve the power efficiency, one common approach is to reduce the PAPR by peak clipping. This technique, however, may distort the transmitted signal and yield spectral regrowth and in-band distortion [18].

For terrestrial and satellite communications, many nonlinear predistortion and equalization techniques have been proposed to combat channel nonlinearities [19], [20]. In the development of these methods, knowledge of the nonlinear channel may facilitate the design of the compensators. This makes the estimation of the nonlinear channel an essential task. The bandpass Volterra series was extensively used to model the nonlinear characteristics of communication channels [21]. In modeling nonlinear channels using the bandpass Volterra series, the channel model can be simplified to a baseband equivalent representation in which the complex envelopes of the channel input and output are related by a baseband equivalent Volterra series [19], [21]. Once the Volterra kernels are obtained, the bandwidth regrowth and in-band distortion caused by the channel nonlinearities can be easily calculated. Due to the large number of the required kernel coefficients for even a relatively low-order Volterra model, the estimation of the Volterra kernel coefficients can be a challenging task.

Early works on identifying Volterra kernels of nonlinear systems were done by assuming the input signal to the nonlinear system was Gaussian [22]–[24]. Although the involved mathematics was greatly simplified by making this assumption, the possible departure from Gaussianity of the input signal can lead to an unsatisfactory estimation result. More general methods based on solving a minimum mean-square error (MMSE) problem were developed later on [25]–[27]. These methods can use a broader class of random signals as the input, however, it requires to estimate various higher-

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order auto-moments (or auto-moment spectra) [28], [29] of the input and calculate an inverse matrix. This suggests that significant computational complexity could be involved for implementing these methods, especially when the number of Volterra kernel coefficients is large.

For the estimation of nonlinear channels in OFDM systems, methods based on the statistical properties of the OFDM signal can be found in the literature [30]–[32]. Specifically, in [30], [31], estimation algorithms were derived by assuming that the OFDM signal was asymptotically i.i.d. complex Gaussian. By using properties of higher-order cumulants of the complex Gaussian signal, simple algorithms for determining the time-domain Volterra kernels were obtained. In [32], properties of the higher-order auto-moment spectra of the OFDM signal were used to derived an algorithm for determining the frequency-domain Volterra kernels. In these methods, the cumulants (or auto-moment spectra) need to be estimated by taking time averages over a realization of the input and output data. Therefore, the estimated higher-order cumulants (or auto-moment spectra) of the OFDM signal can only approach their theoretical values when the amount of data is sufficiently large. Therefore, these methods often require a large amount of data to obtain a satisfactory estimation result, especially when the number of involved Volterra kernel coefficients is large.

For UWA communications, to the best of the authors' knowledge, little work has been done on the estimation of nonlinear UWA channels. Note that the methods in [30]–[32] are not suitable for UWA channels, because these methods assume the channel is time-invariant so that a sufficiently large amount of data can be collected while the channel remains unchanged. This assumption, however, may not be justified in UWA communications because UWA channels are in general time-varying.

In this paper, we propose a frequency-domain channel estimation scheme for nonlinear OFDM UWA communication systems. The nonlinear UWA communication channel model under consideration is composed of a transmitting filter, a nonlinear power amplifier, a physical time-varying UWA channel, and a receiving filter. The Volterra series representation of this channel model is developed and a novel method for estimating the frequency-domain Volterra kernels is derived. Compared to conventional methods, the proposed method requires a significantly less amount of data to accomplish the estimation of the nonlinear channel. This suggests that the proposed method may be suitable for the time-varying channels in UWA communications. Computer simulations are also conducted in this paper to justify the goodness of the proposed method.

II. Baseband Equivalent Nonlinear UWA Communication Channel

A passband communication system can be described by its baseband equivalent model. The block diagram of a baseband equivalent nonlinear UWA communication channel is shown in Fig. 1. The channel input and output are denoted by $x(n)$ and $y(n)$, respectively. For an OFDM modulation scheme, the data to be transmitted are distributed over a group of equally-spaced

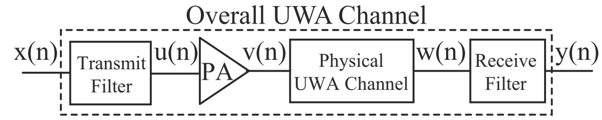

Fig. 1. The block diagram of a nonlinear UWA communication channel.

orthogonal subcarriers. Each subcarrier is modulated with a common modulation scheme like QAM or PSK and can be viewed as a subchannel. The summation of all the subchannel signals forms the baseband OFDM signal. Suppose there are a total of N subcarriers with frequencies $l/T, l = 0, \ldots, N-1$, then the baseband OFDM signal can be expressed as:

$$x(n) = \frac{1}{N}\sum_{m=0}^{N-1} X(m)e^{j2\pi nm/N}, \; n = 0, 1, \ldots, N-1 \quad (1)$$

where $\{X(m)\}$ are the complex data symbols taken from the QAM or PSK symbol alphabets. Note that the above OFDM signal has a waveform duration of $T = NT_s$, where T_s is the sampling period. It is straightforward to see that $x(n)$ is the inverse discrete Fourier transform (IDFT) of $X(m)$.

Let's refer to the data set $\{x(n)|n = 0, \ldots, N-1\}$ as an OFDM symbol frame. The input data to the UWA communication channel are arranged in blocks with each block containing multiple OFDM symbol frames. The training symbols are deployed in the first several frames of each block. The transmit and receive filters are assumed to be time-invariant. The power amplifier (PA) is assumed to be memoryless and can be approximated by a polynomial as follows [19]:

$$v(n) = \sum_{k=0}^{K} a_{2k+1}[u(n)]^{k+1}[u^*(n)]^k \quad (2)$$

where $u(n)$ and $v(n)$ are the baseband equivalent input and output of the PA, respectively. Note that $u(n)$, $v(n)$, and $\{a_{2k+1}\}$ are in general complex-valued. The physical UWA channel is a time-varying channel with Rayleigh-fading tap weights and randomly distributed tap delays. Assuming the block duration of the input data is smaller than the channel coherence time, then the frequency response of the physical UWA channel can be considered as time-invariant within a block duration.

III. Estimation of the Volterra Kernels of the Nonlinear Channel

We assume that the frequency responses of the transmitting and receiving filters in Fig. 1 are known and are denoted by $H_T(m)$ and $H_R(m)$, respectively. By referring to Fig. 1, one can write

$$U(m) = H_T(m)X(m) \quad (3)$$

$$Y(m) = H_R(m)W(m) \quad (4)$$

where $U(m)$, $W(m)$, and $Y(m)$ are the DFTs of $u(n)$, $w(n)$, and $y(n)$, respectively.

By taking the discrete Fourier transform (DFT) of (2), one can obtain the following frequency-domain representation:

$$V(m) = \sum_{k=0}^{K} a_{2k+1} \sum_{m_1=-M}^{M} \cdots \sum_{m_{2k+1}=-M}^{M} \prod_{i=1}^{k+1} U(m_i) \prod_{j=k+2}^{2k+1} U^*(-m_j)\delta(m - m_1 - \cdots - m_{2k+1}) \quad (5)$$

where $V(m)$ is the DFT of $v(n)$, $[-M, M]$ is the range of frequency m in which $U(m) \neq 0$, and $\delta(m)$ is the delta signal. Let's denote the frequency response of the physical UWA channel for the block under consideration by $H_c(m)$. By referring to Fig. 1, one can write

$$W(m) = H_c(m)V(m) \quad (6)$$

Note that the relation between $X(m)$ and $Y(m)$ can be modeled by a frequency-domain Volterra series $Y(m) = F[X(m)]$, where $F[\cdot]$ denotes the Volterra series operator. The Volterra kernels of $F[\cdot]$ can be expressed in terms of $H_T(m)$, $H_R(m)$, $H_c(m)$ and $\{a_{2k+1}\}$. Specifically, by substituting (6), (5), and (3) into (4), one can obtain

$$Y(m) = \sum_{k=0}^{K} a_{2k+1} H_c(m) H_R(m) \sum_{m_1=-M}^{M} \cdots \sum_{m_{2k+1}=-M}^{M} \prod_{i=1}^{k+1} H_T(m_i)X(m_i) \prod_{j=k+2}^{2k+1} H_T(-m_j)X^*(-m_j) \delta(m - m_1 - \cdots - m_{2k+1}) + \varepsilon(m) \quad (7)$$

where $\varepsilon(m)$ is the modeling error caused by channel noise and model imperfection. Note that, due to the spectral broadening effects of the PA, the possible range of m is $[-(2K+1)M, (2K+1)M]$. This Volterra series representation can be expressed as a vector form as follows:

$$Y(m) = \mathbf{H}^T(m)\mathbf{X}(m) + \varepsilon(m) \quad (8)$$

$$\mathbf{H}(m) = [H_0(m)\ H_1(m)\ \cdots\ H_K(m)]^T \quad (9)$$

$$\mathbf{X}(m) = [X_0(m)\ X_1(m)\ \cdots\ X_K(m)]^T \quad (10)$$

where the superscript T denotes the transpose operator and

$$H_k(m) = a_{2k+1} H_c(m) H_R(m),\ 0 \le k \le K \quad (11)$$

$$X_k(m) = \sum_{m_1=-M}^{M} \cdots \sum_{m_{2k+1}=-M}^{M} \prod_{i=1}^{k+1} H_T(m_i)X(m_i) \prod_{j=k+2}^{2k+1} H_T(-m_j)X^*(-m_j) \delta(m - m_1 - \cdots - m_{2k+1}),\ 0 \le k \le K \quad (12)$$

Since the frequency response of the transmitting filter $H_T(m)$ is known, the $X_k(m)$ and hence $\mathbf{X}(m)$ can be calculated for each OFDM symbol frame. The vector $\mathbf{H}(m)$ includes the unknown parameters that are required to describe the Volterra series representation given in (7).

By minimizing the mean square error (MSE) $E[|\varepsilon(m)|^2]$ between the model and actual outputs of the overall UWA channel, one can obtain the optimal MMSE estimate of $\mathbf{H}(m)$, say $\hat{\mathbf{H}}(m)$, as follows:

$$\hat{\mathbf{H}}(m) = E[\mathbf{X}^*(m)\mathbf{X}^T(m)]^{-1} E[\mathbf{X}^*(m)Y(m)] \quad (13)$$

where $m \in [-(2K+1)M, (2K+1)M]$. In practice, the expectation operator in (13) is implemented by taking the average over all available sample functions.

IV. Computer Simulations

To justify the goodness of the proposed channel estimation method for nonlinear OFDM UWA communication systems, we conducted the following computer simulations. The input to the simulated UWA communication system was an OFDM signal with 5 subcarriers. Each subcarrier employed a 16 QAM. In each block of OFDM signal, 2 or 3 frames were used for deploying training symbols. The channel model developed in [33] for high-frequency warm shallow water UWA channels was used for all simulations. This time-varying channel model was a tap-delay-line model with Rayleigh-fading tap weights and Gaussian distributed tap delays. Its input and output are related by [33]:

$$w(t) = A_0(t)v(t) + \sum_{l=1}^{L-1} A_l(t)v(t - \tau_l + J_l(t)) \quad (14)$$

where $A_l(t)$ is a Rayleigh random process, $J_l(t)$ is a Gaussian random process, and τ_l is a delay. In addition, a white Gaussian noise was added to the channel output.

As we mentioned in Section II, we assumed that the block duration of the input data was smaller than the channel coherence time, so that the frequency response of the physical UWA channel could be considered as time-invariant within a block duration.

The PA model used in the simulations was the traveling wave tube (TWT) model developed in [34]. In this model, the AM/AM distortion $A(r)$ and AM/PM distortion $\Phi(r)$ of the PA are given by

$$A(r) = \frac{\alpha_a r}{1 + \beta_a r^2} \quad (15)$$

$$\Phi(r) = \frac{\alpha_p r^2}{1 + \beta_p r^2} \quad (16)$$

for some constants α_a, β_a, α_p, and β_p. The resulting $A(r)$ and $\Phi(r)$ are shown in Fig. 2.

In the first set of simulations, the PA was simulated using (2) with $K = 1$ (i.e., a third-order model). The parameters a_1 and a_3 in (2) were obtained by curve-fitting the $A(r)$ in Fig. 2 to (2). The Volterra model used in (7) was a 3rd-order Volterra model. In this case, both the channel and the model could be expressed as a third-order Volterra series. Therefore, we could compare the actual and the estimated Volterra kernels to verify the correctness of the channel estimation result. Each simulation was conducted using 100 blocks of OFDM signals, and the signal-to-noise ratio (SNR) was either 20, 30, or 40 dB. The normalized MSEs (NMSEs) of the estimated Volterra kernels (denoted by NMSE-H) achieved by using 2 training

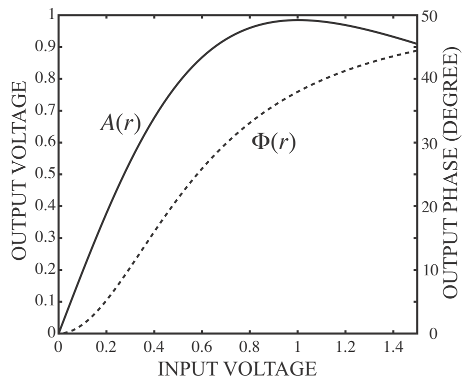

Fig. 2. The AM/AM ($A(r)$) and AM/PM ($\Phi(r)$) characteristics of a typical power amplifier. The $A(r)$ and $\Phi(r)$ denote the amplitude gain and the phase shift of the PA, respectively.

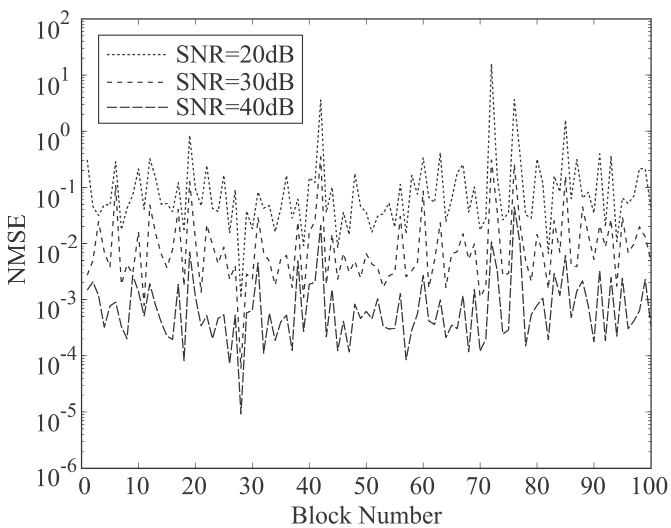

Fig. 3. The NMSEs of the Volterra kernels (NMSE-Hs) for the 100 blocks of OFDM signals using 2 training symbol frames.

symbol frames in each block are shown in Fig. 3. The NMSE-H is defined as

$$\text{NMSE-H} \quad = \quad \frac{||\mathbf{H} - \hat{\mathbf{H}}||^2}{||\mathbf{H}||^2} \tag{17}$$

where $\mathbf{H}$ and $\hat{\mathbf{H}}$ denote the actual and estimated Volterra kernel vectors, respectively, and $||\cdot||$ denotes the Euclidean norm. From Fig. 3 we see that, the NMSEs are small for most cases except for several blocks under SNR=20 dB. This is due to the fact that, the lower the SNR, the larger amount of data would be required by the estimation algorithm to average out the noise effects. This situation can be improved by increasing the number of training symbol frames in each block. To verify this, simulations using 3 training symbol frames in each block were also conducted and the results are shown in Fig. 4. The small NMSEs for all SNR settings in Fig. 4 indicate that excellent estimation results were obtained.

In the second set of simulations, the PA was simulated using the TWT PA model described by (15) and (16), and the Volterra model used in (7) was still a 3rd-order Volterra model. In this case, the actual Volterra kernels of the channel were unknown, hence the goodness of the Volterra kernel estimates was measured using the NMSE of the model output (NMSE-O). The NMSE-O is defined as

$$\text{NMSE-O} \quad = \quad \frac{\sum_m |Y(m) - \hat{Y}(m)|^2}{\sum_m |Y(m)|^2} \tag{18}$$

where $\hat{Y}(m)$ denotes the model output at frequency m, and the summations are taking over all possible values of the output frequency m. The NMSEs of the model output achieved in this case under SNR=20 dB are shown in Fig. 5 (denoted by TWT), where a low NMSE is seen in each block. This suggests that a reasonably good channel estimation result was achieved. Furthermore, for comparison, the NMSEs of the model output achieved by the first set of simulations with 2 training symbol frames per block under the SNRs of 20, 30, and 40 dB are also shown in Fig. 5 (denoted by VOL3). The discrepancy between the TWT (20 dB) and VOL3 (20 dB) results is owing to the fact that the TWT model is higher than the 3rd order. This implies that the performance of the TWT case can be improved by using a higher-order Volterra model in (7). However, a higher-order Volterra model also means a larger number of involved Volterra kernels, which can in turn increase the required computational complexity for obtaining the Volterra kernel estimates.

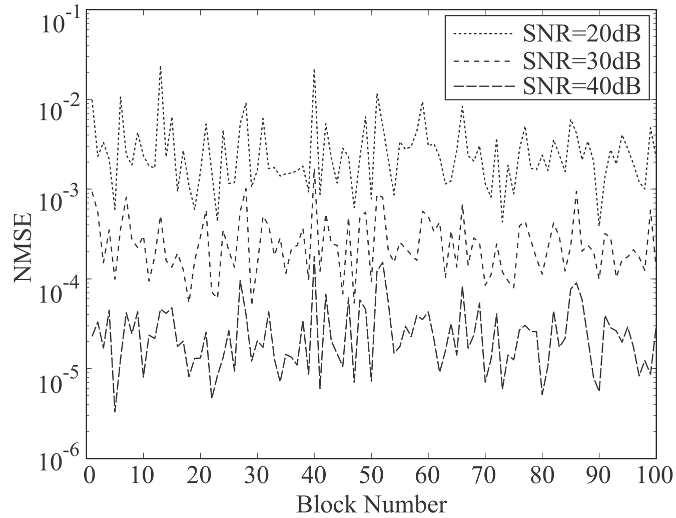

Fig. 4. The NMSEs of the Volterra kernels (NMSE-Hs) for the 100 blocks of OFDM signals using 3 training symbol frames.

V. CONCLUSIONS

In this paper, we have proposed a frequency-domain channel estimation scheme for nonlinear OFDM UWA communication systems. We considered a nonlinear UWA communication channel which was composed of a transmitting filter, a non-linear power amplifier, a physical time-varying UWA channel, and a receiving filter. We first developed a frequency-domain Volterra series representation for this channel and then solved

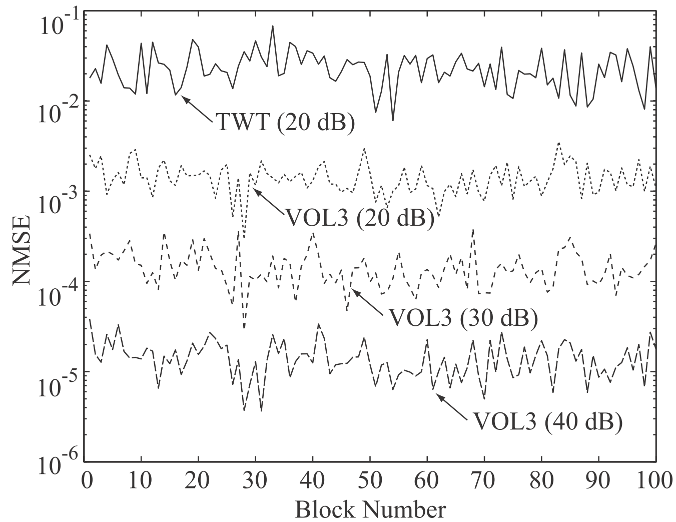

Fig. 5. The model output NMSEs (NMSE-Os) for the 100 blocks of OFDM signals under various simulation settings.

the Volterra kernel estimation problem using the standard MMSE technique. Compared to conventional Volterra kernel estimation methods, the proposed method has the advantage of requiring a significantly less amount of data to accomplish the estimation of the nonlinear channel. We have demonstrated via computer simulations that, when applied to a simulated nonlinear UWA OFDM communication system for high-frequency warm shallow water, the proposed method can achieve good estimation results. This suggests that the proposed method may be suitable for the time-varying channels in UWA OFDM communications.

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