Optimal Spatial Filtering of Real Data from Submarine Sonar Arrays

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***Abstract*- Submarine hydrophone arrays sample the underwater acoustic pressure field in space and time to sense the presence of sources of radiated sound and to extract tactical information from the received sounds. The outputs of the hydrophones are combined by a spatial filter (or beamformer) so that signals from a chosen direction are coherently added while the effects of noise and interference from other directions are reduced by destructive interference. The spatial filter appropriately weights the hydrophone outputs prior to summation so as to enhance the output signal-to-noise ratio, thereby improving the detection, classification, localization and tracking performance of the passive sonar system onboard the submarine. By processing real acoustic data from a submarine hull-mounted array, it is shown that an optimal (adaptive) frequency-domain spatial filter (based on inversion of the observed cross spectral matrix for each frequency bin) enhances the detection of underwater acoustic signals, when compared with conventional spatial filtering (or delay-and-sum beamforming). The degree of improvement, however, depends on the method used to normalize the cross spectral matrix. Superior detection performance for a submarine hull-mounted array occurs when the observed cross spectral matrix for each frequency of interest is normalized using the average of all the single hydrophone output powers at each frequency. It is also shown that the self-noise analysis of a submarine towed array is facilitated by processing the element-level data in the frequency-wave number domain with a constrained optimal spatial filter, which is also referred to as a Minimum Variance Distortionless Response (MVDR) beamformer. This spatiotemporal filtering method is employed to separate signals (surface ship contacts) and tow vessel noise components (direct path and multipath arrivals) according to their directions of propagation. This method is also found to separate spatially-correlated self-noise components that propagate within the towed array structure at a speed that is slower than the speed of sound travel in water. A comparison of the frequency-wave number power spectra estimated using conventional and optimal spatial filtering methods shows that the MVDR spatial filter enables the various sources of acoustic energy that are sensed by the array to be more clearly delineated in frequency-wave number space. The MVDR spatial filter is a data-adaptive spatial filter that is observed to suppress spatial leakage, to enhance the spatial resolution of a towed array through narrower beamwidths, and to provide superdirective array gain at frequencies well below the design frequency of the towed array. Frequency-wave number analysis with optimal spatial filtering is a powerful diagnostic tool for studying the self-noise characteristics of a towed linear array of hydrophones.**

I. INTRODUCTION

The subject of array processing is concerned with the extraction of information collected using an array of sensors [1-4]. Arrays of hydrophones are used in the underwater environment to detect weak acoustic signals, to resolve closely-spaced sources, to estimate the bearing and other properties of signal sources and to identify the spatially-correlated components of the self-noise [5-7]. The spatial filtering of signals and background noise collected with a hydrophone array is also referred to as *beamforming*. Spatial filtering methods include *conventional (delay-and-sum) beamforming* and *adaptive beamforming*. With conventional beamforming in the time domain, the hydrophone outputs are first delayed so as to compensate for the difference in the arrival times of the signal wavefront at the spatially-distributed hydrophones and then the hydrophone outputs are summed together so as to reinforce the signal received from the source. With conventional beamforming in the frequency domain, which is also referred to as *filter-and-sum beamforming* [3], the time domain hydrophone outputs are first transformed to the frequency domain using the fast Fourier transform method. Next, the hydrophone outputs for each narrowband frequency bin are combined to form a beam (after inserting the appropriate phase delay in each narrowband hydrophone output).

Hence the formation of the beams can be implemented in either the time domain or the frequency domain. The time delays that are required for the coherent addition of signals from a given direction in time domain beamforming are replaced by phase delays in frequency domain beamforming. Shading coefficients (real numbers) are sometimes applied to control the sidelobe levels associated with spatial leakage (sidelobes) in a conventional beamformer, otherwise the coefficients are set to unity. With adaptive beamforming, however, the shading coefficients are *complex* numbers (each weight having an amplitude and a phase), which enable the beamformer to steer nulls automatically in the direction of an interfering source and to maximize the output signal-to-noise ratio [3].

This paper is structured so that matrix methods for spatial filtering are reviewed in Section II. The methods are then applied to real hydrophone array data recorded at sea on a submarine for purposes of enhancing the detection performance of a hull-mounted array (in Section III) as well as analyzing the received signals and self-noise characteristics of a towed array (in Section IV). Finally, the conclusions are presented in Section V.

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II. MATRIX METHODS FOR SPATIAL FILTERING

A. Spatial Filtering in Frequency-Bearing Space

Beamforming involves appropriately weighting and delaying the hydrophone outputs of an array prior to summation. In practice, matrix methods for beamforming [1-7] requires estimating the cross spectral matrix $\underline{R}(f)$, which is also referred to as the *cross-power spectral matrix* [6] or the *spatial correlation matrix* [3]. This, in turn, requires decomposing each hydrophone output into its complex spectral components by invoking a fast Fourier transform routine. If $X_j(f)$ denotes the complex narrowband output of the *j*th hydrophone, then the elements of the observed cross spectral matrix are given by

$$R_{ij}(f) = \langle X_i(f) X_j^*(f) \rangle \text{ for } i, j = 1,\ 2,\ \ldots,\ N \quad (1)$$

where $X_j^*(f)$ is the complex conjugate of $X_j(f)$, *N* is the number of hydrophone elements in the array, and $\langle \ldots \rangle$ denotes direct segment (or block) averaging. In block averaging the data from each hydrophone are segmented into *M* blocks and $R_{ij}(f)$ is computed as

$$R_{ij}(f) = \frac{1}{M}\sum_{m=1}^{M} X_i^{(m)}(f) X_j^{*(m)}(f) = \langle X_i(f) X_j^*(f) \rangle \quad (2)$$

where $X_i^{(m)}(f)$ is the Fourier transform of the *m*th data block. Note that *M* is also referred to as the number of integrations used to estimate $R_{ij}(f)$.

The conventional estimate of the output power spectrum of a beam steered in the direction β for an *unshaded* array (where the shading coefficients are all set to unity) is given by

$$P_c(\beta, f) = \underline{v}^H(\beta, f)\underline{R}(f)\underline{v}(\beta, f) \quad (3)$$

where $\underline{R}(f)$ is the cross spectral matrix for a frequency *f*, $\underline{v}(\beta, f)$ is the vector of phase delays required to bring all the hydrophone outputs into phase before summing, and $\underline{v}^H(\beta, f)$ is the complex conjugate transpose of the vector $\underline{v}(\beta, f)$. The angle β, which is sometimes referred to as the *cone angle* [4], is measured with respect to the longitudinal axis of the array, where $\beta = 0^\circ$ is the forward endfire direction, $\beta = 90^\circ$ is the broadside direction and $\beta = 180^\circ$ is the aft endfire direction. For the present case of a line array of uniformly spaced hydrophones, the *j*th element of $\underline{v}(\beta, f)$ is given by

$$v_j(\beta, f) = \exp\left(-i2\pi j(d/\lambda)\cos\beta\right) \quad (4)$$

where *d* is the interhydrophone element spacing and λ is the wavelength of the plane wave arrival. The *optimal* estimate of the output power spectrum for the same steering direction is given by

$$P_o(\beta, f) = \left(\underline{v}^H(\beta, f)\underline{R}^{-1}(f)\underline{v}(\beta, f)\right)^{-1} \quad (5)$$

where *optimal* is used in the sense of maximizing the beamformer's output signal-to-noise ratio. In practice, the observed cross-spectral matrix $\underline{R}(f)$ has both signal and noise components. If $\underline{R}(f)$ is used in an *unconstrained* optimal spatial filter, which requires *a priori* knowledge of the *noise-only* cross spectral matrix, then the signal itself would be treated as noise and would also be suppressed. Equation (5) is derived by maximizing the array gain, subject to the constraint that $\underline{v}^H(\beta, f)\underline{w}_o(\beta, f) = 1$, where $\underline{w}_o(\beta, f) = \underline{R}^{-1}(f)\underline{v}(\beta) / \left(\underline{v}^H(\beta, f)\underline{R}^{-1}(f)\underline{v}(\beta, f)\right)$ is the weight vector for the constrained optimal (MVDR) beamformer. This constraint ensures that the desired signal from the direction of interest passes (unmodified) to the spatial filter's output with unity gain – noise and signals incident on the array from other directions should be suppressed [6]. Note that the term *adaptive* generally refers to beamformers that use estimated cross spectral matrices computed from actual array data, where *optimum* is reserved for beamformers that optimize a certain criterion based on perfect *a priori* knowledge of the true statistics of the array data [4]. Adaptive array processing algorithms vary the complex shading weights according to the signal and noise characteristics prevailing at the time of the observations. In other words, an adaptive spatial filter's characteristics vary depending on the properties of the signal and noise fields at the time that the cross-spectral matrix is estimated. An adaptive array senses the presence of sources of interference noise and automatically self-adjusts (adapts) its array beam sensitivity pattern to suppress these sources, whilst simultaneously enhancing desired signal reception [1].

B. *Spatial Filtering in Frequency-Wavenumber Space*

Rather than restricting the beamforming process to real angles of arrival, the spatial filtering of the acoustic array data can be readily extended to include the whole of wave number space by using the wave vector $\underline{k}$ when computing the output power. This is the preferred approach to spatial filtering when analyzing the self-noise in submarine hydrophone arrays [5]. For a line array, only the component of the wave vector in the direction of the array's axis (denoted by k_x) is required where $k_x = (\sin\theta)/\lambda$ and θ is the angle of incidence, which is measured from the normal to the array axis: $-90^\circ \le \theta \le 90^\circ$. If the speed of sound propagation in the underwater medium is denoted by *c*, then $k_x = (f\sin\theta)/c = (f\cos\beta)/c$. The parameter k_x represents the spatial frequency of the incident plane wave in the direction of the array axis. Note that the real angles of arrival form a subspace within the wave number space and that an alternate definition of k_x includes the factor 2π.

The conventional estimate of the wave number spectrum for an unshaded array is given by

$$P_c(k_x, f) = \underline{w}_c^H(k_x)\underline{R}(f)\underline{w}_c(k_x), \quad (6)$$

where $\underline{w}_c(k_x) = \underline{u}(k_x)/N$ is the conventional weight vector, $\underline{w}_c^H(k_x)$ is the complex conjugate transpose of $\underline{w}_c(k_x)$, and $\underline{u}(k_x)$ is the vector of phase delays required to bring into phase those spectral components of the hydrophone outputs that have the same wave number k_x at a given frequency *f*. For the present case of a linear array with equispaced hydrophone elements, the *j*th element of $\underline{u}(k_x)$ is given by

$$u_j(k_x) = \exp(-i2\pi k_x jd)\,, \qquad (7)$$

where d is the spatial sampling interval (or intersensor spacing), which is equal to the reciprocal of the spatial sampling frequency.

An optimal estimate of the wave number spectrum is given by

$$\begin{aligned} P_o(k_x, f) &= \underline{w}_o^H(k_x, f)\underline{R}(f)\underline{w}_o(k_x, f) \\ &= \left(\underline{u}^H(k_x)\underline{R}^{-1}(f)\underline{u}(k_x)\right)^{-1}, \end{aligned} \qquad (8)$$

where $\underline{w}_o(k_x, f) = \underline{R}^{-1}(f)\underline{u}(k_x)/\left(\underline{u}^H(k_x)\underline{R}^{-1}(f)\underline{u}(k_x)\right)$ is the weight vector for the constrained optimal (MVDR) spatial filter. The constrained optimal (or adaptive) spatial filter selects the set of weights $\underline{w}_o(k_x, f)$ given above so that the output power of the spatial filter is minimized while the spatial response at the wave number of interest is constrained to unity.

The cross spectral matrix plays a significant role in frequency domain spatial filtering [7]. Conceptually, the optimal spatial filter uses the cross terms $\left(R_{ij}, i \neq j\right)$ in the cross spectral matrix $\underline{R}(f)$ to sense the wave number distribution of the noise sources and then shades the array by multiplying the hydrophone outputs by a set of complex weights prior to summation so that the array gain is maximized. With the MVDR spatial filter, a wave traveling with the wave number of interest is considered to be a *signal* by the spatial filter and is unaffected as it passes through the spatial processor, whereas traveling waves with other wave numbers are considered as *noise* and are suppressed by the spatial filter.

III. SUBMARINE FLANK ARRAY – ENHANCED DETECTION PERFORMANCE

A. Flank Array Sonar Data Processing

Conventional and optimal (adaptive) spatial filtering techniques were applied to real data from a linear array of 24 hydrophones that were mounted along the hull of a submarine. The array design (or cut) frequency, which corresponds to an interelement spacing of one-half wavelength, was 750 Hz – only the starboard array is considered here. The digital time series output from each hydrophone was divided into *M* blocks. Each block was decomposed into its complex spectral components, and the cross spectral matrix for each frequency bin $\underline{R}(f)$ was estimated by using (2) with *M*=50 (for a small frequency increment of about 0.81 Hz) or *M*=400 (for a large frequency increment of about 6.51 Hz). Both conventional and optimal spatial filtering methods were implemented in the frequency domain. Conventional and optimal beam output spectra were estimated for the frequency band: 50-1000 Hz for a beam steered in the direction of the source (a surface ship which transmitted underwater tonal signals using sonar projectors).

Two versions of the optimal spatial filter were implemented - these differed only in the way that $\underline{R}(f)$ was normalized. Note that with the normalization of $\underline{R}(f)$, the elements of $\underline{R}(f)$ are adjusted so that the trace of $\underline{R}(f)$ is equal to the number of hydrophones ($N = 24$). For normalization method 1, designated 'ABF (1)', the *ij*th element of $\underline{R}(f)$ is divided by $P'_{ij}(f)$, which denotes the geometric mean of the output power of the *i*th and *j*th hydrophones at a frequency *f*. Similarly, for normalization method 2, designated 'ABF (2)', the *ij*th element of $\underline{R}(f)$ is divided by $P''(f)$, which denotes the power at frequency *f* averaged over all hydrophones in the array. Mathematically,

$$P'_{ij}(f) = \left\langle \left|X_i(f)\right|^2 \right\rangle^{\frac{1}{2}} \left\langle \left|X_j(f)\right|^2 \right\rangle^{\frac{1}{2}}, \qquad (9a)$$

$$P''(f) = \left(\sum_{i=1}^{N} \left\langle \left|X_i(f)\right|^2 \right\rangle \right) / N. \qquad (9b)$$

B. Results

Examples of the output spectra for a beam steered in the direction of the source $\beta = 119^\circ$ are shown in Figs 1-3, and also in Fig. 4 where the sonar array data were recorded nine minutes later when the relative bearing of the source $\beta = 91.5^\circ$. The labels 'CBF', 'ABF (1)' and 'ABF (2)' designate the output power spectra from the conventional beamformer and the two adaptive beamformers, respectively. Also, the label 'HYD' refers to the power spectrum estimated by averaging the spectra from all the hydrophone outputs in the array. The average over the entire array, rather than the output from a single hydrophone, is presented because the self-noise power increases by 10 dB over the aperture of the array. These figures demonstrate the improvement in detection of the sonar projector lines (source signals), which are labelled 'PRO' in Figs 1-4. In Fig. 1, the frequency bin width is large (≈ 6.51 Hz), whereas is Figs 2-4 it is small (≈ 0.81 Hz). The projector tonal corresponding to the array's design frequency of 750 Hz (frequency bin number 116 in Fig. 1 and 923 in Fig. 2) is evident in the spectra of the CBF and both ABFs, but not the single hydrophone HYD spectrum. Similarly, the other projector line at 309 Hz (frequency bin number 381 in Fig. 3 and Fig. 4) is evident in the spectra for all three beamformers but it is indiscernible from the background in the spectrum of a single hydrophone. In Fig. 1, however, this tonal signal (which maps to frequency bin 49.5) is not observed in *any* of the output spectra because the frequency bin width is too large (temporal frequency filtering is too coarse). Note that this tonal signal is near a strong ownship interference line at

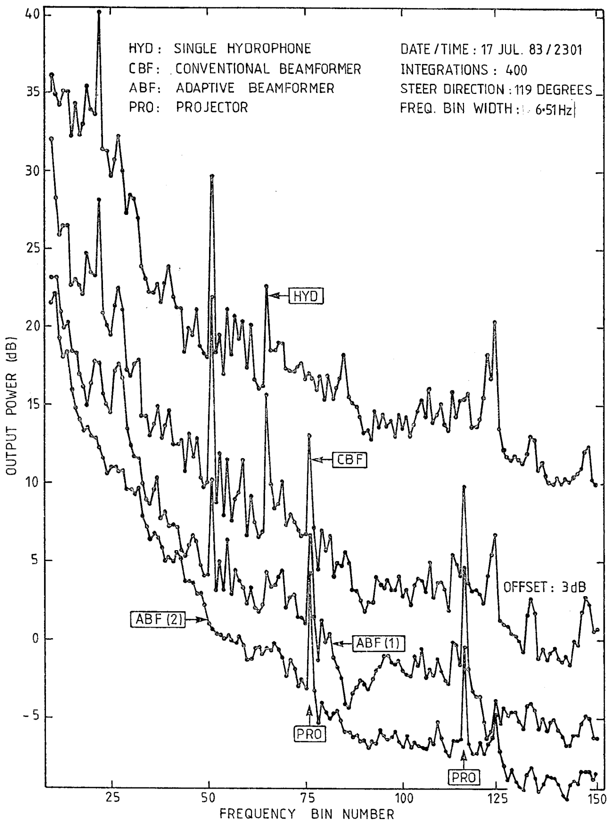

Fig. 1. Comparison of spectra – Wide frequency bin width: 6.51 Hz

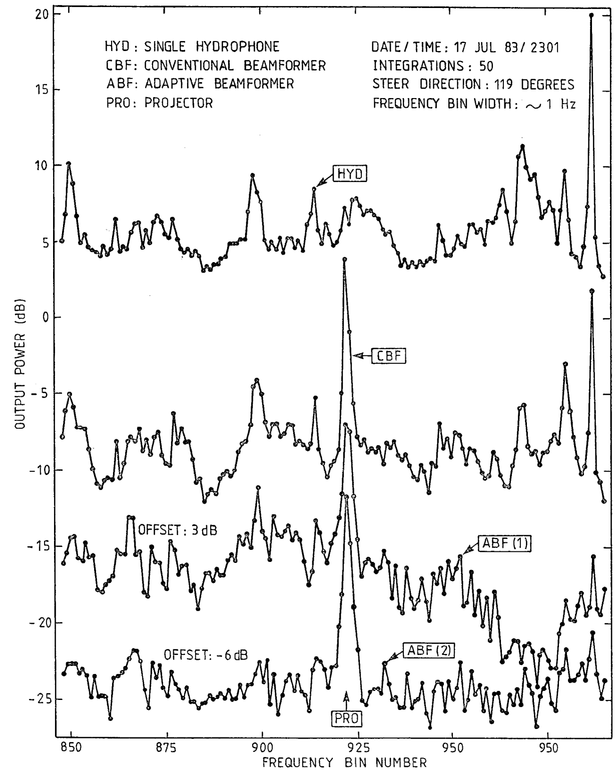

Fig. 2. Comparison of spectra – Narrow frequency bin width: ≈ 0.81 Hz

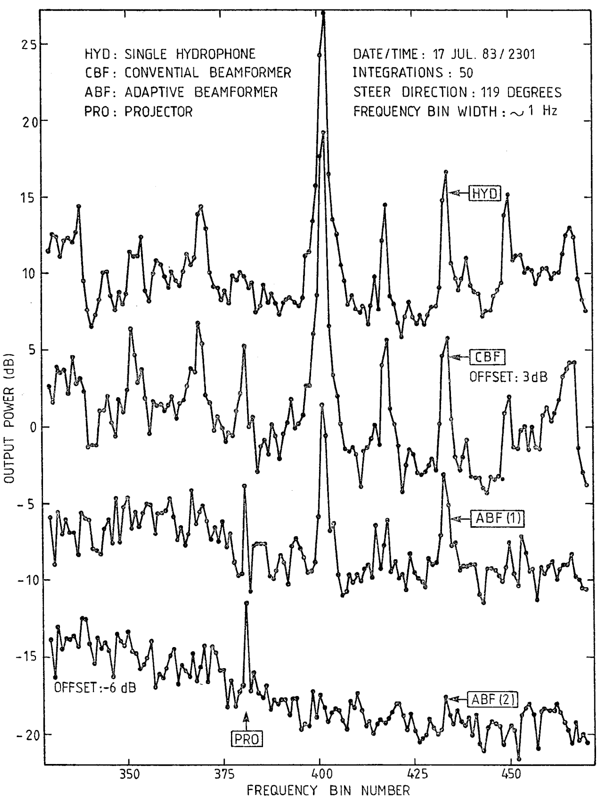

Fig. 3. Comparison of spectra – Narrow frequency bin width: ≈ 0.81 Hz

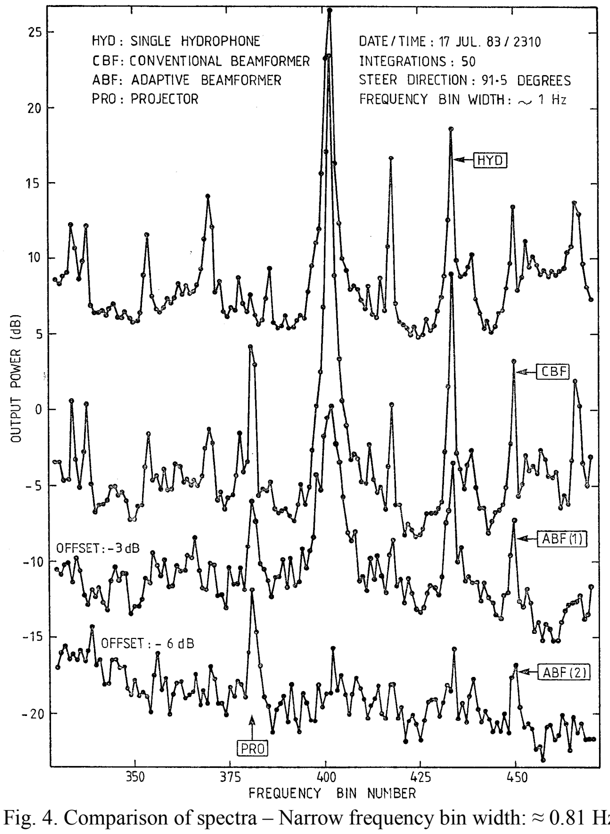

Fig. 4. Comparison of spectra – Narrow frequency bin width: ≈ 0.81 Hz

326 Hz (corresponding to frequency bin number 51 in Fig. 1), which is suppressed *only* by ABF (2). This ownship line is evident in the single HYD hydrophone spectrum and the CBF and ABF (1) beam spectra in Fig. 1. Similarly, this ownship spectral line is dominant in Figs 3 and 4 at frequency bin number 402 in the single hydrophone HYD spectrum, as well as the CBF and ABF (1) beam output power spectra, but it has been suppressed by the ABF (2) spatial filter. The conventional spatial filter CBF detects the projector lines, but the spectra are contaminated by ownship interference lines. Adaptive spatial filtering suppresses the ownship lines with ABF (2) providing better suppression than ABF (1).

Similar results were observed on other submarines fitted with the flank array. The dominant interference for the flank array is ownship noise, which consists of a large number of narrowband components that are strongly coherent over the aperture of the array and a broadband component that is only partially coherent. Theoretically, when compared with the CBF, the statistical variability of the ABF spectral estimates is expected to be higher as discussed by Gray [8]. However, in practice, the detection performance of the ABF is superior to the CBF because the ABF by virtue of its null steering capability is more effective in reducing not only the mean noise power but also its variability. A basic detection algorithm was invoked to provide automatic detections of the tonal signals for all available data sets for the flank array. The results are shown in Fig. 5, where the number of false target detections (that is, detections that do *not* coincide with a projector frequency) can be seen to be far less for ABF (2) than for CBF. This result for submarine flank array data demonstrates the improvement in detection performance and the reduction in false target detection achieved by ABF (2).

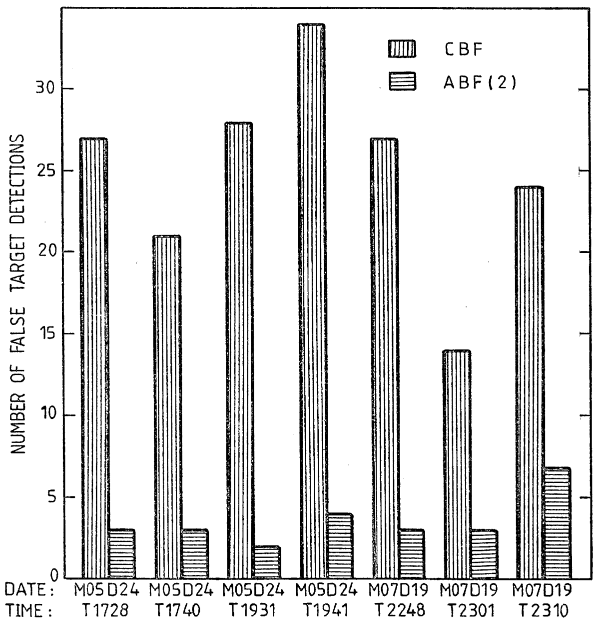

Fig. 5. Comparison of the number of false target detections for the conventional spatial filter CBF and the adaptive spatial filter ABF (2).

IV. SUBMARINE TOWED ARRAY – SELF NOISE ANALYSIS

Conventional and optimal (MVDR) spatial filtering methods were applied previously to real hydrophone data from a submarine slim-line buoyant fiber towed array [5]. The results are reviewed here. The linear array consisted of 24 uniformly spaced hydrophones, each separated by a distance of 7.5 m, which corresponds to the array having a nominal design frequency of 100 Hz. Figures 6(a) and 6(b) show the respective conventional and optimal estimates of the frequency-wave number spectrum for the submarine towed array data with the orientation of the axes shown in Fig. 6(c). In Figs 6(a) and 6(b), the upper frequency is 200 Hz, which corresponds to twice the design frequency. The surfaces in these displays are associated with contacts (surface vessels), the tow vessel (submarine), and the spatially-correlated components of the array self-noise. A comparison of Figs 6(a) and 6(b) demonstrates the advantages of invoking the optimal estimation technique to suppress the spatial leakage (sidelobes) and to improve the spatial resolution performance of the array. Spatial aliasing is also evident in the frequency-wave number power spectra and occurs when the wave number exceeds the spatial Nyquist frequency, that is, $k_x > 1/(2d)$. With spatial aliasing, a wave number with magnitude greater than $1/(2d)$ is transformed so that its magnitude is below $1/(2d)$, and the sign of the wave number is reversed.

Figure 7 is a frequency wave-number plot showing the coordinates (temporal frequency f, spatial frequency k_x) of the peaks in the optimal (MVDR) frequency-wave number power spectrum. The optimal wave number spectrum is preferred because the secondary maxima associated with spatial leakage in the (unshaded) conventional wave number spectrum are suppressed, accounting for a substantial reduction in the number of peak detections. Also, unlike the conventional spatial filter, the optimal spatial filter resolves the two surface vessels near broadside and enhances the resolution between the direct propagation path and the surface reflected multipath of the tow vessel, which is close to forward endfire. The tow vessel and surface ship contacts are broadband sources of acoustic energy and so display a characteristic linear relationship in the frequency-wave number plane. The (f, k_x) coordinates for these broadband sources lie on lines of constant bearing that run radially outward from the origin. The rate of change of wave number with respect to frequency for each of these lines is equal to $(\sin\theta)/c$, where θ is the angle of incidence measured from broadside. The wedge-shaped physical region is bounded by two lines of constant bearing corresponding to the aft and forward endfire directions, and the energy associated with plane-wave signals incident on the array from angles lying between $-90° \le \theta \le +90°$ occurs inside this region, which is referred to as the 'physical region'. Outside the physical region, there is the array self-noise that propagates along the

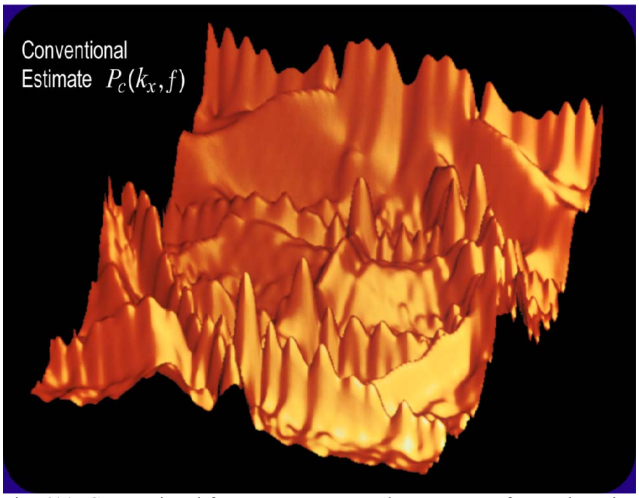

Fig. 6(a). Conventional frequency-wave number spectrum for a submarine towed array.

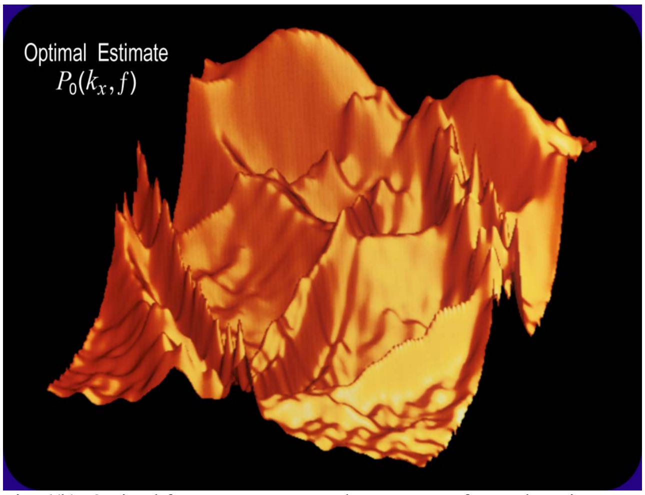

Fig. 6(b). Optimal frequency-wave number spectrum for a submarine towed array.

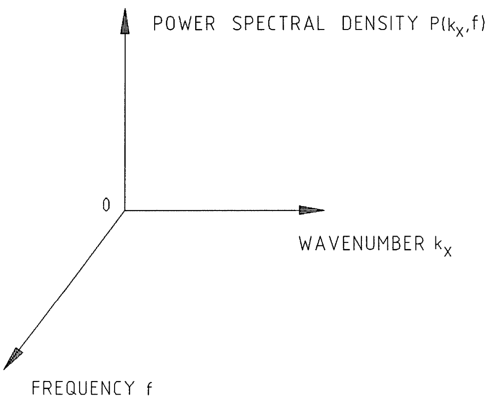

Fig. 6(c). Rectangular Cartesian coordinate system showing the orientation of the axes for the frequency-wave number spectra displayed in Figs 6(a) and (b). For the temporal frequencies: $0 < f \le 2f_d = 200$ Hz and for the spatial frequencies (wave numbers): $-1/(2d) \le k_x \le 1/(2d)$, where $d = 7.5$ m

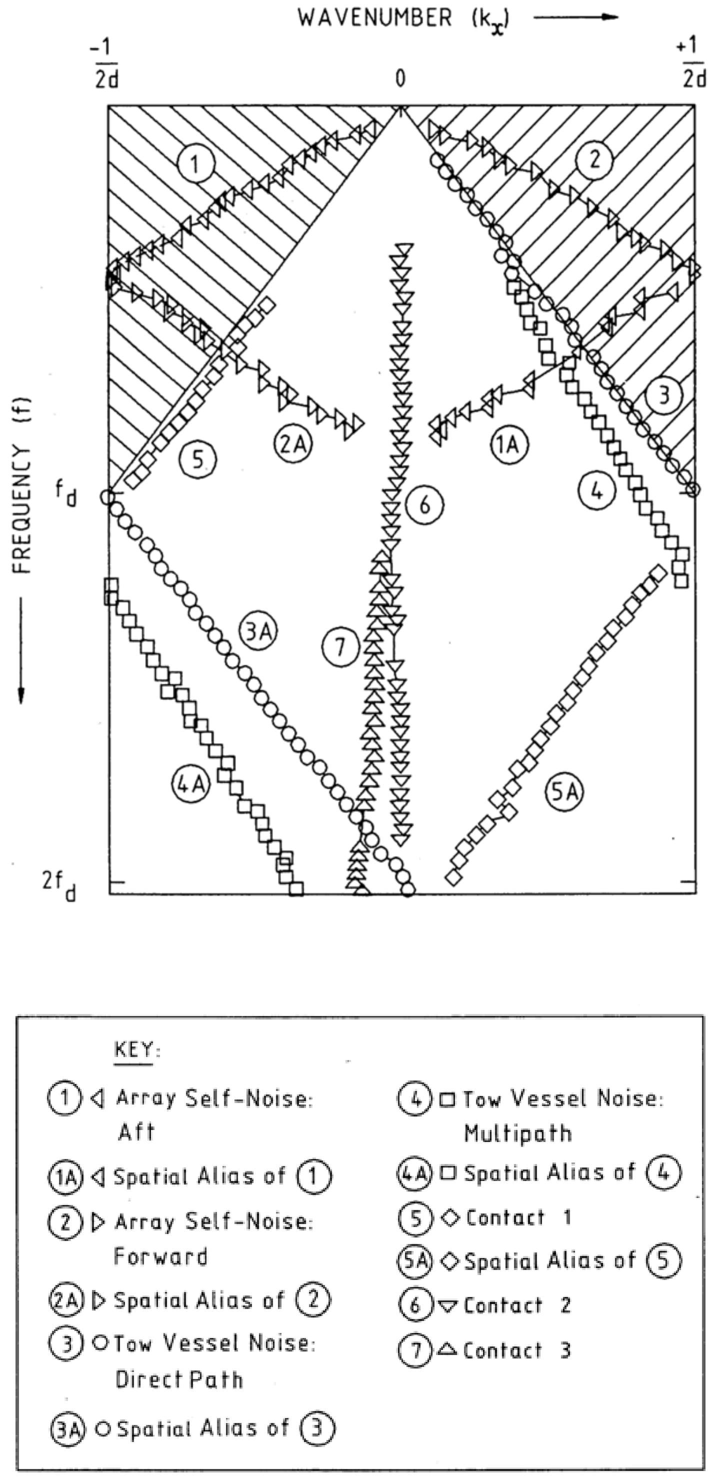

Fig. 7. Frequency- wave number plots for the submarine towed array data showing the wave number bins corresponding to maxima in the optimal frequency-wave number spectrum for each temporal frequency bin. The spacing between the array elements is $d = 7.5$ m and the design frequency is $f_d = 100$ Hz.

array structure with a phase speed (derived from Fig. 7) that is about half the speed of sound propagation in the underwater medium. Array self-noise phenomena are common in all submarine hydrophone arrays, whether hull-mounted or towed, and are observed when vibrations propagate along the array structure and excite the hydrophones. The array self-noise is symmetric in wave number space because the vibrations correspond to structural (extensional) waves propagating with the same speed along the array in both axial directions. Spatial aliasing of the structural waves is readily observed in Fig. 6(b) and Fig. 7.

V. CONCLUSIONS

Compared with conventional spatial filtering (delay-and-sum beamforming), optimal (adaptive) spatial filtering of submarine sonar data improves the detection performance of a hull-mounted array and the resolution and superdirective performance of a towed array.

Conventional spatial filtering of submarine hull-mounted array data results in beam output spectra that are characterized by a combination of projector (signal) spectral lines and ownship (interference) spectral lines. With adaptive spatial filtering, the ownship lines tend to be rejected, which facilitates the detection of the projector lines. Hence, for the submarine hull-mounted array, an enhancement in detection performance is achieved by implementing an adaptive spatial filtering algorithm based on inversion of the cross spectral matrix at each frequency bin of interest. The degree of improvement depends on the method used to normalize the cross spectral matrix.

Optimal estimation of the frequency-wave number power spectrum by the MVDR beamformer enables sources of acoustic energy to be readily discerned in frequency-wave number space. It enables both temporal and spatial frequency characteristics of each source to be quantified, especially the correlated components of the self-noise in the towed array. The advantages of optimal spatial filtering over its conventional counterpart are the suppressed sidelobes, enhanced spatial resolution of the array, and improved superdirective array gain at frequencies well below the design frequency of the towed array.

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