Sonar Uncertainty and Sensitivity Analysis Techniques

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***Abstract*-Current sonar performance prediction models provide outputs such as probability of detection. It is desirable to give measures of *uncertainty* in the output due to uncertainty in all the inputs. This process is *uncertainty analysis* (UA). It is also desirable to indicate the *sensitivity* of the output to uncertainty in *each* of the inputs. This process is *sensitivity analysis* (SA). In this paper, a set of techniques has been incorporated into a sonar uncertainty and prediction tool (SUST) which carries out the main steps of UA/SA: (1) sampling of inputs, (2) propagation of inputs through a model to yield outputs, (3) UA from outputs and (4) SA from outputs. In the following, two enhancements to a previous UA/SA tool are presented: (a) empirical orthogonal functions, a general method of representing uncertainty in the sound speed profile. and (b) Latin hypercube sampling, a more efficient means of sampling the input space. This paper demonstrates the use of the techniques by performing UA/SA for an environment offshore from Sydney, Australia.**

I. INTRODUCTION

Current sonar performance prediction models provide outputs such as transmission loss (TL) and probability of detection (P_d). It is desirable to give measures of *uncertainty* in the output due to uncertainty in all the inputs – this process is *uncertainty analysis* (UA). It is also desirable to indicate the *sensitivity* of the output to uncertainty in *each* of the inputs – this process is *sensitivity analysis* (SA).

The main steps in UA and SA are:

1. Generate an ensemble of model inputs with distributions that characterize their uncertainties.
2. Propagate the inputs through the forward model (e.g. sound propagation model) to yield an ensemble of outputs, e.g. P_d .
3. Estimate the uncertainty in the output.
4. Estimate the sensitivity of the output to uncertainties in each input.

In a previous paper [1], we presented the Underwater Acoustic Sensitivity Tool (UAST) that implements the above four steps. It uses the methodology of Jansen [2], adapted to underwater acoustics and sonar performance prediction. The model inputs (step 1) are sonar system and ocean scenario parameters, and the model outputs (step 2) are acoustic TL and sonar P_d . The tool estimates uncertainty and sensitivity in acoustic TL and sonar P_d (steps 3 and 4).

Further development of these methods has continued, including:

- Latin hypercube sampling [3]: a method of sampling the input parameter space more efficiently in step 1. This is required to give good estimates of uncertainty and sensitivity
- Empirical orthogonal functions [4]: a general technique which is applied to representing the uncertainty / variability in the water column sound speed profile (SSP)

These developments have been included in an enhanced tool called the Sonar Uncertainty and Sensitivity Tool (SUST).

Note that there are many environmental and system parameters that affect sonar performance – SSP, seabed parameters, source level, target strength, noise level, etc. In this paper, we only consider sonar uncertainty and sensitivity related to key environmental parameters that affect acoustic propagation, viz. SSP and seabed parameters.

The paper is structured as follows. In Section II, the algorithms used to carry out sonar UA and SA are described. The section is split into four subsections, one for each of major steps in UA/SA, including the new material introduced above. In Section III input data for analysis in Section IV is described; in the latter section, some UA and SA results are presented for a particular location to demonstrate the use of the techniques. Some discussion of the results is given. In the final section, some conclusions are drawn, and plans for future work are given.

II. SONAR UNCERTAINTY AND SENSITIVITY ANALYSIS ALGORITHMS

The algorithms used in SUST for sonar UA and SA will now be described. The analysis only considers sonar uncertainty and sensitivity related to seabed properties and the water column SSP information. Correlations between inputs have been ignored. In future work, we will consider uncertainties in other parameters that affect sonar performance, as well as correlations between inputs.

A. *Generation of input ensemble*

Seabed parameters

We assume that the seabed consists of a number of sedimentary layers and a basement. Each sedimentary layer has four parameters which affect sound propagation:

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- Density
- Compressional wave (P-wave) sound speed
- P-wave attenuation
- Layer thickness

The basement has only the first three of these parameters – its thickness is considered to be infinite.

In general, the seabed parameters are uncertain, because of errors in the bottom sampling process, the variability of the bottom, and the limited sample density.

To generate an input ensemble of seabed parameters, one requires the above parameters and their uncertainties. This information can be extracted from a seabed database. Typically, the uncertainties are characterized by uniform or Gaussian distributions.

Single-parameter characterization of the seabed

A seabed with several layers typically has 10-20 parameters, so the input ensemble of seabed parameters has a very large dimension. If this ensemble were used directly in UA/SA, a very large number of forward model evaluations (step 2 of UA/SA) would be required – this would be computationally expensive. However, the following technique can be used to characterize the seabed by just *one* parameter.

The acoustic properties of the seabed can be characterized by bottom loss (BL) versus grazing angle (β) curves, denoted by BL(β). Jones *et al* [5] have shown that for many seabed types, these curves may be characterized by just the *slope*, g, of BL(β) at low grazing angles. They show that the magnitude of BL can be approximated by

$$\mathrm{BL} \cong g\beta \tag{1}$$

and that the reflection phase shift, φ, can be approximated by

$$\varphi \cong -\pi + \pi g\beta / 6, \quad \beta \le 6/g, \tag{2a}$$

$$\cong 0 \quad \text{otherwise.} \tag{2b}$$

So if the distribution of g is known, an ensemble of uncertain complex bottom reflection coefficients (BRCs) can be generated using (1) and (2).

To estimate the distribution of g, one carries out the following steps:

Algorithm 1

- Generate a large ensemble of seabed parameters according to their distributions obtained from the seabed database
- For each sample of the ensemble, compute BL(β) using Bartel's [6] algorithm, then estimate g. This process yields an ensemble of g's.
- Estimate the distribution of g's from the ensemble

An example distribution is given in Section III. These distributions are typically highly skewed.

Water column SSP

LeBlanc and Middleton [4] model the uncertainty in the water column SSP by fitting a number of depth-dependent *orthogonal functions* with randomly varying amplitudes to the SSP uncertainty. They are *empirical* orthogonal functions (EOFs) as they are derived from measured data. The use of EOFs is a new development in the SUST.

(i) LeBlanc and Middleton's EOF representation

From [4], the SSP can be described by a mean profile $c_0(z)$ (z: depth) plus zero-mean random perturbations $\Delta c(z)$. $\Delta c(z)$ can be decomposed in terms of a set of orthogonal basis functions $\{\psi_l(z)\}$ so that

$$\Delta c(z) = \sum_{l=1}^{L} w_l \psi_l(z) \tag{3}$$

where the scaling coefficients w_l are independent random variables. The functions $\{\psi_l(z)\}$ are called *orthogonal* because the w_l vary independently, i.e., the variations in the scale of one function are independent of the variations in the scale of all others.

Because the SSP is only available at a set of discrete depths, a discretized version of (3) must be used :

$$\Delta\mathbf{c} = \mathbf{\Psi w} \tag{4}$$

where $\Delta\mathbf{c}$ is the depth-discretized vector of SSP perturbations, $\mathbf{\Psi}$ is a matrix whose columns, $\{\boldsymbol{\psi}_l\}$, are the depth-discretized EOFs, and $\mathbf{w}$ is the L-element vector of the $\{w_l\}$.

(ii) Computation of the EOFs and scaling variables from measured variability data

We assume that a set of SSPs $\mathbf{c}$ has been extracted from a database for the location, season and time-of-day of interest. The corresponding set of perturbations $\Delta\mathbf{c}$ can be computed by $\Delta\mathbf{c} = \mathbf{c} - \langle\mathbf{c}\rangle$, where $\langle\mathbf{c}\rangle$ is the mean of $\mathbf{c}$. The covariance matrix $\mathbf{K}_{\Delta\mathbf{c}}$ of the $\Delta\mathbf{c}$, that is the matrix of correlations between one depth and another (or itself), can be estimated from the set of $\Delta\mathbf{c}$. Reference [4] shows that the vectors $\{\boldsymbol{\psi}_j\}$ (4) and the variances $\{\lambda_j\}$ of the $\{w_l\}$(3) are given by the first L columns of the eigenvalue decomposition of $\mathbf{K}_{\Delta\mathbf{c}}$

$$\mathbf{K}_{\Delta\mathbf{c}}\mathbf{\Psi} = \mathbf{\Psi\Lambda}$$

where $\mathbf{\Lambda} = \mathrm{diag}\{\lambda_j\}$ and $\mathbf{\Psi} = [\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \ldots \boldsymbol{\psi}_L]$. Here L is the number of "significant" eigenvalues, and is often much less than N_Z , the number of discrete depth points. Each eigenvector $\boldsymbol{\psi}_j$ represents one mode of SSP variation, and each eigenvalue λ_j indicates the amount of energy (variance) in the mode.

An ensemble of L-element scaling vectors $\{\mathbf{w}^k\}$ whose components $\{w_j^k\}$ are independent and Gaussian with variances $\{\lambda_j\}$, is generated using

$$\mathbf{w}^k = \mathbf{\Lambda}^{1/2}\mathrm{rand}(L)$$

where the function rand(L) generates a vector of L independent uniform random variables with unity variance. From (4), the ensemble of SSPs $\{\mathbf{c}^k\}$ whose perturbations have covariance $\mathbf{K}_{\Delta\mathbf{c}}$ is generated using

$$\mathbf{c}_k = \langle\mathbf{c}\rangle + \Delta\mathbf{c}_k \quad \text{where} \quad \Delta\mathbf{c}_k = \mathbf{\Psi w}^k .$$

An example of a set of measured SSPs, and a corresponding (larger) ensemble of SSPs whose perturbations $\Delta\mathbf{c}_k$ have the same statistics as those of the measured set perturbations is given in Section III.

Sampling methods

In drawing a multidimensional ensemble of samples, several sampling methods can be used. In UAST, we used *random sampling*, and in SUST, we introduce *Latin hypercube sampling*. We discuss these next.

(i) Random sampling

In UAST, each sample of the input space (e.g. SSP, BL) is drawn *independently* of the previous sample. This type of sampling is called *random sampling*. Advantages of random sampling are that it is easy to implement and yields unbiased estimates of means, variances and PDFs. However, random sampling may not cover the input space adequately unless a very large number of samples is drawn. This may not be computationally feasible for forward models that are expensive to evaluate. Latin hypercube sampling can be used in these cases.

(ii) Latin hypercube sampling

Latin hypercube sampling has the effect of spreading the sampling more uniformly across the sample space, that is, it requires fewer samples than random sampling to cover the sample space adequately. We describe the algorithm in the following.

Let q be the number of parameters (x_1, x_2 , … x_q) to be sampled, and let N be the desired number of samples. The procedure is:

1. Divide the range of each parameter into N intervals of equal probability.
2. Select one value of x_1 at random from each interval.
3. Pair the N values of x_1 at random and without replacement with N values of x_2.
4. Combine these pairs randomly and without replacement with N values of x_3 until N triplets are formed.
5. Continue until N q-tuples are formed.

Fig. 1(a) shows an example of random sampling for q = 2 and N = 10. Note that there are no samples in the top-left quadrant. Fig. 1(b) shows a corresponding example of Latin hypercube sampling. Note the more uniform spread of samples across the sample space for Latin hypercube sampling.

B. Forward model

The forward model used in this paper computes TL and P_d . Here, we only consider the effects of acoustic propagation uncertainty due to uncertainty in the SSP and BL. There are other uncertain parameters in the estimation of TL and P_d (e.g. acoustic absorption coefficient, surface loss, target strength, noise level), but we do not consider uncertainties in these in this paper. Later versions of SUST will consider uncertainties in these other parameters.

(a)

(b)

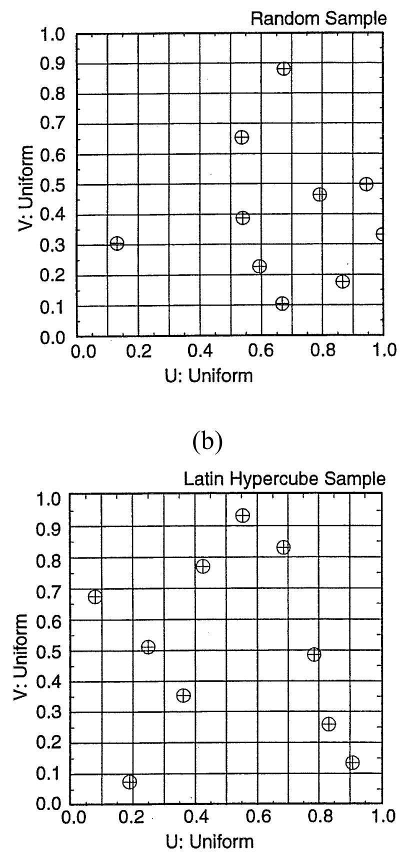

Figure 1. Examples of (a) random sampling and (b) Latin hypercube sampling for q=2 (parameters U and V) and N =10.

For the computation of TL, the inputs required are the ocean depth, the SSP, BL(β), the absorption coefficient, the surface loss and the source and receiver positions. The latter four inputs are assumed known. TL can be computed using a variety of models – Porter's [7] *Acoustics Toolbox* contains a number of codes, including BELLHOP, a ray-tracing code. UAST and SUST use BELLHOP which is fast for shallow water modelling and accurate at active sonar frequencies.

The probability of detection P_d of a sonar system depends on two parameters – the *signal-to-noise* ratio (SNR) at the input of the detection system, and the value of the *false alarm probability* P_{fa} . The SNR (in decibels) of an active system can be computed by

$$\mathrm{SNR} = \mathrm{SL} + \mathrm{DI} - 2 \cdot \mathrm{TL} + \mathrm{TS} - \mathrm{NL} \tag{5}$$

where SL is the source level, DI is the receiver directivity index, TL is the transmission loss, TS is the target strength and NL is the noise level. P_{fa} is usually set to a predetermined level, typically 10^{-4}. P_d can be computed from P_d versus P_{fa} curves, one for each value of SNR. Such families of curves are called *receiver operating characteristic (ROC)* curves, which we denote

$$P_d = P_d(P_{fa};\, \mathrm{SNR}) \qquad (6)$$

Note that all the terms in (5) apart from TL are assumed known.

The steps to calculate ensembles of TLs and P_d's are:

1. Generate ensembles of SSPs and BRCs as described in subsection A.
2. Generate an ensemble of TLs using BELLHOP as described above.
3. Generate an ensemble of P_d's using (5) and (6).

C. Uncertainty Analysis

The measure of uncertainty is the *total variance,* estimated from the ensemble of outputs $\{y_j\}$ (i.e. TL or P_d) by

$$\mathrm{VTOT} = \frac{1}{N-1}\sum_{j=1}^{N}(y_j - \bar{y})^2$$

where N is the number of samples in the ensemble and $\bar{y}$ is the sample mean of the ensemble.

One method of estimating *sensitivity* (see subsection D) requires *two* ensembles of outputs. In this case, VTOT should be estimated from

$$\mathrm{VTOT} = \sqrt{v_1 v_2}$$

where v_1 and v_2 are the variances estimated from ensembles 1 and 2 respectively.

D. Sensitivity Analysis

SUST uses the *top marginal variance (TMV)* as a measure of sensitivity to uncertainty in input parameters. The TMV due to an input parameter (or a group S of input parameters) is the expected variance reduction that would occur if one knew S perfectly. It can be shown [8] that the TMV of output y due to S , denoted by TMV_S is given by

$$\mathrm{TMV}_S = \mathrm{Var}[\mathrm{E}(y|S)] \qquad (7)$$

where Var and E denote variance and expectation respectively. Suppose y depends on two uncertain parameters a_1 and a_2. From (9), the TMV of y due to a_1 is

$$\mathrm{TMV}(y;a_1) = \int p_1(a_1)\left[\int p_2(a_2)\,y(a_1,a_2)\,da_2 - \bar{y}\right]^2 da_1 \qquad (8)$$

where $p_1(a_1)$ is the probability density function (PDF) of parameter a_1, $p_2(a_2)$ is the PDF of a_2 and $\bar{y}$ is the mean of y.

Note that (8) is computationally intensive to evaluate, and would be even more so with more uncertain parameters. However, Saltelli *et al* [9] have devised a *resampling method* which computes an approximation to the TMV due to a group of uncertain parameters more efficiently. Their technique is as follows:

1. Construct two independent ensembles of inputs $\mathbf{X}_1$ and $\mathbf{X}_2$. $\mathbf{X}_1$ and $\mathbf{X}_2$ can be represented as matrices with N rows and m columns, where m is the number of uncertain inputs.
2. Suppose the TMV due to inputs $\{i_1, \ldots, i_p\}$ is to be estimated. Set columns $\{i_1, \ldots, i_p\}$ of $\mathbf{X}_2$ equal to columns $\{i_1, \ldots, i_p\}$ of $\mathbf{X}_1$.
3. Using the chosen forward model (e.g. Subsection B above), compute two output ensembles $\mathbf{y}_1$ and $\mathbf{y}_2$ (e.g. TL or P_d) from $\mathbf{X}_1$ and $\mathbf{X}_2$ respectively.
4. Compute the two sample variances v_1 and v_2 from $\mathbf{y}_1$ and $\mathbf{y}_2$ respectively. Estimate VTOT by the geometric mean of v_1 and v_2.
5. Let $\mathrm{corr}(\mathbf{y}_1, \mathbf{y}_2)$ be the Pearson correlation coefficient of $\mathbf{y}_1$ and $\mathbf{y}_2$ [10]. It can be shown [9] that $\mathrm{corr}(\mathbf{y}_1, \mathbf{y}_2)$ is an estimate of the *relative* TMV

$$\mathrm{TMV}_S / \mathrm{VTOT} \cong \mathrm{corr}(\mathbf{y}_1, \mathbf{y}_2)$$

SUST follows Saltelli *et al*'s resampling method to estimate the relative TMV due to the set S of inputs. Note that TMV_S / VTOT has values between 0 and 1 – for values near 0 (1), the output is insensitive (very sensitive) to uncertainty in the set S.

III. Environmental Model and Input Data

We now describe the environmental model and data used for sonar UA and SA at a location off Sydney, Australia, where the water depth is approximately 150 m. Both raw and processed SSP and seabed data are described. Fig. 2 illustrates the environmental model used.

A. SSP data

The SSPs were extracted from the World Ocean Database [11] within a 0.5°x0.5° box for the area of interest for the months of January, February and March (oceanic summer in

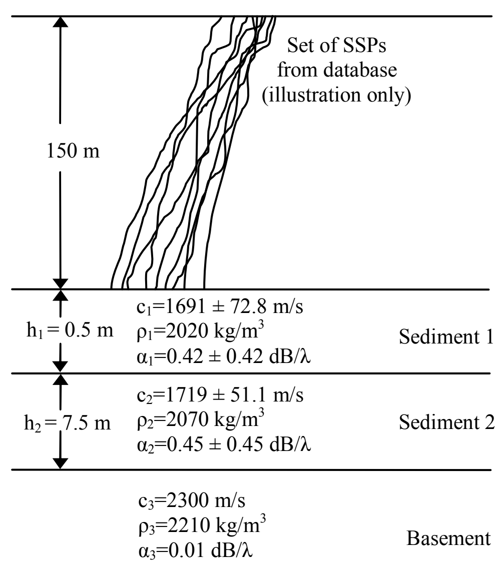

Figure 2. Schematic diagram showing SSP and seabed parameters at the location of interest

the Southern Hemisphere. The blue curves in Fig. 3 show the raw SSP data. The yellow curves show the ensemble with the same statistics as the blue curves. These were generated using EOFs $\mathbf{\Psi}$ and associated variances $\mathbf{\Lambda}$ as indicated in II.A. Fig.4 shows a 20-SSP subset of the generated ensemble. Note that the curves in the subset have generally the same features as the raw SSPs – downward refracting with a weak surface duct.

B. Seabed parameters

Fig. 2 shows the raw seabed parameters extracted from the location of interest. The notation for the seabed properties is as follows, where the subscripts "i" indicate the layer numbers.

h_i: Layer thickness
c_i: Compressional sound speed
ρ_i: Density
α_i: Compressional attenuation

The distributions of the seabed properties are assumed Gaussian and independent. The values after the ± sign are standard deviations of the parameters. Negative values of α_1 and α_2 (possible with the parameters in Fig. 2) are truncated to zero.

Figure 5 shows the probability density function of the BL gradient, estimated from the values and distributions of the seabed properties, as indicated in Algorithm 1 of Section IIA. Note that the distribution is highly skewed towards zero, with a long tail.

IV. UA AND SA RESULTS FOR LOCATION OFF SYDNEY

Here we demonstrate the UA/SA techniques presented in II using the environment presented in III.

In the analysis the following deterministic parameters were used:

- Frequency: 5000 Hz
- Ocean depth: 150 m
- Source depth: 7 m
- Receiver depth: 15 m

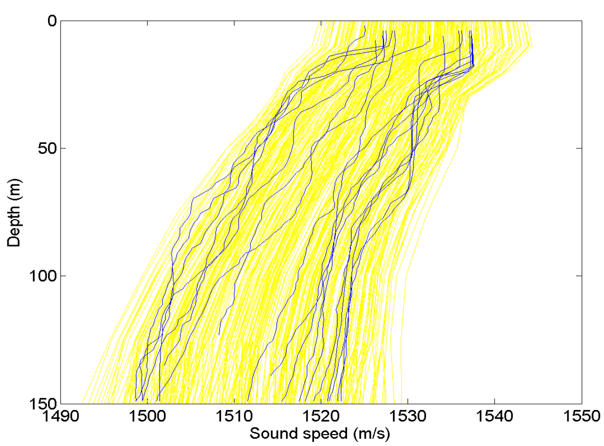

Figure 3. Raw SSPs (blue), and SSP ensemble (yellow), generated using EOFs, location off Sydney, January to March.

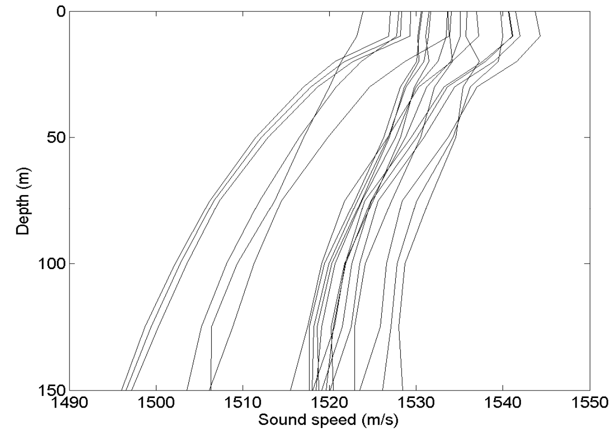

Figure 4. 20-SSP subset of ensemble of Fig,3

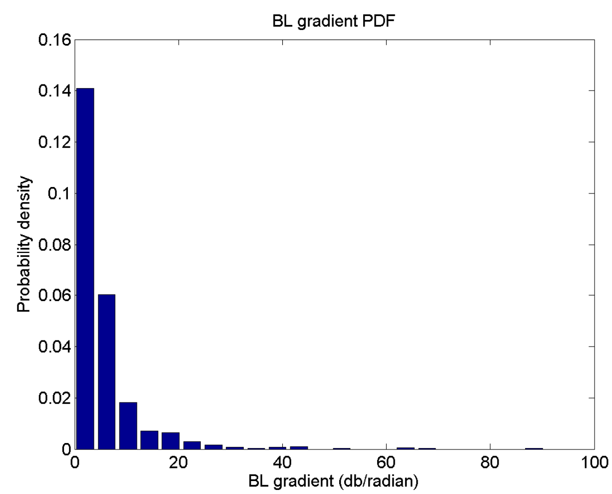

Figure 5. Probability density function of seabed loss gradients derived from seabed parameters and uncertainties of Fig. 2.

TL results

Fig. 6 shows the TL uncertainty and sensitivity curves for the location in summer. The solid curve in the upper plot shows the mean ensemble incoherent TL versus range. The upper dashed curves show plots of mean TL±√VTOT, i.e. mean TL ± one standard deviation. VTOT has been computed from decibel values of TL. The standard deviations vary between 2 dB and 8 dB, and increase with range.

The SA results for this environment are shown in Fig. 6(b). The solid and dashed curves show the relative TMVs due to SSP and BL uncertainty respectively – we denote these by TMV(SSP) and TMV(BL). They are plotted as percentages, i.e. 100·TMV/VTOT. The TMV due to BL is greater than that due to SSP at all ranges. This can be attributed to the fact that the SSP is strongly downward refracting in summer, and that changes in SSP are insufficient to change it to isospeed or upward refracting. Because of the downward refraction, uncertainty in TL is dominated by the uncertainty in BL gradient.

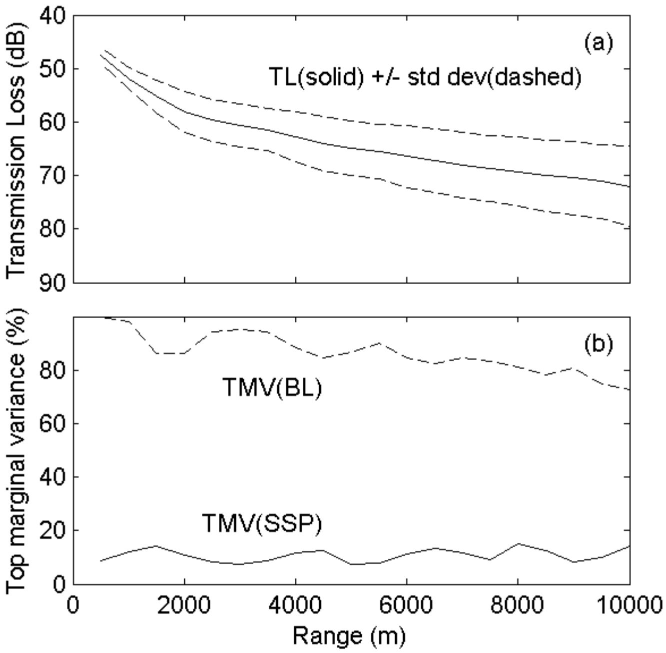

Figure 6. (a) UA and (b) SA of TL off Sydney in summer, using Latin hypercube sampling

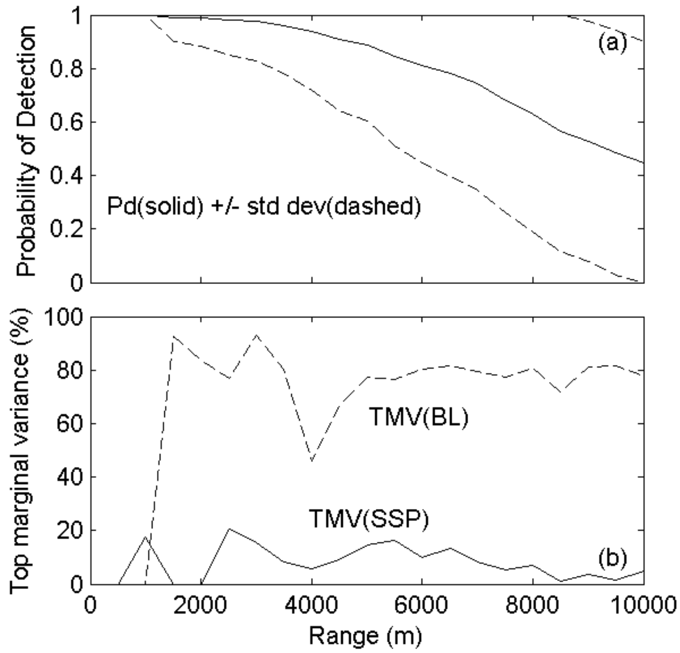

Figure 7.(a) UA and (b) SA of P_d off Sydney in summer using Latin hypercube sampling,

P_d results

Fig. 7(a) shows the P_d uncertainty curves for the location in summer. The standard deviations become very large at long range, up to 0.45. This very wide spread could be due to the fact that the P_d versus TL curve may transition rapidly from near unity to near zero over a short TL interval.

The sensitivity results for this environment are shown in Fig. 7(b). The TMV due to BL is greater than that due to SSP at most ranges. This can be attributed to the fact that the TMV of TL due to BL is greater than that due to SSP at all ranges, and the greater sensitivity of TL due to BL feeds through to the sensitivity of P_d .

The low TMVs due to BL at ranges ≤ 1 km are artifacts due to the fact that P_d is unity or close to it at these ranges. In these cases, the variance of P_d is zero or close to it, causing an indeterminate relative TMVs.

V. CONCLUSIONS

We have presented a sonar uncertainty analysis tool (SUST) which implements a set of algorithms to carry out the four main steps of UA/SA, viz. (i) generation of input ensemble, (ii) propagation of the input ensemble through a forward model to yield an ensemble of outputs such as TL or P_d, (iii) uncertainty analysis and (iv) sensitivity analysis. This paper introduces two main enhancements to a previous UA/SA tool (UAST), a more general representation of SSP using empirical orthogonal functions, and more efficient sampling of the input space using Latin hypercube sampling.

We have demonstrated the UA/SA techniques by applying them to the environment offshore from Sydney, Australia. Results obtained included uncertainty and sensitivity versus range for TL and P_d using Latin hypercube sampling.

The SUST will be further developed to assess sensitivity to other uncertain parameters, such as source level, target strength and noise level. It will also be generalized to assess sensitivity to *groups* of parameters, and enhanced to allow consideration of *correlations* between inputs, for example, the parameters in a given sediment layer.

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