Improving Resolution and SNR of Correlation Function with the Increase in Bandwidth of Recorded Noise Fields During Estimation of Bottom Profile of Ocean

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Abstract- **Cross-correlation of coherent wave fronts of acoustic ambient noise fields gives an estimation of the bottom profile of ocean. Hydrophones are used to record the noise fields. Averaging time needed for estimating depth of sea bed depends on the number of coherent wave fronts extracted from the noise fields. In the current approach of depth estimation if only two hydrophones are used to record the acoustic noise fields ,then it takes large amount of averaging time to extract sufficient amount of coherent wave fronts for estimating depth of sea bed. Use of a vertical array of hydrophones for noise field recording and later by combining the cross- correlation of all hydrophone pairs of the vertical array a stronger signature can be obtained and this greatly reduces averaging time. Reduction of averaging time can be expressed as increase in signal to noise ratio (SNR) and resolution of correlation function. Main goal of this paper is to increase SNR and resolution of correlation function using only two hydrophones and proper use of bandwidth of recorded noise fields. It is shown that recording large bandwidth signals in two hydrophones greatly increases SNR and resolution of correlation hence reduces the averaging time needed for estimating depth of sea bed. Results of this paper were verified using mathematical expression and simulation.**

I. INTRODUCTION

Cross-correlation of acoustic ambient noise has lots of applications in underwater communication such as estimation of the bottom profile of the sea bed, retrieval of the time domain Green's function and so on. The accuracy of all these applications depends on the SNR and resolution of the cross-correlation function. The goal of this paper is to investigate the factors that can improve the SNR and resolution of the correlation function.

Lots of work has been done related to the cross-correlation of ocean ambient noise in recent years. An image of bottom profile can be estimated using cross-correlation of the acoustic noise fields recorded at two sensors [1]. It is also known that the time domain Green's function between two points can be recovered from the cross-correlation of acoustic ambient noises measured at two sensors [3,5,6,8]. Retrieval of the Green's function depends on the amount of coherent wave fronts extracted from acoustic ocean noise fields [3,4]. Coherent wave fronts are those signals that pass both the sensors with time delay. The more amount of coherent wave fronts extracted from ambient noise fields indicates better estimation of Green's function which is equivalent to improvement of SNR of correlation function. For extracting sufficient amount of coherent wave fronts, ambient noises need to be recorded for large amount of time if only two sensors are used. Then this large time slot data is divided into small slots and processed individually and averaged together to get better SNR of correlation [5,9]. But recording of ambient noises for large amount of time is a time consuming process hence not an effective technique for improving SNR .

M Siderius, C.H. Harrison , and M.B. Porter demonstrated in their work of depth estimation of sea bed that use of vertical array of sensors and beam forming each pair of sensors improves SNR of correlated signal hence results better estimation of sea bed [1]. They used an array of 32 sensors to improve estimation. The problem of using array processing technique is that signal processing for beam forming of pair of sensors is very complex [1,7]. Also it needs a number of sensors in the array that is costly and hard to deploy underwater.

In this paper we have shown that SNR and resolution of the cross-correlation of background noise fields can be improved by increasing the bandwidth of recorded signals and using only two sensors instead of an array of sensors. Mathematical formulation of frequency domain cross-correlation has been done for finite bandwidth which establishes a relationship between bandwidth and SNR and between bandwidth and resolution. Also time domain cross-correlation function has been analyzed in the simulation.

II. NOISE FIELDS AT WATER SURFACE

Let's consider that ambient noise sources are distributed on the surface like an infinite sheet of noises. Two sensors s_1 and s_2 are positioned at (z_1, r_1) and (z_2, r_2) in a cylindrical co-ordinate system respectively where depth of sea bed was considered as H. Surface noises coming from different directions are recorded by s_1 and s_2 sensors .Most

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of the surface noises are generated due to the wind action at surface water and these noise sources can be considered as the infinite sheet of point sources located just below the surface at depth z' [1].

Geometry of the sheet sources, sensors and bottom image of sources is shown in Fig. 1.

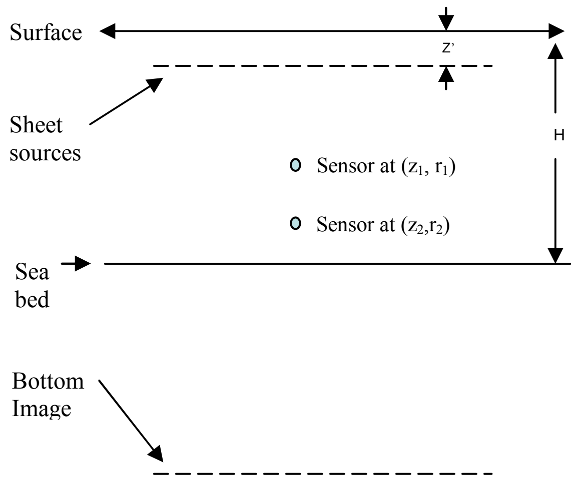


Figure 1. Geometry of noise sheet source , bottom image and sensors [1].

Pressure field received at sensor s_1 at depth z_1 can be expressed as follows [1]-

$$P(\omega,R,z_1)=\int_0^\infty g(k_r,z_1)J_0(k_rR)k_rdk_r \tag{1}$$

Pressure field received at sensor s_2 at depth z_2 can be expressed as follows [1]-

$$P(\omega,R,z_2)=\int_0^\infty g(k_r,z_2)J_0(k_rR)k_rdk_r \tag{2}$$

Where $g(k_r,z_1)$ and $g(k_r,z_2)$ are the Green's functions at s_1 and s_2 sensors respectively from sheet noise sources at z' and its bottom reflection image and k_r is the horizontal wave-number. As sensors are separated vertically, so $J_0(k_rR)=1$ where $R=r_1-r_2$ in the cylindrical co-ordinate system [1].

In the geometry of Fig. 1 sea bed is considered as a rigid boundary. So Green's function [11,1] retrieved at two sensors s_1 and s_2 due to sheet noise sources and its bottom image can be expressed respectively as follows -

$$g\ (k_r,z_1)=\left[\frac{e^{ik_z(z_1-z')}}{4\pi ik_z}+\frac{e^{ik_z[(H-z_1)+(H-z')]}}{4\pi ik_z}\right] \tag{3}$$

$$g\ (k_r,z_2)=\left[\frac{e^{ik_z(z_2-z')}}{4\pi ik_z}+\frac{e^{ik_z[(H-z_2)+(H-z')]}}{4\pi ik_z}\right] \tag{4}$$

Where k_z is the vertical wave-number. Positive sign in the second term of the Green's function indicates that after reflecting at rigid sea bed there is no change of phase of the incident signal [12]. As R dependency of pressure field disappears due to vertically separated sensors, pressure field expressions don't have Bessel function any more. Substituting the expressions for Green's functions from (3) and (4) into (1) and (2) pressure fields can be rewritten as follows-

$$P(\omega,z_1)=\int_0^\infty\left[\frac{e^{ik_z(z_1-z')}}{4\pi ik_z}+\frac{e^{ik_z[(H-z_1)+(H-z')]}}{4\pi ik_z}\right]k_rdk_r \tag{5}$$

$$P(\omega,z_2)=\int_0^\infty\left[\frac{e^{ik_z(z_2-z')}}{4\pi ik_z}+\frac{e^{ik_z[(H-z_2)+(H-z')]}}{4\pi ik_z}\right]k_rdk_r \tag{6}$$

We know, vertical wave number, $k_z=\sqrt{k^2-k_r^2}$

So,

$$dk_z=\frac{1}{2\sqrt{k^2-k_r^2}}\times(-2k_rdk_r)$$

$$\Rightarrow dk_z\sqrt{k^2-k_r^2}=-k_rdk_r$$

$$\Rightarrow k_zdk_z=-k_rdk_r$$

When a dependent variable of the integration is changed , limit of integration also need to be changed .

When $k_r=0$, $k_z=k$

and

when $k_r=k$, $k_z=0$

because maximum value of k_r can be equal to k .

Pressure field at sensor s_1 can be expressed as follows-

$$\begin{aligned}P(\omega,z_1)&=-\int_k^0\left[\frac{e^{ik_z(z_1-z')}}{4\pi ik_z}+\frac{e^{ik_z[(H-z_1)+(H-z')]}}{4\pi ik_z}\right]k_zdk_z\\&=\frac{1}{4\pi}\left[\frac{1}{(z_1-z')}+\frac{1}{[(H-z_1)+(H-z')]}\right.\\&\left.-\frac{e^{ik(z_1-z')}}{(z_1-z')}-\frac{e^{ik[(H-z_1)+(H-z')]}}{[(H-z_1)+(H-z')]}\right]\end{aligned} \tag{7}$$

Noise pressure field at sensor s_2 can be expressed as follows-

$$\begin{aligned}P(\omega,z_2)&=-\int_k^0\left[\frac{e^{ik_z(z_2-z')}}{4\pi ik_z}+\frac{e^{ik_z[(H-z_2)+(H-z')]}}{4\pi ik_z}\right]k_zdk_z\\&=\frac{1}{4\pi}\left[\frac{1}{(z_2-z')}+\frac{1}{[(H-z_2)+(H-z')]}\right.\\&\left.-\frac{e^{ik(z_2-z')}}{(z_2-z')}-\frac{e^{ik[(H-z_2)+(H-z')]}}{[(H-z_2)+(H-z')]}\right]\end{aligned} \tag{8}$$

Since noise pressure field is a function of frequency, small constant terms $\frac{1}{(z_1-z')}+\frac{1}{[(H-z_1)+(H-z')]}$ and $\frac{1}{(z_2-z')}+\frac{1}{[(H-z_2)+(H-z')]}$ can be neglected from the mathematical expressions due to mathematical simplicity.

After this approximation noise pressure fields recorded at sensors s_1 and s_2 can be expressed respectively as follows-

$$P(\omega,z_1)=\frac{-1}{4\pi}\left[\frac{e^{ik(z_1-z')}}{(z_1-z')}+\frac{e^{ik[(H-z_1)+(H-z')]}}{[(H-z_1)+(H-z')]}\right] \quad (9)$$

$$P(\omega,z_2)=\frac{-1}{4\pi}\left[\frac{e^{ik(z_2-z')}}{(z_2-z')}+\frac{e^{ik[(H-z_2)+(H-z')]}}{[(H-z_2)+(H-z')]}\right] \quad (10)$$

III. Cross-correlation of Finite Bandwidth Signals

To get the finite bandwidth signal a filter needs to be applied on the recorded noise fields [2]. Here a Brick wall low pass filter was applied whose frequency response can be expressed as a rectangular function that is-

$$H(\omega)=rect(\frac{\omega}{2\omega_0}) \quad (11)$$

Where ω_0 is the cut-off frequency of the filter. As we have applied low pass filter on the noise field, cut-off frequency ω_0 and bandwidth of the noise fields after filtering, B is same. Therefore (11) can be rewritten as follows-

$$H(\omega)=rect(\frac{\omega}{2B})$$

After applying low pass filter pressure fields can be expressed as follows-

$$P(\omega,z_1)=\frac{-1}{4\pi}\left[\frac{e^{ik(z_1-z')}}{(z_1-z')}+\frac{e^{ik[(H-z_1)+(H-z')]}}{[(H-z_1)+(H-z')]}\right]\times rect(\frac{\omega}{2B}) \quad (12)$$

$$P(\omega,z_2)=\frac{-1}{4\pi}\left[\frac{e^{ik(z_2-z')}}{(z_2-z')}+\frac{e^{ik[(H-z_2)+(H-z')]}}{[(H-z_2)+(H-z')]}\right]\times rect(\frac{\omega}{2B}) \quad (13)$$

We know cross-correlation between two signals in time domain is equivalent to multiplication by complex conjugates in frequency domain. Therefore, cross spectral density, $C(\omega,z_1,z_2)$ for finite bandwidth noise fields can be represented as follows-

$$\begin{aligned}C(\omega,z_1,z_2)&=P(\omega,z_1)\times P^*(\omega,z_2)\\&=\frac{(-1)}{4\pi}\left[\frac{e^{ik(z_1-z')}}{(z_1-z')}+\frac{e^{ik[(H-z_1)+(H-z')]}}{[(H-z_1)+(H-z')]}\right]\\&\times\frac{(-1)}{4\pi}\left[\frac{e^{-ik(z_2-z')}}{(z_2-z')}+\frac{e^{-ik[(H-z_2)+(H-z')]}}{[(H-z_2)+(H-z')]}\right]\\&\times\left[rect(\frac{\omega}{2B})\right]^2\end{aligned} \quad (14)$$

After applying inverse Fourier transform time domain cross-correlation function can be expressed as follows-

$$\begin{aligned}c(\tau,z_1,z_2)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}C(\omega,z_1,z_2)e^{-i\omega\tau}d\omega\\&=\frac{1}{(2\pi)(4\pi)^2}\times\int_{-\infty}^{\infty}\left[\frac{e^{ik(z_1-z')}}{(z_1-z')}+\frac{e^{ik[(H-z_1)+(H-z')]}}{[(H-z_1)+(H-z')]}\right]\\&\times\left[\frac{e^{-ik(z_2-z')}}{(z_2-z')}+\frac{e^{-ik[(H-z_2)+(H-z')]}}{[(H-z_2)+(H-z')]}\right]\\&\times rect(\frac{\omega}{2B})\times e^{-i\omega\tau}d\omega\end{aligned}$$

As, $\left[rect(\frac{\omega}{2B})\right]^2=rect(\frac{\omega}{2B})$.

After the multiplication of two terms in the above integration $c(\tau,z_1,z_2)$ can be expressed as follows-

$$\begin{aligned}&c(\tau,z_1,z_2)\\&=\frac{1}{(2\pi)(4\pi)^2}\times\int_{-\infty}^{\infty}\Big[\frac{e^{ik(z_1-z_2)}}{(z_1-z')(z_2-z')}\\&+\frac{e^{ik(z_1+z_2-2H)}}{(z_1-z')[(H-z_2)+(H-z')]}\\&+\frac{e^{ik(2H-z_1-z_2)}}{[(H-z_1)+(H-z')](z_2-z')}\\&+\frac{e^{ik(z_2-z_1)}}{[(H-z_1)+(H-z')][(H-z_2)+(H-z')]}\Big]\\&\times rect(\frac{\omega}{2B})\times e^{-i\omega\tau}d\omega\end{aligned} \quad (15)$$

Wave-number k can be represented as $k=\omega/c$, where c is the speed of sound under water.

Finally after substituting value of k in (15) and doing some calculation time domain cross-correlation function can be expressed as follows-

$$\begin{aligned}&c(\tau,z_1,z_2)\\&=\frac{1}{(2\pi)(4\pi)^2}\times\int_{-\infty}^{\infty}\Big[\frac{e^{-i\omega[\tau-1/c(z_1-z_2)]}}{(z_1-z')(z_2-z')}\\&+\frac{e^{-i\omega[\tau-1/c(z_1+z_2-2H)]}}{(z_1-z')[(H-z_2)+(H-z')]}\\&+\frac{e^{-i\omega[\tau-1/c(2H-z_1-z_2)]}}{[(H-z_1)+(H-z')](z_2-z')}\\&+\frac{e^{-i\omega[\tau-1/c(z_2-z_1)]}}{[(H-z_1)+(H-z')][(H-z_2)+(H-z')]}\Big]\\&\times rect(\frac{\omega}{2B})d\omega\end{aligned} \quad (16)$$

We know inverse Fourier transform of rectangular function [13] is –

$$\int_{-\infty}^{\infty}rect(\frac{\omega}{2B})e^{-i\omega\tau}d\omega=2B\sin c(B\tau)$$

So time domain cross-correlation function can be expressed as follows-

$$
\begin{aligned}
&c(\tau, z_1, z_2) \\
&= \frac{1}{(2\pi)(4\pi)^2} \times \left[\frac{2B\sin c(B[\tau - \frac{1}{c}(z_1 - z_2)])}{(z_1 - z')(z_2 - z')} \right. \\
&+ \frac{2B\sin c(B[\tau - \frac{1}{c}(z_1 + z_2 - 2H)])}{(z_1 - z')[(H - z_2) + (H - z')]} \\
&+ \frac{2B\sin c(B[\tau - \frac{1}{c}(2H - z_1 - z_2)])}{[(H - z_1) + (H - z')](z_2 - z')} \\
&\left. + \frac{2B\sin c(B[\tau - \frac{1}{c}(z_2 - z_1)])}{[(H - z_1) + (H - z')][(H - z_2) + (H - z')]} \right]
\end{aligned} \tag{17}
$$

From the time domain cross-correlation function of noise fields recorded at two vertically separated sensors it is seen that four peaks are generated after correlation. Among them second and third sinc function gives the peaks due to correlation between direct and bottom reflected signals. These two peaks gives idea about bottom depth . From (17) it is clear that peaks in the correlation are directly proportional to bandwidth, B .

IV. Relation Between SNR and Bandwidth

SNR of the cross-correlation function can be defined as the ratio of the power of the peak generated by the cross-correlation of coherent wavefronts and power of the cross-correlation of uncorrelated background noises.

Let's suppose-

$C_{CN} =$ Cross-correlation due to Coherent noise signals

$C_{UCN} =$ Cross-correlation due to uncorrelated noise signals

So, $SNR = \frac{C_{CN}^p}{C_{UCN}^p}$

Where p in the superscript of C indicates power.

The time domain cross-correlation function in (17) is actually the cross-correlation function between coherent signals received at two sensors. Presence of only four sinc functions in (17) verifies that $C_{CN} = c(\tau, z_1, z_2)$.

From (17) it can be said that C_{CN} is directly proportional to the bandwidth of recorded noise field.

$$C_{CN} \propto B$$

Therefore power of the correlation peak is directly proportional to the square of bandwidth.

$$\text{So, } C_{CN}^p = c^p(\tau, z_1, z_2) = k_1 B^2 \tag{18}$$

Where, $\tau = \pm \frac{1}{c}(z_1 + z_2 - 2H)$ and $\pm \frac{1}{c}(z_1 - z_2)$, B is bandwidth and k_1 is a constant that can be calculated from (17).

We know covariance of uncorrelated random noises is zero. So cross-correlation of uncorrelated noises is equivalent to white noise. And according to the properties of the white noise, its power is directly proportional to the bandwidth.

$$C_{UCN}^p = k_2 B \tag{19}$$

Finally SNR of the cross-correlation of band limited background noises can be expressed as follows-

$$SNR = \frac{k_1 B^2}{k_2 B} = \frac{k_1}{k_2} B = kB \tag{20}$$

Where k is a constant. From (20) we can say that SNR of the cross-correlation of noise signals recorded at two sensors is directly proportional to bandwidth of recorded signal.

V. Relation between Resolution and Bandwidth

Resolution of the cross-correlation function can be defined as the width of correlation peak [2]. Here resolution of correlation was expressed as the full width of the main lobe of correlation peak. From the mathematical expression of the time domain cross-correlation in (17) it is clear that peaks in the correlation are generated due to presence of sinc function in the expression.

So width of correlation peak is dependent on the cut-off frequency of low pass filter, ω_0 [2].If cut-off frequency of low pass filter is ω_0, then half main lobe width of the correlation peak is $\frac{1}{2\omega_0}$ [2] in time unit.

So full width of main lobe of the correlation peak is $\frac{1}{\omega_0}$ in time unit.

In case of low pass filter ω_0 is equal to bandwidth. So bandwidth, $B = \omega_0$. So width of main lobe of correlation peak in meter [2],

$$R = \frac{c}{\omega_0} = \frac{c}{B} \tag{21}$$

Where c is the speed of sound .As we know reduction of width of correlation peak means increase in resolution. So resolution of correlation increases with the increase in bandwidth.

VI. Numerical Simulation and Results

In simulation 100,000 surface noise sources were considered to be distributed uniformly over a circular area whose diameter was 5km. Depth of sea bed was 10m and two sensors were placed vertically at 6m and 6.2m depth from the surface. The sea bed was considered as a rigid boundary hence after reflecting phase of incident signal is not changed [12]. Noises coming from different directions were recorded at two sensors at 48 KHz sampling rate. In the simulation 20 seconds of data was used and speed of sound under water was considered as 1500 m/s. Received signals at sensors consist of direct signals from sources and bottom reflected signals after reflecting the signals at sea bed. To find out the effect of bandwidth on cross-correlation function a Brick wall low pass filter was applied on the noise fields. Cross-correlation of the recorded noise fields for different bandwidth is shown in Fig. 2. As sensor separation was only 20cm, two centre peaks were very close to one another [2]. These two peaks are generated due to cross-correlation of direct paths and cross-correlation of reflected paths with itself received at two sensors. Two clearly visible leftmost and rightmost peaks are generated due to the cross-correlation between direct signals at first sensor and reflected signals at second sensor and cross-correlation between reflected signals at first sensor and direct signals are second sensors.

From Fig. 2 it is obvious that peak values of the correlation function are increased with the increase of bandwidth with respect to the random noise part of the correlation that satisfies the time domain cross-correlation function of (17). This random part of correlation function is generated due to the cross-correlation between uncorrelated signals. In this simulation SNR is defined as the square of the peak value of cross-correlation function that is produced due to cross-correlation of correlated signals and the variance of the random part of cross-correlation function that is produced due to cross-correlation of uncorrelated signals. Power of correlation peak that is responsible for depth estimation and variance of uncorrelated part of cross-correlation function with respect to bandwidth is shown in Fig. 3 which satisfies (18) and (19). SNR of the cross-correlation function with respect to the bandwidth of recorded noise fields is shown at Fig. 4(a) that clearly shows that SNR of cross-correlation function is directly proportional to the bandwidth of recorded signals at two sensors which satisfies the mathematics of (20).

Resolution of the cross-correlation function was defined as the full width of main lobe of the correlation peaks. Width of correlation peaks with respect to bandwidth of recorded signals is shown in Fig. 4(b) for both simulation and theory .This figure shows that width of the peaks of correlation decreases with the increase of bandwidth equivalently resolution of cross-correlation function is directly proportional to bandwidth of recorded noise fields which satisfies (21).

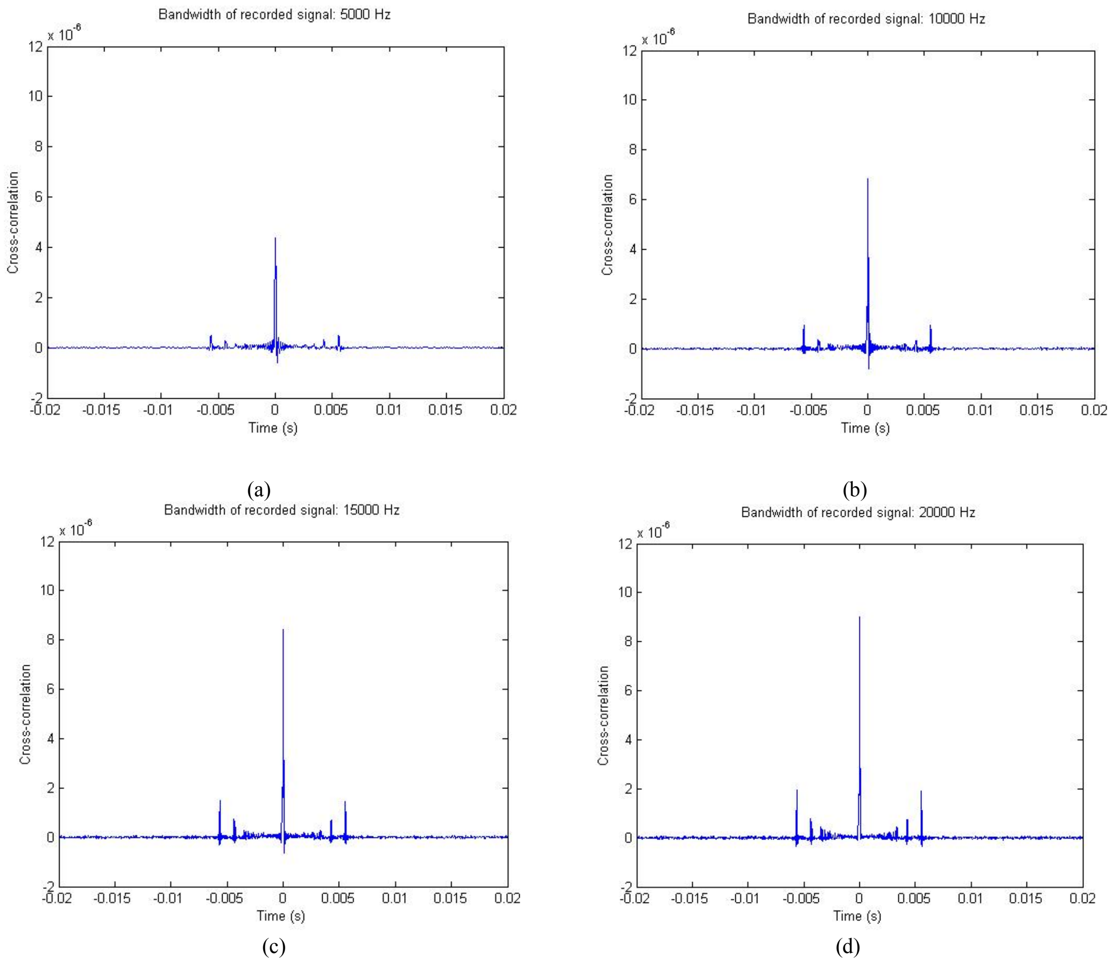
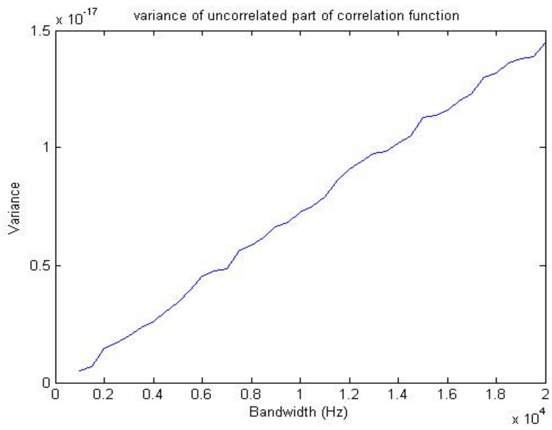
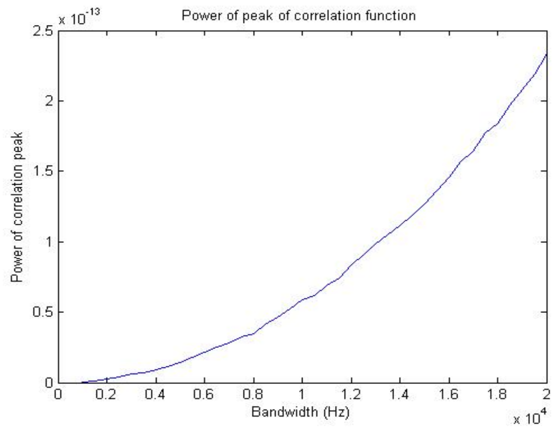


Figure 2.(a), (b) ,(c) and (d) corresponds to cross-correlation of noise fields recorded at two sensors having 5KHz , 10KHz , 15KHz and 20KHz bandwidth respectively.

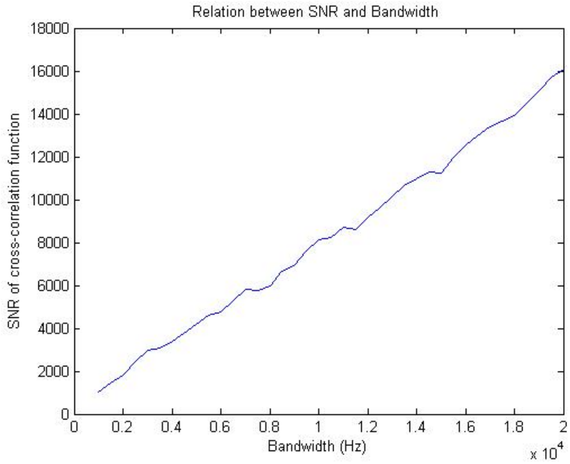


(a)

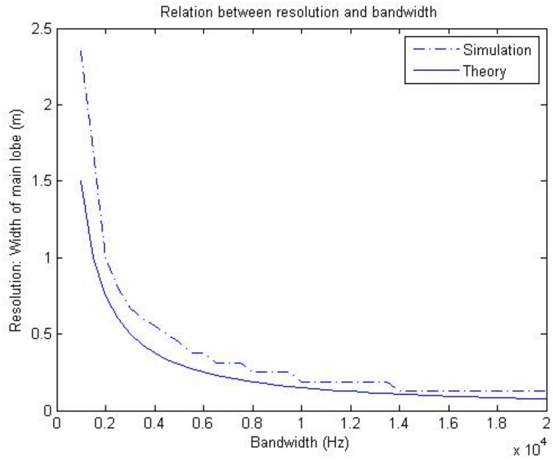


(b)

Figure 3.(a) corresponds to the variance of uncorrelated part of cross-correlation function with respect to bandwidth and (b) corresponds to the peak power of correlation function that is responsible for depth estimation with respect to bandwidth .



(a)



(b)

Figure 4.(a) and (b) corresponds to the SNR and resolution of correlation function with respect to bandwidth of noise fields.

VII. CONCLUSION

From the above discussion it can be said that SNR and resolution of cross-correlation of acoustic ambient noise fields recorded at two sensors are directly proportional to the bandwidth of noise fields. That means SNR and resolution increases with the increase in bandwidth. This results have been verified using both mathematics and simulation. Increase of SNR and resolution of cross-correlation function indicate that exact positions of the correlation peaks can be determined more accurately. We know positions of the leftmost and rightmost peaks is responsible for estimation of depth of sea bed [1] . So recording a large band noise fields at two sensors results better estimation of depth of sea bed and it's because of the increase of SNR and resolution of cross-correlation function of the recorded noise fields.

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