Robust Discrete Fourier Transform based Receivers for Continuous Phase Modulation

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***Abstract*— This paper proposes and investigates a discrete Fourier transform based receiver for continuous phase modulation. The architecture is robust and thus can be used even if the channel exhibits Doppler spreading. e.g. due to fading and relative motion as in an wideband underwater communication channel. The proposed receiver performance is compared with other widely used receivers such as the Viterbi algorithm based and matched filter based architecture.**

Keywords — continuous phase modulation, instantaneous frequency estimation, discrete Fourier transform (DFT), digital receivers, Viterbi algorithm.

I. Introduction

Continuous phase modulation is a both spectral and power efficient modulation scheme which has been widely used in GSM based mobile communications as well as in Bluetooth and DECT transmitter standards [1], [2]. Due to the constant envelope which makes them much more favorable than orthogonal frequency division multiplexing, cheap and nonlinear power amplifiers can be used in transmission instead of expensive, linear ones [3]. The optimum decoding of CPM often requires a maximum-likelihood sequence estimation (MLSE) by means of Viterbi algorithm (VA) [2]. However, the total number of states in trellis is equal to pM^{L-1} , where p is the denominator of modulation index, M is the number of modulation levels and L is the length of frequency response in symbol intervals, making the implementation of Viterbi demodulation very complicated. Several suboptimal demodulation methods have been proposed to reduce complexity at the expense of some performance. Osborne in [4] has discussed the use of matched filter method in coherent and noncoherent detection of CPM. The bit error probability of this method has been calculated by approximating the phase tree, and then using a reduced state Viterbi receiver [[2],chapter 8]. Since Laurent in [5] decomposed the CPM signal as a combination of linear pulse amplitude modulated (PAM) pulses, many research has aimed in designing reduced complexity receivers using few PAM pulses [6]. The differential sequence detection has also been reported to reduce the receiver complexity [7]. The methods mentioned above have achieved good performances and reduced complexity, in particular for binary transmissions, but the receivers are heavily dependent on modulation index h.

Signals having a constant amplitude but time varying instantaneous phase such as CPM signals are common in many other engineering disciplines, e.g. radar and sonar signal processing. Such signals belong to the broader category of frequency modulated (FM) signals and can be characterized using the instantaneous frequency (IF), which is the time derivative of the instantaneous phase [8]. The IF estimation is a well discussed topic over the last few decades and many efficient IF estimation techniques have been developed based on the peak location of the Discrete Fourier Transform (DFT) or the Wigner-Ville distribution (WVD) [8], [9]. A simpler, computationally efficient, autocorrelation based IF estimation algorithm has been proposed in [10]. The algorithm is recursive and ideally suitable for hardware implementation. CPM demodulation using IF estimation via LMS algorithm based frequency tracking has been discussed in [11]. The advantage of the use of IF estimation is that the resulting receiver is Doppler tolerant. Furthermore, in a composite CPM (i.e. muti-component CPM) environment having over-lapping spectra, the receiver design is only possible through IF tracking [11]. However, a comparison of the use of IF estimation and tracking with other methods in either performance or complexity, has not been reported in the literature. The aim of this paper is to investigate the merits and demerits of using the DFT and IF estimation in a CPM receiver design.

It is well known that the VA based receivers are complicated, in particular when the modulation index is an irrational number. Furthermore, if the modulation index is unknown at the receiver, and requires estimation, the VA algorithm performs rather poorly [12]. As such, if the channel exhibit Doppler spreading (e.g. due to fading and relative motion as in an underwater communication channel) the VA based demodulation would be quite unacceptable. In here, as an alternative, we investigate a CPM receiver architecture based on an orthogonal DFT filter bank as shown in Figure 1, where $\{f_0(t)...f_{Q-1}(t)\}$ are Q analog filters representing the DFT kernel functions. The proposed architecture has the following advantages and some of these would be elaborated in subsequent sections.

- The hardware is re-useable as it is independent of the modulation index, h and the number of modulation levels, M. Other receivers (e.g. based on Laurent's or Gram-Schmidt decompositions) need hardware redesign and reconfiguration. Furthermore, no additional analog

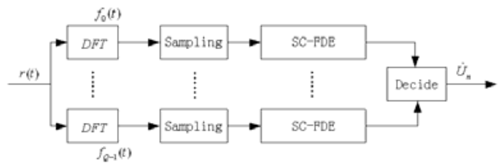

Fig. 1: Proposed CPM demodulator architecture based on the DFT.

filtering (e.g. for anti-aliasing) is necessary.

- As CPM signals have very low normalized bandwidth (< 5), only a few DFT filters are required. Thus, in conjunction with simple, efficient algorithms existing in literature, the IF could be efficiently estimated.
- By implementing receiver at an over-sampled rate, IF variation can be obtained for Doppler estimation (for evaluating h) and/or timing synchronization.
- The modulation index, h can be non-rational: this would ease the receiver design for multi-h transmission environments.
- Performs can be improved by resorting to matched filtering that can be simply implemented via the DFT.
- As with other methods, simple frequency domain equalization can be performed using cyclic prefixing, e.g. using the SC-FDE technique as noted in [13].

II. Instantaneous Frequency Estimation

Consider a constant amplitude frequency modulated (sampled) signal $z(n)$ in the presence of an additive complex white noise sequence $\{\xi(n)\}$ i.e.,

$$z(n) = \alpha e^{j\phi(n)} + \xi(n), \tag{1}$$

where $\phi(n)$ is the instantaneous phase of the signal and is the sampling index. The constant term α is the complex signal amplitude. The SNR for the signal in equation 1 is defined as $SNR = |\alpha|^2/\sigma^2$, where σ^2 is the noise variance. The instantaneous phase can be related to the signal instantaneous frequency $f(n)$, via the following expression.

$$f(n) = \frac{1}{2\pi}\frac{d\phi(n)}{dn} \tag{2}$$

When the IF is slowly varying, an efficient way to estimate $f(n)$ is to use the peak of the DFT, that also meets the Cramer-Rao bound (CRB) for frequency estimation [14]. When the signal behavior is non-stationary (e.g. the instantaneous frequency is time varying), it is well known that the peak search on the Wigner-Ville distribution (WVD) is the more appropriate IF estimation method [9]. A recursive IF estimation algorithm based on the autocorrelation of the signal has been recently reported in [10]. The method is extremely efficient for hardware implementation but has $3dB$ performance loss in comparison to the DFT based estimation. In the following section we show how IF estimation can be used in the design of CPM receivers.

III. Continuous Phase Modulation(CPM)

A CPM signal (analog) received in an additive, white, Gaussian (AWGN) channel can be expressed as

$$r(t) = s(t,\alpha) + v(t); s(t,\alpha) = \sqrt{\frac{E_s}{T}} e^{j\phi(t,\alpha)} \tag{3}$$

where T is the symbol duration and E_s is the energy per symbol, $v(t)$ is a Gaussian random process having zero mean and noise power spectral density N_0 . The transmitted information is contained in the signal phase as given by

$$\phi(t,\alpha) = 2\pi h \sum_n a_n \cdot q(t-nT); q(t) = \int_{-\infty}^{t} g(\tau)d\tau \tag{4}$$

where h is the modulation index, $q(t)$ is the phase response, $g(t)$ is the (instantaneous) frequency pulse and $\alpha(n)$ is an M-ary data symbol sequence selected from the integer set $\alpha \in \{\pm1, \pm3, ..., \pm(M-1)\}$. In the following discussion we consider 2 types of frequency pulse shapes: the rectangular (REC) and the raised cosine (RC) shape. These two frequency pulses are described respectively as,

$$g(t) = \begin{cases} 0 & t < 0 \\ \frac{0.5}{LT} & 0 \le t \le LT \\ 0 & t \ge LT \end{cases}$$
$$g(t) = \begin{cases} 0 & t < 0 \\ \frac{0.5}{LT}(1 - cos\frac{2\pi t}{LT}) & 0 \le t \le LT \\ 0 & t \ge LT \end{cases} \tag{5}$$

In equation 5, the positive integer L is the duration of the instantaneous frequency pulse in symbols. If $L = 1$ the transmission is a full response system, otherwise it is a partial response (see [2] for details). The use of the REC pulse result in CPFSK transmission while RC pulse (with $L = 3$) results in a scheme resembling the popular modulation techniques used in DECT and GSM standards [15].

As noted earlier, the optimum CPM demodulator is the maximum likelihood sequence estimator (MLSE), where the received signal $r(t)$ is observed over the entire time axis for all possible sequences. The MLSE can be efficiently implemented using the Viterbi algorithm, and the observation is usually limited to about 15 symbol durations [2]. In contrary to the traditional approach, in here we investigate the possibility of the detection of frequency pulse $g(t)$ by estimating the IF. Simulations results will be shown to evaluate symbol error performances in order to compare the IF based demodulation method with other conventional receivers.

Let the receiver be implemented in digital form using a sampling frequency of f_s, and the over-sampling rate is defined as $P = f_s T$. For the analysis, using 3, we first relate the signal SNR to the energy of transmission per bit, γ_b , which can be used to evaluate the symbol (or bit) error rate.

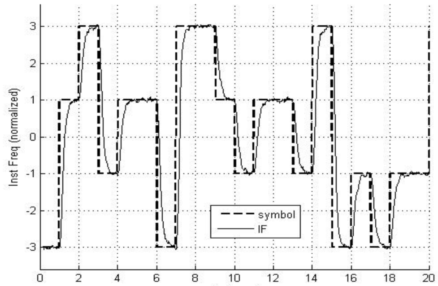

Fig. 2: Transmitted symbol and the estimated normalized IF for $SNR = 20dB$, Quaternary CPFSK signaling, $h = 0.9$

$$\gamma_b = \frac{E_b}{N_0} = \frac{E_s}{(log_2 M)N_0} = \frac{1}{log_2 M}\frac{E_s}{T}\frac{Tf_s}{N_0 f_s} = \frac{SNR \times P}{log_2 M} \tag{6}$$

As an example, figure 2 shows the estimated IF (normalized) when the frequency pulse is selected as a full response rectangular pulse with $M = 4$ (Quaternary CPFSK) and $h = 0.9$. The receiver is implemented by sampling at 100 times the symbol rate, i.e. $f_s = 100/T$ corresponding to an over-sampling rate, $P = f_s T = 100$. The IF is estimated using a sliding window of length $w = P/2 = 50$. The transmitted symbols are also shown in the figure.

Figure 3 shows the eye pattern resulting from the estimated IF signal at the receiver for two different (full and partial response) RC signaling schemes. The eye pattern can be used for timing synchronization at the receiver. Furthermore, the transmitted symbol can be detected by observing the centre of the eye resulting from the IF signal. Note that in the shown partial response signaling, the eye consists of 8 different tracks although the transmission symbol is binary. This is a result due to the fact that the pulse $g(t)$ spans 3 symbols. The symbol decision can be obtained by observing 3 successive eye patterns.

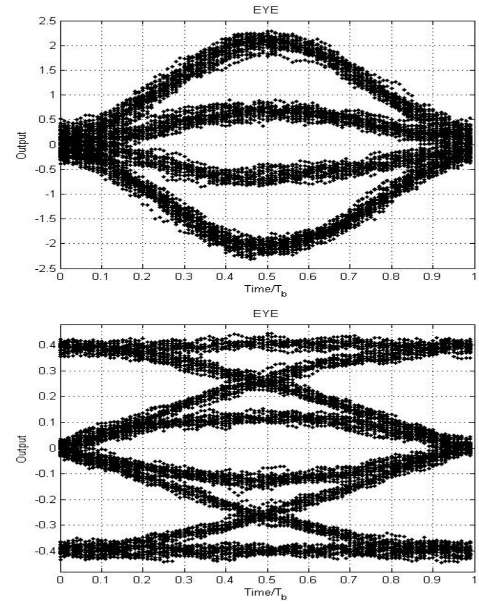

Fig. 3: The eye pattern (top) Quaternary $1 - RC$ signaling, $h = 0.8$, $SNR = 5dB$, (below) Binary $3 - RC$ signaling, $h = 0.8$, $SNR = 20dB$.

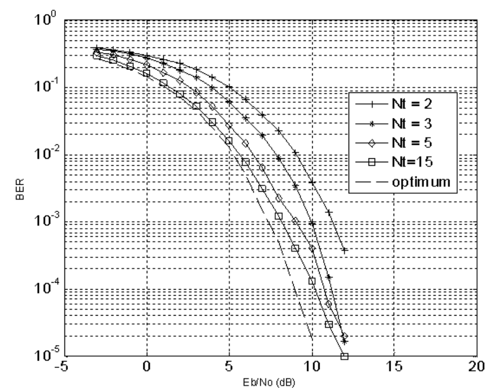

Fig. 4: The BER performance of Viterbi Algorithm based demodulator for MSK transmission, $h = 0.5$, for different observation intervals.The channel is an AWGN channel.

IV. Performance of viterbi algorithm based receivers

This section provides a symbol error analysis in order to compare the use of the IF estimation technique in a CPM demodulator with other decoders in use. At first, the performance of the VA based demodulator is evaluated with a binary CPFSK transmission with $h = 0.5$. This scheme is a widely used CPM modulation method which is also known as the MSK. Figure 4 shows the bit error rates (BER) obtained from simulation of the receiver implemented using a sample rate of $f_s = 16/T$. (At least 10 bit errors were collected for BER evaluation). The VA based algorithm was implemented with varying number observation intervals, Nt. It is noted in [2] that VA based decoding at best performs $0.8dB$ lower than the theoretical BER for binary-PSK (BPSK) transmission. This is shown as the '*optimum*' in the Figure. It can be seen that VA demodulator requires at least 15 observation intervals for satisfactory performance.

it is well known that the receiver complexity can be reduced by using pre-coding at the transmitter [6]. the performance would be then equal to the BPSK transmission and this could be achieved using 2 observation intervals, resulting in an extremely efficient receiver.

In the following we would evaluate the performance of the receivers in Figure 4 with a pre-coded receiver in a fading environment. Fading was simulated as Rayleigh fading with a maximum Doppler spread as $f_s = 0.01/T$. Figure 5 shows the BER performance of different receivers. It can be noted that the pre-coded receiver performance, although superior in an AWGN channel, suffers largely due to the fading. Hence, the VA based demodulators are the suitable choice in fading

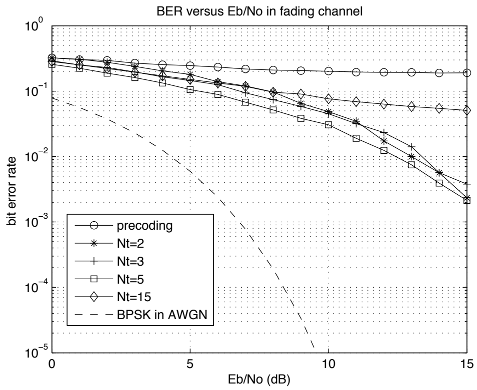

Fig. 5: The BER performance of viterbi algorithm based demodulator and pre-coded demodulator for MSK transmission, $h = 0.5$, for different observation intervals. The channel exhibits Rayleigh fading. Performance of a receiver with precoding in an AWGN channel is also shown.

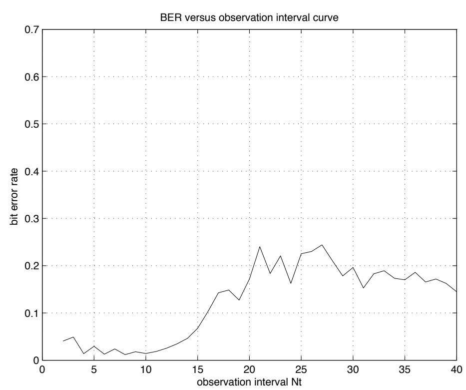

Fig. 6: The BER performance of viterbi algorithm based demodulator for MSK transmission, $h = 0.5$, for different observation intervals. the channel exhibits Rayleigh fading. $Eb/No = 10dB$.

environments.

Figure 6 shows the BER performance variation with the observation length when Eb/No is 10dB. It can be seen that increasing the observation length improves the performance only up to a certain value and beyond this the performance deteriorates. The knowledge of the Doppler spread would be useful in determining an optimum observation length. It is noted here that the VA based detection and matched filter (MF) detection are similar in performance as the VA is an efficient implementation of the MF detector. However, VA based receivers are more complicated than the MF receivers when the modulation index is not a ratio of integers. (The relative motion of the transmitter and receiver may cause non integer modulation indices at the receiver.) Therefore, it is concluded that the more robust receiver in the presence of fading and modulation index variations are the MF based detection.

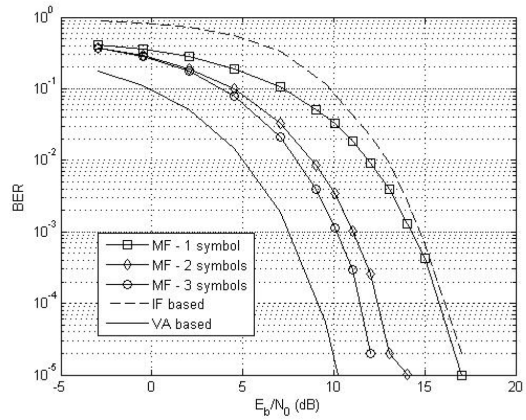

Fig. 7: The BER performance of IF and MF based demodulator using the architecture in Figure 1, for MSK transmission ($h = 0.5$).

V. Instantaneous Frequency Estimation and Matched Filter Based Receivers

Figure 7 shows the performance of the receiver based on the architecture shown in Figure 1. The DFT filter outputs are sampled at $f_s = 16/T$ and the IF was estimated using the DFT peak search. (Note: If equalization is required the signals are taken after the equalizer.) Efficient DFT peak search algorithms, e.g. as noted in [14] can be used for the IF estimation. (Alternatively, efficient correlation based algorithms can be used by directly sampling the received signal before the DFT filters [10].) The DFT filter outputs can also be used efficiently in implementing the demodulator as a matched filter (MF). In this approach, the DFT filter outputs are cross correlated with all possible outputs to find the highest match. Performance of MF based demodulation using 1, 2 and 3 symbol duration observations are shown in Figure 7. It can be seen that, at high E_b/N_0 values, the IF based detection performs almost similar to the MF detection with 1 symbol observations. The MF detection with 3 symbol durations performs well having $\sim 3dB$ higher Eb/No compared to the 'optimum' VA based detection.

The performance of a quaternary CPFSK transmission with $h = 0.5$ is shown in Figure 8. The detection performance of IF based and MF based implementations are shown. As before, the performance of IF based estimation is similar to MF using 1 symbol interval. The MF using 2 symbols has superior performance and only about $1dB$ higher Eb/No than the performance that can be achieved by the VA based demodulation in MSK transmission. Note that quaternary CPFSK with $h = 0.5$ has approximately the same bandwidth as the MSK transmission [2]. Thus, the quaternary transmission with 2 symbol MF demodulation using the architecture shown in Figure 1 presents an attractive CPM demodulation scheme that has good BER performance as well as a low complexity receiver. Further improvements for IF based estimation can be obtained with octal transmissions.(Note: Figure 8 shows symbol error rates, SER instead of BER. It is assumed

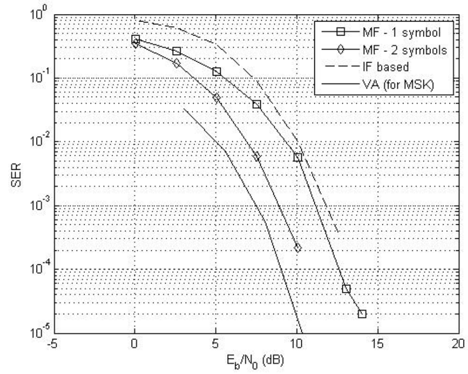

Fig. 8: The SER performance of IF and MF based demodulator using the architecture in Figure1, for quaternary CPFSK transmission, $h = 0.5$.

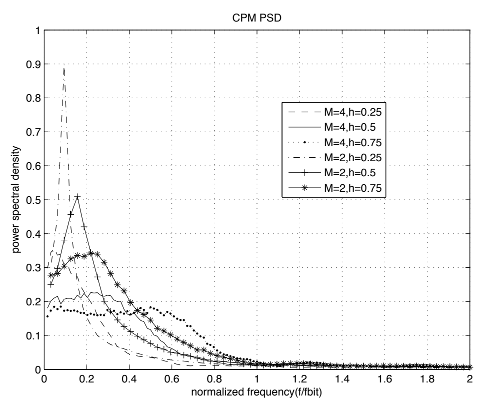

Fig. 9: Power spectral density for binary and quaternary MSK transmissions,for different values of modulation index, $h = 0.25, 0.5, 0.75$. Horizontal axis is normalized frequency fT.

here that that the SER is approximately same as BER due to use of Gray coding.) Figures 7 and 8 also show that the quaternary transmission has better performance for IF estimation than the binary transmission. This means that the timing synchronization in the quaternary transmission is easier than in the binary transmission. (It may be assumed that a BER value of 10^{-2} is sufficient for the keeping the eye open in Figure 3 so that timing synchronization could be achieved.)

As seen from figures 7 and 8, choosing quaternary transmission over binary transmission improves the IF based estimators, however, with a loss in bandwidth efficiency. Figure 9 shows a comparison between required bandwidth for binary and quaternary transmissions for different values of the modulation index.

VI. Conclusion

This paper has discussed the use of a discrete Fourier transform filter bank based architecture for implementing a continuous phase modulation receiver. Using the DFT the instantaneous frequency of the received signal can be estimated for the purpose of timing synchronization. The DFT architecture can also be used in implementing matched filter based symbol detection. Simulations have shown that the proposed demodulator performance reach that of a more complicated VA based CPM receiver for MSK, especially at quaternary (or higher) level modulations. It is possible to show that for proper IF estimation the energy per bit should be greater than $24/log_2 M$ [14]. As seen in the simulations, above these γ_b values, symbol detection using IF estimation reach the performance of a matched filter using 1 symbol duration for detection, as well as providing sufficient eye opening for timing synchronization. Therefore, symbol detection using architecture in Figure 1 is attractive at higher levels of modulations as the value of M is large. It is worth noting that higher levels of modulation provides superior performance for CPM transmission, but in such schemes the VA type demodulation becomes extremely complicated in implementation [2]. Only a limited number of reduce complexity receivers for higher levels of modulation are available in the literature [e.g. [16]], which justifies the approach used in this paper working independent of the modulation index,h. Furthermore, the IF based receivers have robust performance in fading environments.

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