Searching for Freak Waves from In-situ Buoy Measurements

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***Abstract*-There are several criteria to determine a freak wave in the ocean. These criteria are independent but the results are often not the same. This study tests on four presented criteria including the ratios of $H/H_{1/3}$ and H/Hs, the kurtosis *k*, and the groupiness factor GF, and add one more condition (Hs>1m) on the searching of freak wave in the database of data buoy during typhoon from 1998 to 2009. The water elevation is inversed from buoy measured acceleration signal by wavelet transform. It has still not enough knowledge to know how a freak wave looks like in the real world. The result shows more confident cases of freak waves by the joint assessment of 5 conditions. In this study, we find 18 freak wave cases within the 9122 time series during Typhoons.**

I. Introduction

Freak waves (also known as rogue waves) are relatively large and spontaneous ocean surface waves that are a threat even to large ships. They are not necessarily the biggest waves found at sea; they are, rather, surprisingly large waves for a given sea state. They have steep wave form, asymmetry and are highly non-linear. Freak wave has high impacts to navigation ships in the ocean, ocean engineering structure and also to people at the coast when it propagates to the coastal area. Occurrence of freak waves is so far largely unpredictable, and they can arise suddenly, even in moderate seas. Consequently a great deal of research is being performed to better understand how freak waves are formed. For example, it may be that nonlinear effects, multidirectional effects, focuing, or other phenomena can cause them. Recently, energy self-focusing by wave modulational instability is assumed to be the most possible cause. Many researches use Benjamin-Feir instability theory to simulate the freak waves [1][2][3]. Reference [4] derived the analytical solution of the probability function of occurrence of freak waves. It is not possible to forecast the height of the freak wave. However, the prediction of the occurrence probability of freak waves now looks possible.

Numerical simulations or laboratory experiments are popular for freak wave studies. Theoretical derivation is achieved under many assumptions, such as the surface elevation follows Gaussian distribution. The field data are therefore essential for better understanding of freak waves and validation of numerical or physical models. To study the freak wave, to define a freak wave is the first process. In oceanography, a definition of a freak wave is one that is 2.0 times higher than the significant wave height (SWH). Some studies presented that when a freak wave happens, the corresponding nonlinearity and wave groupiness of the sea-state are both higher. The Groupiness Factor (GF) is therefore also an index for looking for the freak waves. Recently, the kurtosis of the surface elevation time series becomes a popular index for freak wave definition. The occurrence probability of freak waves is significantly enhanced by the kurtosis increase as a consequent of the wave-wave interactions [4]. In addition, Reference [5] presented that for long crest lengths the statistics of freak waves is strongly non-Gaussian and the group length (or Benjamin–Feir index, BFI) is a good indicator of increased freak wave activity. Such many indexes using for freak wave definition raises confusion and brings uncertainty for relative studies. Second, field investigation of freak wave is relative few in this field. It is the motivation to search for the freak wave from the database of in-situ measurement, and discusses the difference of the results by various criteria. These criteria include H/H_s, $H/H_{1/3}$, κ (kurtosis), and GF (group factor).

II. Surface Elevation Derivation

Field measurements must be performed to increase the practical knowledge of waves. Apart from remote sensing devices, buoys and vessels are the only platforms to satisfy the wave measurement in deep water areas [6]. The data buoy is designed to measure wind waves and swell in any depth. The accelerometer installed inside the buoy measures its vertical acceleration, and the heave motion of a buoy. Ocean wave energy distribution is obtained from a data buoy by the directional wave spectrum theory. In additional to obtain the wave parameters from directional spectrum, the record of the sea surface elevation is also essential for studying several topics, such as the freak waves [7]. To obtain the sea surface elevation information from the acceleration record, the Fourier and inverse Fourier transform was used [8], in additional to the process of twice integration with the heave acceleration. However the stationary assumption is necessary for Fourier analysis. Reference [9] applied the wavelet transform to inverse to the surface elevation from buoy measured acceleration signal.

Based on the theory of one dimensional continuous wavelet transform, the acceleration signal can be broken into

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various wavelets which are scaled and shifted versions of a pre-chosen mother wavelet function. The acceleration signal $A_c(t)$ corresponds to the acceleration value of each time point t. The continuous wavelet transform $W_{Ac}(b,a)$ of acceleration signal $A_c(t)$ for a transformed mother wavelet $\psi_{b,a}$ is:

$$W_{Ac}(b,a) = \frac{1}{\sqrt{C_\psi}} \left\langle \psi_{b,a} \middle| A_c(t) \right\rangle \tag{1}$$

in which the scaling parameter a is related to the dilated frequency in the time domain. It gives all dilated versions of the mother wavelet the same energy, that is, it is the ratio of the size of the dilated wavelet to the size of the mother wavelet. The translation parameter b corresponds to the position of the wavelet as it shifts through the time domain. The function $\psi_{b,a}$ must have finite energy and satisfy the admissibility condition, which can be expressed by:

$$C_\psi = (2\pi) \int_{-\infty}^{\infty} \frac{\left|\hat{\psi}(\omega)\right|^2}{|\omega|} d\omega < \infty \tag{2}$$

where $\hat{\psi}$ is the Fourier space of function ψ; it means the function in frequency domain. ω is the frequency. (1) can also be expressed as:

$$W_{Ac}(b,a) = C_\psi^{-1/2} a^{-1/2} \int \psi_{b,a}^*(t) \cdot A_c(t) dt \tag{3}$$

$$\psi_{b,a}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \tag{4}$$

where ψ is the complex conjugate of the wavelet function ψ. The Morlet wavelet function is chosen in this study for detecting the wave information from the acceleration signal. The Morlet mother wavelet function and its function in the Fourier (spectral) space, as defined in (5) and (6).

$$\psi(t) = e^{(i\omega_0 t)} \cdot e^{(-0.5t^2)} \tag{5}$$

$$\hat{\psi}(\omega) = (2\pi)^{-0.5} \cdot e^{-0.5(\omega-\omega_0)^2} \tag{6}$$

where ω_0 is a constant that forces the admissibility condition to be satisfied. It was suggested to set up as 5.5. For calculating the sea surface elevation time series η(t) from the energy, the inverse wavelet transform is shown in (7).

$$\eta(t) = \frac{1}{C_\psi} \int_0^\infty \int_{-\infty}^{\infty} W_\eta(b,a) \cdot \psi_{b,a}(t) db \frac{da}{a^2} \tag{7}$$

where $W_\eta(b,a)$ is the continuous wavelet transform of the sea surface elevation time series η(t).

Figure 1 shows the result of validation. The relative differences between the theoretical values and the wavelet results were less, except for the locations near the margins of the wave record. These differences might come from the algorithm, in which we discretized the continuous wavelet transform for analyzing the digital wave records. This kind of error should be improved by increasing the sampling rate of the wave data. In addition, errors occur in the marginal areas because of the biased energy in the marginal area of the scalogram.

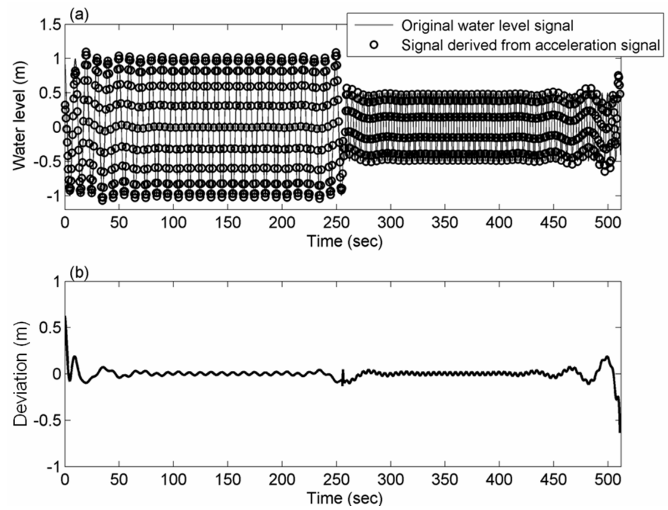

Figure 1. Validation of the present methodology to transfer the acceleration signal to surface elevation. (a) the validation; (b) the error distribution

III. THE DATASET

A coastal ocean monitoring network was established around the Taiwanese Waters. The instruments include data buoy, pile station, tide station, coastal weather station and X-Band radar system [10]. The data from Suao buoy (see Fig. 2) are used in this study due to the frequent announced coastal freak wave events at this area. For preliminary study, only the data during typhoons are adopted. There are totally 67 typhoon alarms by Central Weather Bureau from 2000 to 2009. The average data collection rate at Suao Buoy during typhoons is higher than 85%, making it a rich database for freak wave study. Ten minutes measurements per hour on the buoy motion is design by the sampling rate of 2Hz. Present methodology to transfer from the buoy acceleration signal to surface fluctuation data is applied. Since it is normally 2 to 4 days warning period of a typhoon, there are totally 9122 time series analyzed in this study.

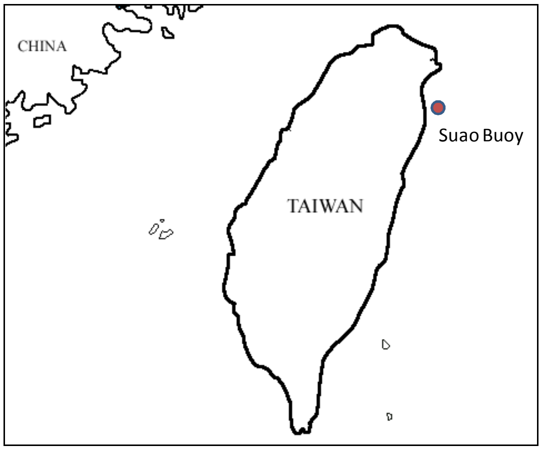

Figure 2 Location of Longdong Buoy

IV. RESULTS

A. *Freak waves culled by the wave height ratio*

Within the 9122 time series, there are 238 of them contain at least one freak wave in the time series, based on the criterion $H/H_{1/3}$>2.0. The significant wave height, $H_{1/3}$ is the average wave height of the one-third largest waves, derived by zero-up cross method. According to the validation result in section 2, the inversed surface elevation may have numerical error at the edges of the time series. The cases that freak waves are found in the edge of the time series are eliminated. It remains 179 freak wave cases found by this criterion. This is a large proportion (179/9122=2.0%) on the occurrence of freak waves in the open sea. However some of them are challenged the correctness by using subjective vision, such as the case in Figure 3. When the ratio is changed to 2.2, it reduces to only 75 cases satisfying the criterion (55 cases for $H/H_{1/3}$>2.4). This presents high sensitivity on the value of the ratio.

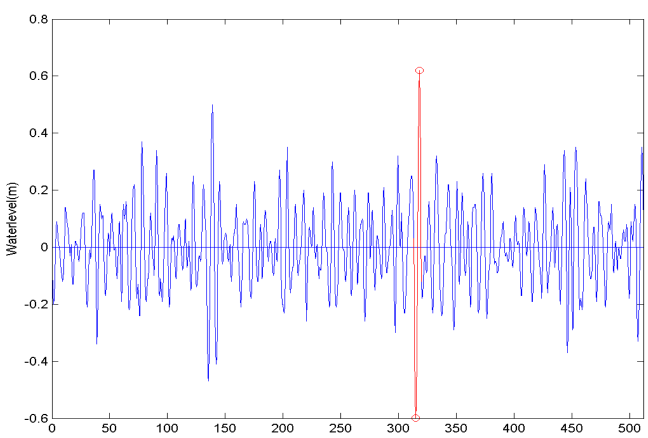

Figure 3 An incredulous freak wave case found by the criterion H/H1/3>2 (the ratio is 2.12 in this case. Suao Buoy/2000/9/8 00:00)

When we replace the significant wave height to it derived from spectral energy in the criterion of searching for freak wave, i.e. H/Hs. There are 111, 67 and 50 cases found by the ratio values 2.0, 2.2 and 2.4, respectively. Compare to the results above, the found freak wave cases are close when the ratio is high, as shown in Figure 4.

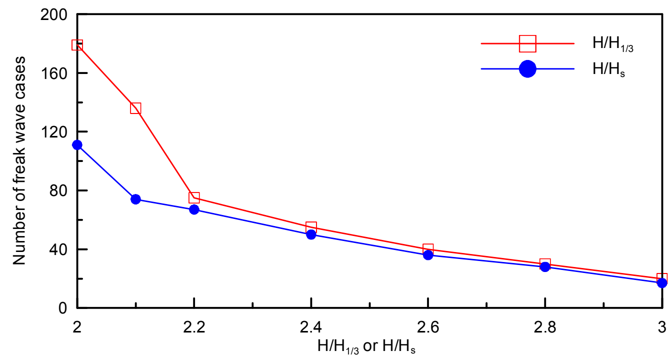

Figure 4 Freak wave cases culled by two different SWH ratios

B. *Freak waves culled by the kurtosis*

Kurtosis is a measure of whether the data are peaked or flat relative to a normal distribution. That is, data sets with high kurtosis tend to have a distinct peak near the mean, decline rather rapidly, and have heavy tails. Data sets with low kurtosis tend to have a flat top near the mean rather than a sharp peak. The kurtosis for a standard normal distribution is 3.0. Freak wave simulation is always base on the assumption of the surface elevation is according to the normal distribution. One of the important features of freak waves is that the crest height is much greater than the depth to the trough of the wave. Therefore the kurtosis of the time series containing a freak wave is higher than 3. For instance, the famous Draupner freak wave has a kurtosis of 4.0648 [11].

We test the kurtosis values 3.8, 4.0, 4.5 and 5.0 for the database in this study. There are 285, 113, 93, and 72 cases according. A case of the freak wave culled by high kurtosis is shown in Figure 5. However the statistical parameter kurtosis is sensitivity for the noise in the signal. Some cases are with high kurtosis but induced by the noise.

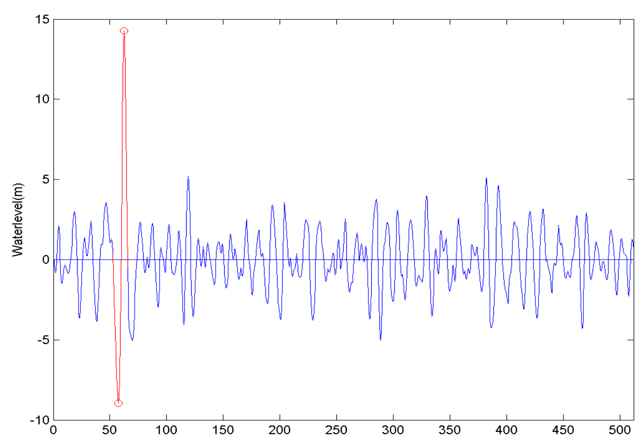

Figure 5 A freak wave case culled by the high kurtosis (Suao Buoy 2009/8/7 13:00. Kurtosis=9.76)

C. Freak waves culled by GF

Measurements of waves usually show a tendency of grouping between waves that is; high waves; often seem to be grouped together. Examination of the sea surface profile records indicates that wave heights are not uniform and they occur in successive groups of higher or lower waves. Though there are a number of different methods of measuring the degree of grouping in waves, only two are used extensively: the average run length, and the groupiness factor [12]. The groupiness factor (GF) is a measure of the variance in wave energy. The Smoothed Instantaneous Wave Energy History (SIWEH), which employs a Bartlett filter to smooth the wave energy, is used to determine the variance in wave energy. Since the measure of GF is in frequency domain, the GF values are sometime large due to large energy density of grouping wave induced by typhoons. It is culled 358 freak wave cases when applying the GF>1.0. It reduces to 70 and 38 cases according to GF>1.5 and GF>2.0. Figure 6 shows a freak wave case with GF=1.09.

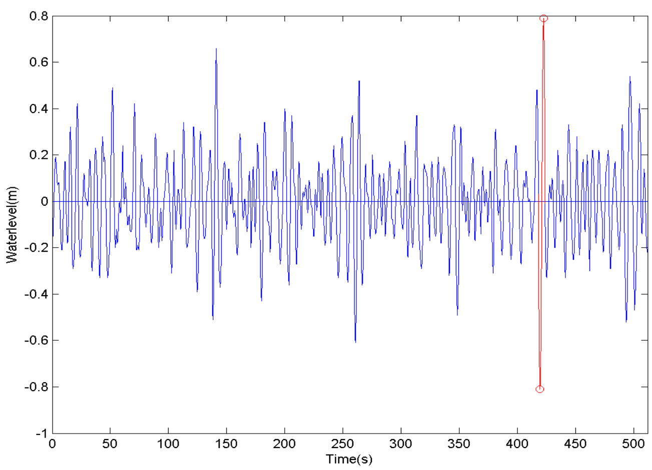

Figure 6 A freak wave case with high GF
(Suao Buoy: 2006/8/8 04:00. GF=1.09)

D. Discussion

Different criterion is used for culling freak wave in the field measurement or applied for simulation of freak wave in the numerical or laboratory experiments. Above results show that there is a quite large variance on the results with each criterion. In this study, it is proposed to have a joint assessment on the searching of freak wave in the database of field measurement. The idea is to apply above four indexes in order to cull more likely freak wave cases, i.e. $H/H_{1/3}>2$, $H/H_s>2$, $k>4.0$, and GF>1.5. It has been proofed an uncertain result because of the definition of SWH ($H_{1/3}$ or H_s) [11]. Therefore these two ratios are both included and set to larger than 2.0 according to the critical value of the derivation in wave theory. Kurtosis is a relative new index for deciding a freak wave. It is an experience to decide the critical kurtosis as 4.0 (The well-known Draupner freak wave has a kurtosis of 4.0648). For the groupiness factor (GF) in frequency domain, it is decide the critical value as 1.5 since the data used in this study are typhoon waves that having higher energy density than normal wind waves. In addition, we implement a new criterion that is the SWH should larger than 1.0m. This is to avoid the possible enhancement of the ratios because of small denominator. Furthermore we are more interested in the large waves. Figure 7 shows the results of jointly applying above criteria. It is found totally 18 cases satisfied with the five criteria. These 18 cases are shown in Figure 8.

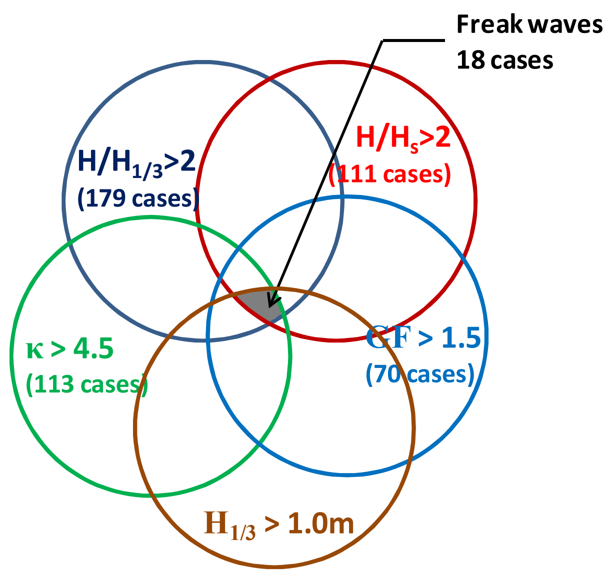

Figure 7 Joint assessments on the freak wave cases according to different criteria

V. SUMMARY

Time series of water elevation are derived from buoy measured acceleration signal during Typhoons from 1998 to 2009. There are 9122 time series tested in this study. With the criterion of $H/H_{1/3}>2$, we find 179 freak wave cases within them, as well as 111 cases by H/Hs>2, 113 cases by the kurtosis k>4.5, 70 cases by GF>1.5. This study defines that a freak wave should follow above four criteria, in additional to an external condition that Hs>1m. We find finally 18 freak wave cases in the data pool. The occurrence probability of freak wave is therefore 0.19% according to the database used in this study. We find the estimations of kurtosis and GF (by SIWEH) are sensitivity with the time series properties such as the variation and amplitude of surface fluctuation of water during typhoon are large inducing often large kurtosis and GF. This is why these two criteria are defined as the high value. This makes the well-known Draupner freak wave doesn't satisfy the definition in this study. It needs to consider this in the further study.

ACKNOWLEDGEMENTS

This study is supported by the National Science Council of Taiwan (NSC 98-2611-M-019-018). The raw data of the buoy are provided by Central Weather Bureau, Taiwan and the Coastal Ocean Monitoring Center of National Cheng Kung University. Thanks for Kelly Tzeng and Alice Chang for the data quality check.

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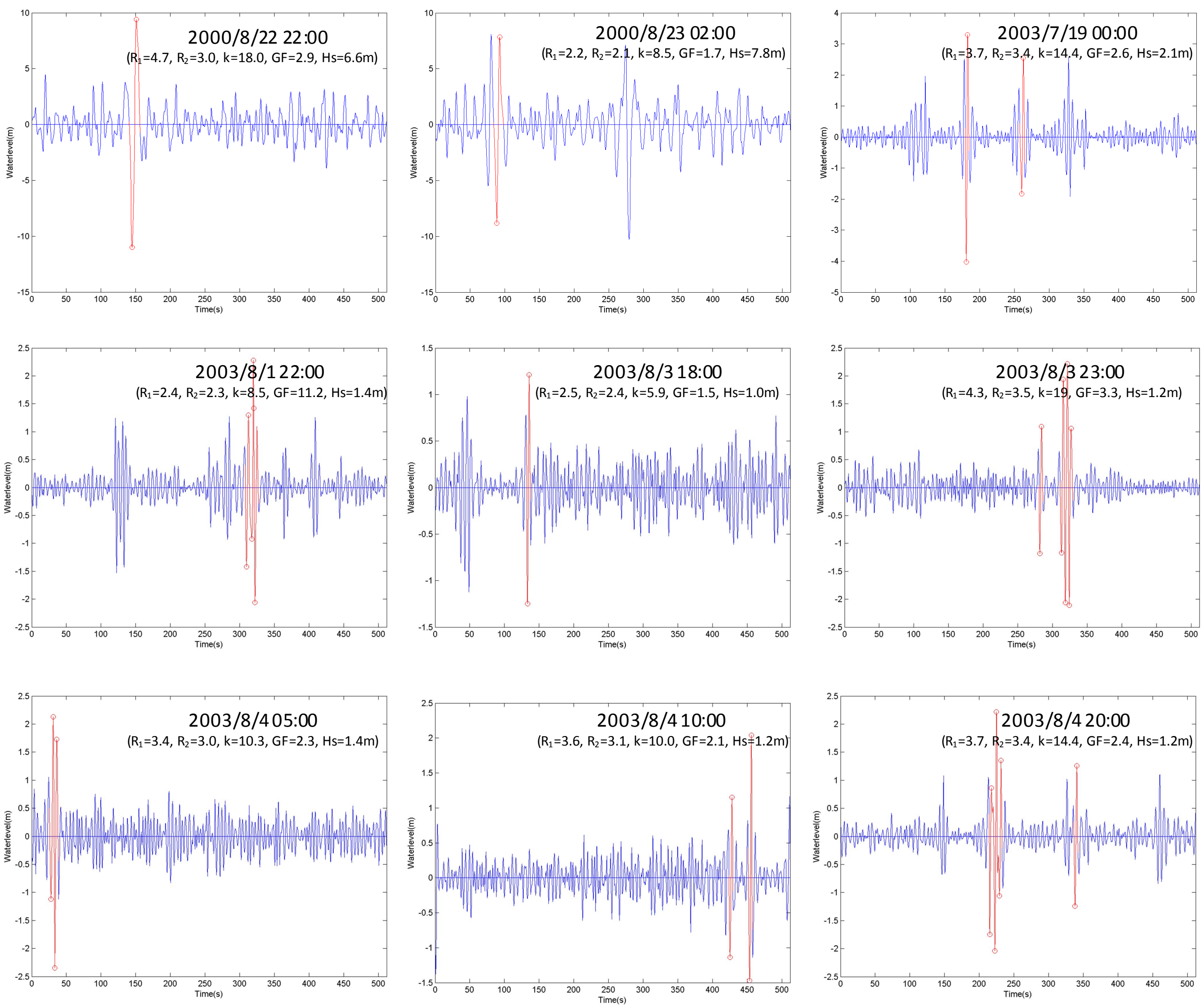

Figure 8. Culled freak wave cases in this study (R_1: $H/H_{1/3}$; R_2: H/Hs; k: kurtosis; GF: groupiness factor; Hs: Significant wave height)

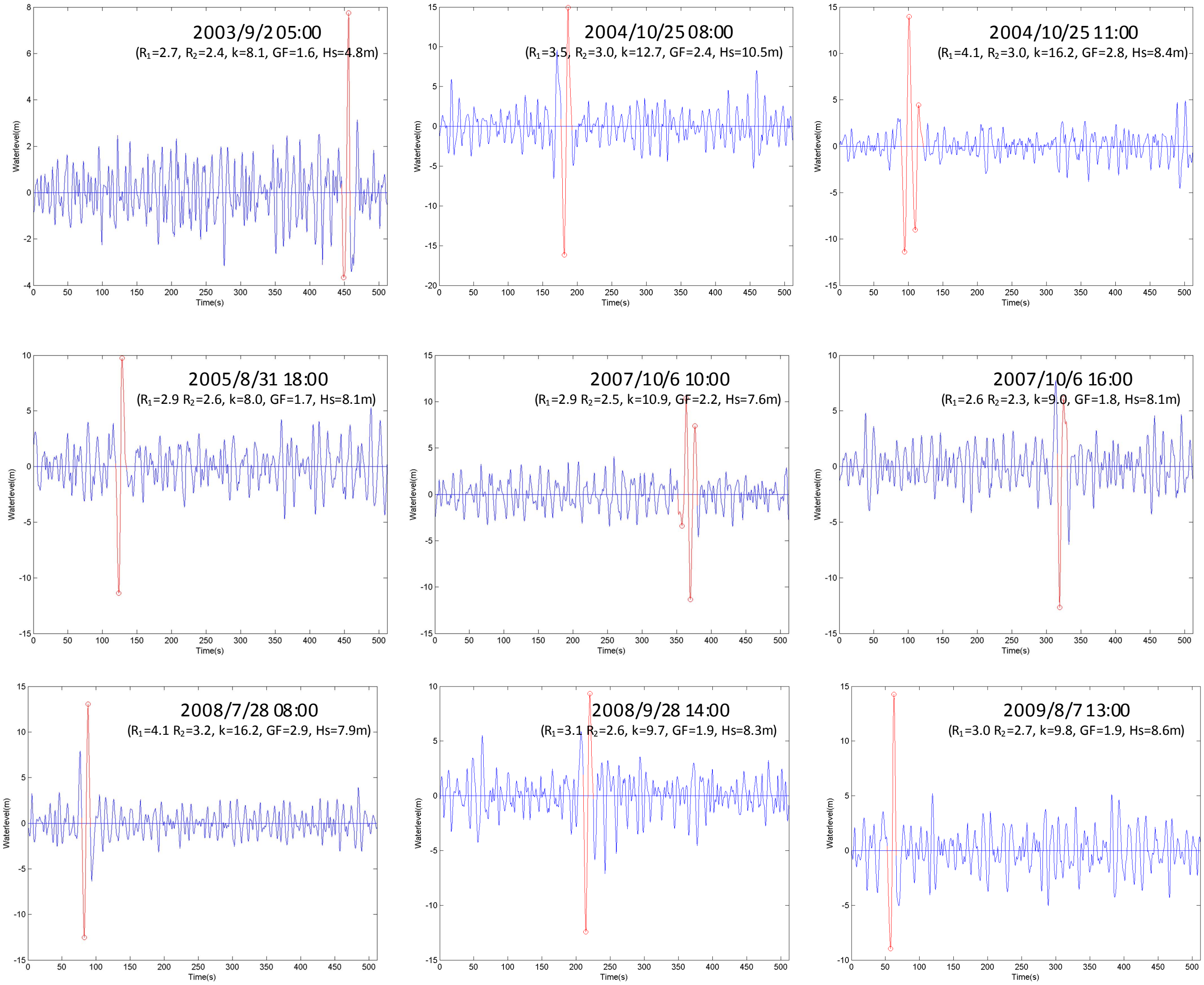

Figure 8 (cont.). Culled freak wave cases in this study. (R_1: $H/H_{1/3}$; R_2: H/Hs; k: kurtosis; GF: groupiness factor; Hs: Significant wave height)