An improvement to the pulse compression scheme

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***Abstract*—The pulse-compression is a technique mainly used in sonar, radar and echography to augment the range resolution as well as the signal to noise ratio. This is achieved using a matched-filtering of the received signal with the bandpass transmitted signal. Taking into account the main assumptions of the matched filter theory, the use of the bandpass transmitted pulse as matched filter's impulse response is only available if the useful signal is well known and if the noise is white, which is not the case in practice. For this reason, we propose to improve the classical pulse-compression technique using the stochastic matched filter, which ensures a maximization of the signal to noise ratio, when the useful signal is a realization of a random process and the disturbing signal a colored noise. Results obtained on synthetic and real data are proposed and discussed.**

***Index Terms*—Pulse-compression; matched filter; stochastic matched filter; de-noising process; linear chirp**

I. INTRODUCTION

In several domains of engineering technologies, such as telecommunication, sonar imaging [1], positioning systems, radar [2], [3], medical imaging [4]..., the main problem is to identify a transmitted useful pulse in a noise-corrupted received signal. A solution to this problem consists in using a modulated pulse (a chirp for example) for emission and a matched filter for reception [5]. Such a concept is known as pulse-compression. It has been shown that this technique enhances the range resolution as well as the signal to noise ratio. Nevertheless, the use of a matched filter, with the transmitted pulse as impulse response, supposes that the signal of interest is well known (i.e. deterministic) and that the disturbing terms are well represented by a white noise. But, if we consider an underwater context, it is well known that the disturbing signal has different origins:

- Anthropogenic noises (industrial and ship activities), biologic activities, ice cracking events, precipitations, sea state, bubbles and spray (surface agitation) ... ,
- Electronic noises (Johnson-Nyquist noise, Shot noise, Flicker noise, Burst noise ...),
- Noises produced by the ship which carry the receiver system (mechanical noise, fluid circulation and the noise produced by other carried electronics systems ...)

and, so, depending on the bandwidth used, the noise is not really white. Furthermore, the limited bandwidth of the transducers and the one of the receiver chain suggest that, even if ambient noise is considered as white, the resultant noise can not be considered as a realization of a white Gaussian noise. Moreover, because, on the one hand, of the accuracy of the electronic components used to construct the pulse and, on the other hand, of the signal deformation due to the propagation channel (Doppler effect, attenuation ...), the useful signal can not be considered as well known. For all these reasons, the classical use of the pulse-compression technique is under optimal. That is why, we propose in this paper to modify the classical pulse-compression scheme using a de-noising process. This de-noising process corresponds to the stochastic matched filtering method [6] and allows obtaining a great noise reduction. Indeed, this filtering method consists in expanding the received noise-corrupted signal into series of functions with uncorrelated random variables for decomposition coefficients. The basis functions are obtained by maximizing the signal to noise ratio, described here as a generalized Rayleigh quotient, where the kernels are the signal and the noise autocorrelation functions. So that, each basis function contributes to an improvement of the signal to noise ratio after processing.

After a brief presentation of the classical pulse-compression method, we present in a third section the context of this study. Next, in a fourth section, we present the stochastic matched filter in a de-noising context. This section is followed by the description of the new pulse-compression scheme. In the last part of this article, we confront this new concept to simulated and to real data coming from an IFREMER campaign.

II. THE PULSE-COMPRESSION

A. Principle

In sonar and radar, in order to augment the range resolution as well as the signal to noise ratio, it sounds wise to resort to pulse-compression. This one is achieved by modulating the transmitted pulse and then correlating the received signal with the transmitted pulse. That corresponds to a matched filtering of the received signal. Let $Z(t)$ be a noisy data corresponding to the superposition of a useful well known signal $s(t)$ and a noise $N(t)$:

$$Z(t) = s(t) + N(t). \tag{1}$$

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The aim of the matched filter theory is to determine the impulse response $h(t)$ of a filter ensuring a maximization of the signal to noise ratio $\rho(t)$ after process:

$$\rho(t) = \frac{|s * h(t)|^2}{E\{|N * h(t)|^2\}}, \tag{2}$$

where $*$ denotes the convolution product.
If the disturbing signal is a white noise, one can show that $\rho(t)$ is maximized when $h(t)$ is defined as follows:

$$h(t) = \frac{k}{\sigma_N^2} s^*(\tau - t), \tag{3}$$

where τ is an arbitrary delay and k a constant generally taken equal to σ_N^2. In these conditions, the signal to noise ratio is maximized, when the received signal $Z(t)$ is convolued with $h(t)$ equal to $s^*(-t)$; in other words, the cross-correlation of the received signal with the transmitted signal is computed. Let $Z^{PC}(t)$ be the pulse-compressed received data, it comes:

$$Z^{PC}(t) = Z(t) *_t s(t) = s^{PC}(t) + N^{PC}(t), \tag{4}$$

where $*_t$ denotes the correlation operator.
If the pulse corresponds to a linear chirp, having a duration T, with a unitary amplitude and linearly sweeping the frequency band Δf centered on carrier f_0, it can be shown that $s^{PC}(t)$ takes the following expression:

$$s^{PC}(t) = \sqrt{T}\left(1 - \frac{|t|}{T}\right) \operatorname{sinc}\left(\pi \Delta f \left(1 - \frac{|t|}{T}\right) t\right) e^{2i\pi f_0 t}, \tag{5}$$

for all $t \in [-T; T]$.
The maximum of $s^{PC}(t)$ is reached at 0. Around zero, $s^{PC}(t)$ behaves as the cardinal sinus term. The $-3\,dB$ temporal width of the main lobe is more or less equal to $1/\Delta f$. So a linear chirp having a duration T is compressed to a signal which duration is $1/\Delta f$.
Concerning $N^{PC}(t)$, one can show that the power of the noise $N^{PC}(t)$ is equal to the one of $N(t)$. Before and after the pulse compression step, the received signal energy remains the same. However, before pulse compression, the energy E_s is spread on the whole duration of the signal and it is located now in the main lobe of the cardinal sine, whose width is approximatively $1/\Delta f$. If we denote P_s the signal power before compression and $P_{s^{PC}}$ the signal power after compression, it comes:

$$E_s = P_s T = P_{s^{PC}}/\Delta f. \tag{6}$$

As a consequence, the signal power can be considered as being amplified by $T\Delta f$. Thus, the noise power being not changed by the compression, the gain on the signal to noise ratio of the compressed data is equal to $T\Delta f$.
All these advantages make the pulse-compression a very powerful tool when the noise is white and the useful signal well known.

B. Problem

If we consider an underwater context, it is well known that the disturbing signal has different origins, such as anthropogenic noises, electronic noises, noises produced by the ship which carry the receiver system ... and, so, depending on the bandwidth used, the noise is not really white. Furthermore, the limited bandwidth of the transducers and the one of the receiver chain suggest that, even if ambient noise is considered as white, the resultant noise can not be considered as a realization of a white Gaussian noise. Moreover, because, on the one hand, of the accuracy of the electronic components used to construct the pulse and, on the other hand, of the signal deformation due to the propagation channel (Doppler effect, attenuation ...), the useful signal can not be considered as well known.
As a first example, let consider two received signals $\boldsymbol{Z_1}$ and $\boldsymbol{Z_2}$. The first one corresponds to a linear chirp disturbed by a white Gaussian noise and the second one to the same pulse in the presence of a colored noise corresponding to some marine signatures. The two received signals have the same signal to noise ratio equal to $0\,dB$ for this experiment. The linear pulse has the following characteristics: $T = 1\,s$, $\Delta f = 1.5\,kHz$ and $f_0 = 1.25\,kHz$.

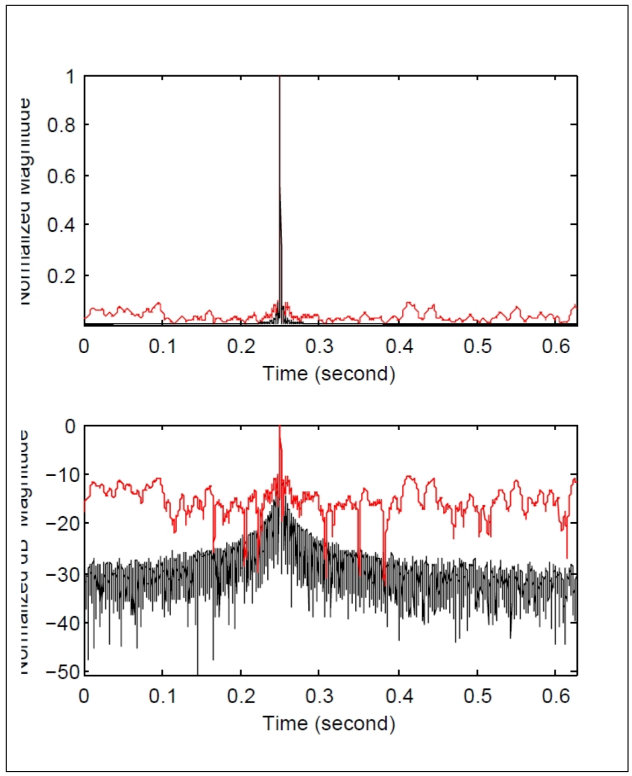

Fig. 1. *Pulse-compressed data for a RSB equal to* $0\,dB$ *for two different noise cases. Top: Normalized pulse-compressed data in linear magnitude; white noise case in black lines and colored noise case in red lines. Bottom: Normalized pulse-compressed data in* dB *magnitude*

The use of a pulse-compression on these signals gives the results presented on figure 1 in linear ($\boldsymbol{Z_1^{PC}}$ and $\boldsymbol{Z_2^{PC}}$) and dB magnitude ($10\log_{10}\boldsymbol{Z_1^{PC}}$ and $10\log_{10}\boldsymbol{Z_2^{PC}}$). An analysis of this figure clearly reveals a degradation of the performances in terms of signal to noise ratio gain, when the noise is not white. Let considerer, as second example, the case where the useful signal is not deterministic but a realization of a random process. To evaluate qualitatively the influence of the random nature of the useful signal, we have simulated a Doppler effect on it and we have computed the cross-correlation between this dopplerised pulse with the deterministic pulse (with no Doppler shift). Two different cases were studied:

- $0.2\,\%$ Doppler shift: $T = 1.002\,s$, $\Delta f = 1.4970\,kHz$, $f_0 = 1.2475\,kHz$;
- $-0.1\,\%$ Doppler shift: $T = 0.999\,s$, $\Delta f = 1.5015\,kHz$, $f_0 = 1.2513\,kHz$.

The results obtained are presented on figure 2. On the same graph, we have reported the pulse-compressed data obtained without Doppler effect.

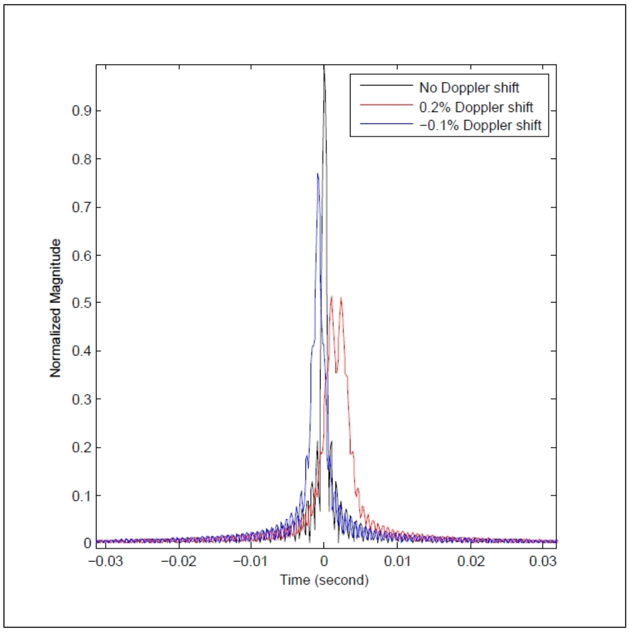

Fig. 2. *Pulse-compressed data in presence of a Doppler effect*

This experiment reveals a decrease of the SNR gain with the increase of the Doppler shift. Moreover the width of the main lobe increase with the Doppler shift, so that the range resolution is degraded.

Concerning the noise, one can use a whitening process before pulse-compression, so that the white noise assumption will be verified. Nevertheless, there is no method allowing to transform a random signal into a deterministic one. As consequence, the pulse-compression technique describes in this section appears under-optimal even though this technique remains robust in most cases.

III. Context of the Study

In order to locate its underwater devices (such as AUVs, ROVs and NAUTILE), IFREMER, the French research institute for the sea exploitation, uses two acoustic positioning systems based on the Ultra-Short-Base-Line principle: the first one to position down to $3000\,m$ under the sea surface and the second one to position down to $6000\,m$ under the sea surface. In those systems, a pulse is emitted from the submarine device, propagated on the water and received by the ship as described on figure 3.

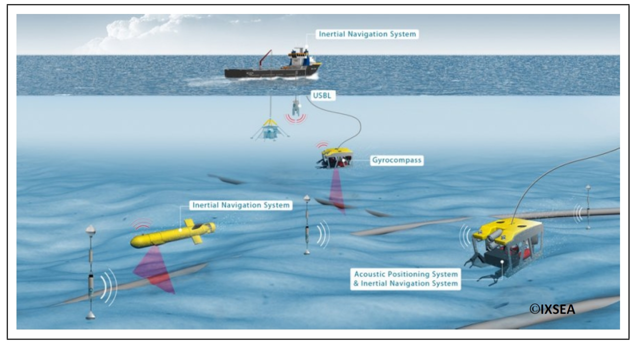

Fig. 3. *IFREMER positioning systems*

IFREMER develops a new acoustic positioning system. In this context, the purpose is to increase its performances in terms of range resolution and signal to noise ratio gain. The receivers used by these acoustic positioning systems exploit the pulse-compression technique to concentrate the useful signal energy in the time domain ensuring some good performances at the detection step. Nevertheless, while analyzing the measures obtained by the DIVACOU campaign, it appears that the received pulse greatly differs from the theoretical transmitted pulse (i.e. linear chirp: $T = 10.5\,ms$, $\Delta f = 8.6\,kHz$, $f_0 = 26\,kHz$) as shown in figures 4 and 5.

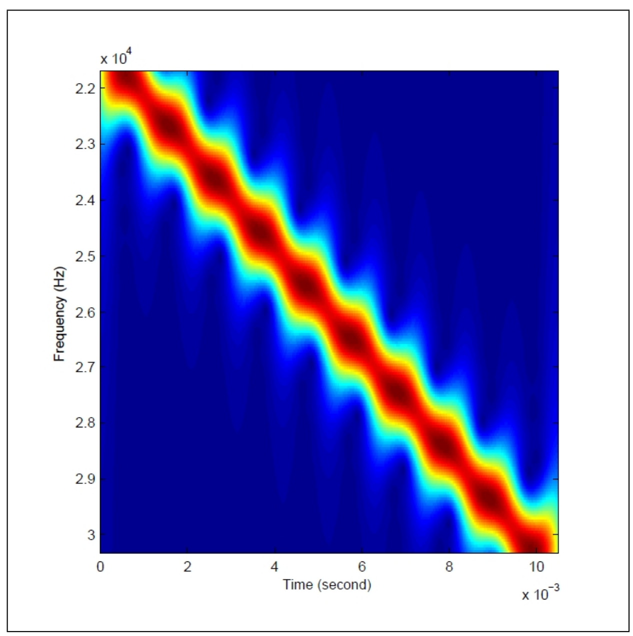

Fig. 4. *Time-frequency plane of the theoretical transmitted pulse*

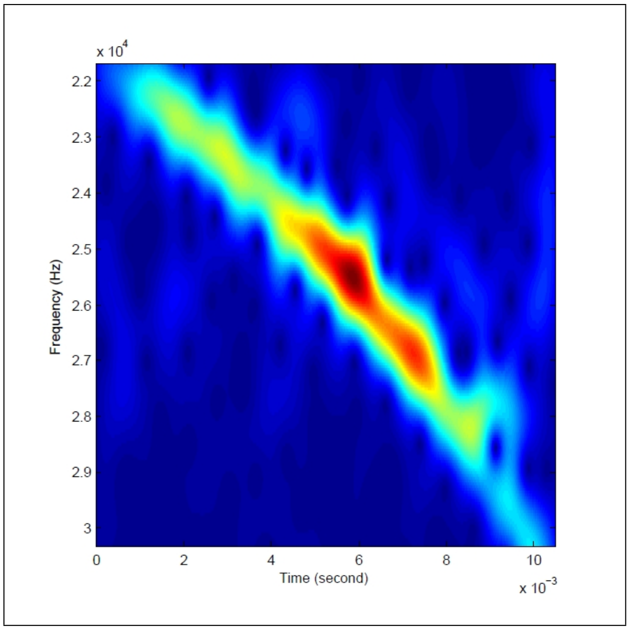

Fig. 5. *Time-frequency plane of the received pulse*

These differences may be explained by several physical phenomenons such as:

- the underwater propagation effects,
- Doppler effects caused by the relative movement of system carriers,
- electronics errors and uncertainties at emission and reception.

Such differences between emission and reception will induce some degradations on the pulse-compression performances in terms of range resolution and signal to noise ratio gain. As a consequence, the positioning acoustic systems will give under-optimal results. For these reasons, it appears of great interest to develop a new pulse-compression concept taking into account the random nature of the pulse and the coloration of the noise.

IV. The Stochastic Matched Filter (SMF)

The Stochastic Matched Filter theory, first proposed by Cavassilas in [6], allows to provide a second-moment characterization of a noise-corrupted signal in terms of uncorrelated random variables. Doing so, the noisy data could be described by its expansion into a weighted sum of known vectors by uncorrelated random variables. Depending on the choice of the basis vectors, some random variables are carrying more useful signal informations than noise. In these conditions, it is possible to highly improve the signal to noise ratio using the most appropriate basis vectors.

Let us consider a noise-corrupted signal $\boldsymbol{Z}$, made of M successive samples and corresponding to the superposition of a signal of interest $\boldsymbol{S}$ with a colored noise $\boldsymbol{N}$. If we consider the signal and noise variances, σ_S^2 and σ_N^2, we have:

$$\boldsymbol{Z} = \sigma_S \boldsymbol{S_0} + \sigma_N \boldsymbol{N_0}, \tag{7}$$

with $E\left\{\boldsymbol{S_0}^2\right\} = 1$ and $E\left\{\boldsymbol{N_0}^2\right\} = 1$. In the previous relation, reduced signals $\boldsymbol{S_0}$ and $\boldsymbol{N_0}$ are assumed to be independent, stationary and zero-meaned.

It is possible to expand noise-corrupted signal $\boldsymbol{Z}$ into a weighted sum of known vectors $\boldsymbol{\Psi_m}$ by uncorrelated random variables z_m, as follows:

$$\boldsymbol{Z} = \sum_{m=1}^{M} z_m \boldsymbol{\Psi_m}. \tag{8}$$

Considering a basis $\{\mathbf{\Phi_m}\}$ made up with M-dimensional deterministic vectors, random variables z_m can be expressed using the following scalar product:

$$z_m = \boldsymbol{Z}^T \boldsymbol{\Phi_m}. \tag{9}$$

So, the determination of these random variables depends on the choice of basis $\{\mathbf{\Phi_m}\}$. We will use the basis, which provides the uncorrelation of the random variables, i.e.:

$$E\left\{z_n z_m\right\} = E\left\{z_m^2\right\} \delta_{n,m}.$$

Under this condition, it is possible to show that $\mathbf{\Phi_m}$ are solution of the following generalized eigenvalues problem:

$$\mathbf{\Gamma_{S_0S_0}} \mathbf{\Phi_m} = \lambda_m \mathbf{\Gamma_{N_0N_0}} \mathbf{\Phi_m}, \tag{10}$$

where $\mathbf{\Gamma_{S_0S_0}}$ and $\mathbf{\Gamma_{N_0N_0}}$ represent signal and noise reduced covariances respectively.

One can show that λ_m equals $\frac{E\left\{s_m^2\right\}}{E\left\{\eta_m^2\right\}}$, where s_m and η_m correspond to the random variables coming from the signal and noise expansion respectively.

When the eigenvectors $\boldsymbol{\Phi_m}$ are normalized as follows:

$$\boldsymbol{\Phi_m}^T \boldsymbol{\Gamma_{N_0N_0}} \boldsymbol{\Phi_m} = 1, \tag{11}$$

it comes $E\left\{\eta_m^2\right\} = 1$ and $E\left\{s_m^2\right\} = \lambda_m$. In these conditions, the deterministic vectors $\boldsymbol{\Psi_m}$ of noise-corrupted signal expansion are given by:

$$\boldsymbol{\Psi_m} = \boldsymbol{\Gamma_{N_0N_0}} \boldsymbol{\Phi_m}. \tag{12}$$

In this context, it is possible to show that the signal to noise ratio ρ_m of component z_m corresponds to the native signal to noise ratio times eigenvalue λ_m:

$$\rho_m = \frac{\sigma_S^2}{\sigma_N^2} \lambda_m. \tag{13}$$

So, an approximation $\widetilde{\boldsymbol{S}}_{\boldsymbol{Q}}$ of the signal of interest (the filtered noise-corrupted signal) can be built by keeping only Q components associated to eigenvalues greater than a certain threshold, in any cases greater than one:

$$\widetilde{\mathbf{S}}_{\mathbf{Q}} = \sum_{m=1}^{Q} z_m \mathbf{\Psi_m}. \tag{14}$$

In these conditions, such a technique applied to an observation $\boldsymbol{Z}$ with an initial signal to noise ratio $\frac{S}{N}\big|_{\boldsymbol{Z}}$ substantially enhances the signal of interest perception (see [7]). Indeed, after processing, the signal to noise ratio $\frac{S}{N}\Big|_{\widetilde{\boldsymbol{S}}_{\boldsymbol{Q}}}$ becomes:

$$\left.\frac{S}{N}\right|_{\widetilde{\boldsymbol{S}}_{\boldsymbol{Q}}} = \left.\frac{S}{N}\right|_{\boldsymbol{Z}} \frac{\sum_{m=1}^{Q} \lambda_m \left\|\boldsymbol{\Psi_m}\right\|^2}{\sum_{m=1}^{Q} \left\|\boldsymbol{\Psi_m}\right\|^2}. \tag{15}$$

This last equation reveals that the higher the number Q, the smaller the signal to noise ratio gain.

The choice of truncature order Q depends on the criterion retained for the Stochastic Matched Filter application. When it is question of signal de-noising, a relevant criterion could lie in the minimal mean square error between the signal of interest $\boldsymbol{S}$ and its approximation $\widetilde{\boldsymbol{S}}_{\boldsymbol{Q}}$. Let $\epsilon(Q)$ be this mean square error:

$$\epsilon(Q) = E\left\{\left(\boldsymbol{S} - \widetilde{\boldsymbol{S}}_{\boldsymbol{Q}}\right)^T \left(\boldsymbol{S} - \widetilde{\boldsymbol{S}}_{\boldsymbol{Q}}\right)\right\}. \tag{16}$$

It has been shown [8] that $\epsilon(Q)$ is minimized when Q corresponds to the number of eigenvalues λ_m verifying the following inequality:

$$\lambda_m \left.\frac{S}{N}\right|_{\boldsymbol{Z}} > 1 \quad \Leftrightarrow \quad \rho_m > 1. \tag{17}$$

Consequently, if the noisy data presents a high enough signal to noise ratio, many $\boldsymbol{\Psi_m}$ will be considered for the filtering (so that $\widetilde{\boldsymbol{S}}_{\boldsymbol{Q}}$ tends to be equal to $\boldsymbol{Z}$), and in the opposite case, only a few number will be chosen.

V. A NEW PULSE-COMPRESSION SCHEME

A. Principle

Let K be the number of samples of the useful signal used at emission and let consider the K-dimensional vector $\boldsymbol{Z_k}$ corresponding to the data extracted from a window centered on index k of the noisy data, i.e.:

$$\boldsymbol{Z_k}^T = \left\{Z\left[k - \frac{K-1}{2}\right], \ldots, Z[k], \ldots, Z\left[k + \frac{K-1}{2}\right]\right\}.$$

As the SMF is applied using a sliding sub-window processing, only the sample located in the middle of the window is estimated, so that relation (14) becomes:

$$\widetilde{S}_{Q[k]}[k] = \sum_{m=1}^{Q[k]} z_{m,k} \Psi_m \left[\frac{K+1}{2}\right], \tag{18}$$

with:

$$z_{m,k} = \boldsymbol{Z_k}^T \boldsymbol{\Phi_m} \tag{19}$$

and where $Q[k]$ corresponds to the number of eigenvalues λ_m times the signal to noise ratio of window $\boldsymbol{Z_k}$ greater than one, i.e.:

$$\lambda_m \left.\frac{S}{N}\right|_{\boldsymbol{Z_k}} > 1. \tag{20}$$

To estimate the signal to noise ratio of window $\boldsymbol{Z_k}$, the signal power is directly computed from the window's data and the noise power is estimated on a part of the received data $\boldsymbol{Z}$, where no useful signal *a priori* occurs. This estimation is realized using the maximum likelihood principle.

Using relations (18) and (19), the estimation of the de-noised sample value is realized by a scalar product:

$$\widetilde{S}_{Q[k]}[k] = \boldsymbol{Z_k}^T \underbrace{\sum_{m=1}^{Q[k]} \Psi_m \left[\frac{K+1}{2}\right] \boldsymbol{\Phi_m}}_{\boldsymbol{h_{Q[k]}}}. \tag{21}$$

Using these values, a K-dimensional vector $\widetilde{\boldsymbol{S}}_{\boldsymbol{k}}$ made up with samples $\widetilde{S}_{Q[m]}[m]$, $\forall m \in \mathbb{N}$ such as $m \in \left[k - \frac{K-1}{2}; k + \frac{K-1}{2}\right]$ is constructed.

Let $\boldsymbol{C}$ be the useful signal used at emission. The pulse-compression principle consists in making the correlation between $\boldsymbol{C}$ and the received data $\boldsymbol{Z}$. As the use of the SMF modifies the nature of the data, it is necessary to modify the same way the useful signal $\boldsymbol{C}$.

Let $\widetilde{\boldsymbol{C}}_{\boldsymbol{q}}$ be the result obtained applying the SMF on $\boldsymbol{C}$, considering for the whole signal the same number q of eigenvalues, i.e.:

$$\widetilde{C}_q[k] = \boldsymbol{C_k^0}^T \boldsymbol{h_q} \quad \forall k = 1 \ldots K, \tag{22}$$

where:

$$\boldsymbol{C_k^0}^T = \left\{C^0\left[k - \frac{K-1}{2}\right], \ldots, C^0[k], \ldots, C^0\left[k + \frac{K-1}{2}\right]\right\},$$

$\boldsymbol{C^0}$ being a $(2K-1)$-dimensional vector constructed from $\boldsymbol{C}$ completed with zero on its edges.

Next the k^{th} pulse-compressed sample $Z^{\text{SMF-PC}}[k]$ is obtained using the scalar product between vector $\widetilde{\boldsymbol{S}}_{\boldsymbol{k}}$ and $\widetilde{\boldsymbol{C}}_{\boldsymbol{q=Q[k]}}$, i.e.:

$$Z^{\text{SMF-PC}}[k] = \widetilde{\boldsymbol{S}}_{\boldsymbol{k}}^T \widetilde{\boldsymbol{C}}_{\boldsymbol{q=Q[k]}}. \tag{23}$$

Doing so, first, window $\boldsymbol{Z_k}$ is de-noised using, for each sample and depending on the signal to noise ratio, the most appropriate number of eigenvalues and, next, the data is compressed using the pulse approximation obtained using the same number of eigenvalues than for the middle sample of the studied window.

B. Algorithm

The algorithm leading to the pulse-compression of the M-dimensional noisy data $\boldsymbol{Z}$, by the way of the stochastic extension of the matched filter, is presented below.

1) Computation or estimation of reduced covariances $\boldsymbol{\Gamma_{S_0S_0}}$ and $\boldsymbol{\Gamma_{N_0N_0}}$ of signal of interest and noise respectively.
2) Determination of eigenvectors $\boldsymbol{\Phi_m}$ by solving the generalized eigenvalue problem described in (10).
3) Normalization of eigenvectors $\boldsymbol{\Phi_m}$ in respect of relation (11).
4) Computation of basis vectors $\boldsymbol{\Psi_m}$ according to (12).
5) Determination of vectors $\boldsymbol{h_q}$, described in (21), for all q values (raised by the number of eigenvalues λ_m).

6) Computation of approximations $\widetilde{\boldsymbol{C}}_{\boldsymbol{q}}$ for all q values (see relation (22)).
7) Estimation of the noise power in an homogeneous area of noisy data $\boldsymbol{Z}$.
8) For $k = 1$ to M do:
   a) Sub-window $\boldsymbol{Z_k}$ data acquisition.
   b) Computation of the sub-window signal power.
   c) Estimation of the sub-window signal to noise ratio $\frac{S}{N}\big|_{\boldsymbol{Z_k}}$.
   d) Determination of $Q[k]$ according to (20).
   e) $\widetilde{S}_{Q[k]}[k]$ computation (see relation (21)) and construction of vector $\widetilde{\boldsymbol{S}}_{\boldsymbol{k}}$.
   f) When the number of elements in $\widetilde{\boldsymbol{S}}_{\boldsymbol{k}}$ is equal to K compute $Z^{\text{SMF-PC}}[k]$ according to relation (23).

VI. Experimentations

In this section, we present results obtained on simulated and real signals. We compare the results obtained applying the SMF based pulse-compression scheme with those obtained using the classical pulse-compression.

A. Simulated data

To illustrate this new SMF based pulse-compression scheme, let us consider as transmitted pulse a linear chirp (duration: $10.5\,ms$, central frequency: $26\,kHz$, bandwidth: $8.6\,kHz$). The simulated received data $\boldsymbol{Z}$ corresponds to an additive mix of the useful signal $\boldsymbol{S}$ (the pulse) and a colored noise $\boldsymbol{N}$ corresponding to some marine signatures coming from an IFREMER campaign (the DIVACOU campaign, September 2009). In these conditions, the received pulse appears $0.16\,ms$ after its emission. In order to control the signal to noise ratio, we introduce a parameter g as follows:

$$\boldsymbol{Z} = \boldsymbol{S} + g\boldsymbol{N}.$$

The signal to noise ratio of the received data corresponds to the ratio between the signal and the noise powers in the same spectral bandwidth, i.e.:

$$\frac{\displaystyle\int_{\Delta\nu} \gamma_{SS}(\nu)d\nu}{\displaystyle\int_{\Delta\nu} g^2\gamma_{NN}(\nu)d\nu},$$

where $\Delta\nu$ corresponds to the intersection of the signal and noise spectral bandwidths and with $\gamma_{SS}(\nu)$ and $\gamma_{NN}(\nu)$ as signal and noise power spectral densities (see figures 6 and 7). While varying the control parameter g, several realizations of the observation were built, with the SNR taking values between $-5\,dB$ and $10\,dB$.

The received signal is filtered by an anti-aliasing filter with a cutoff frequency equal to $32.5\,kHz$ and is sampled using a $75\,kHz$ sampling frequency. Doing so the number of samples to describe the pulse is equal to 788 and will correspond to the K value afterwards.

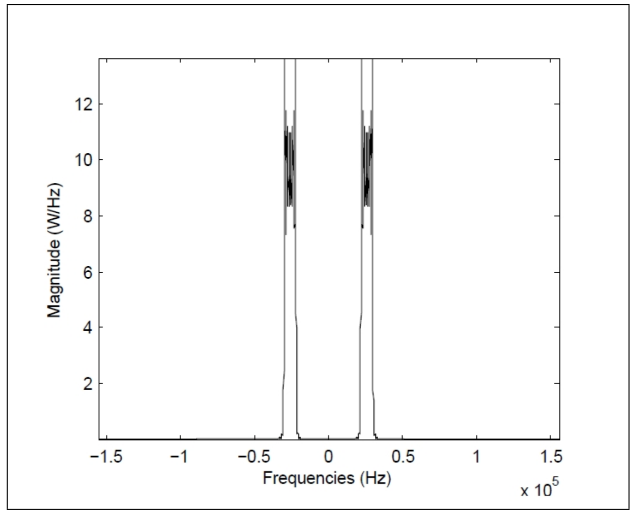

Fig. 6. *Signal Power Spectral Density in a $0\,dB$ SNR case*

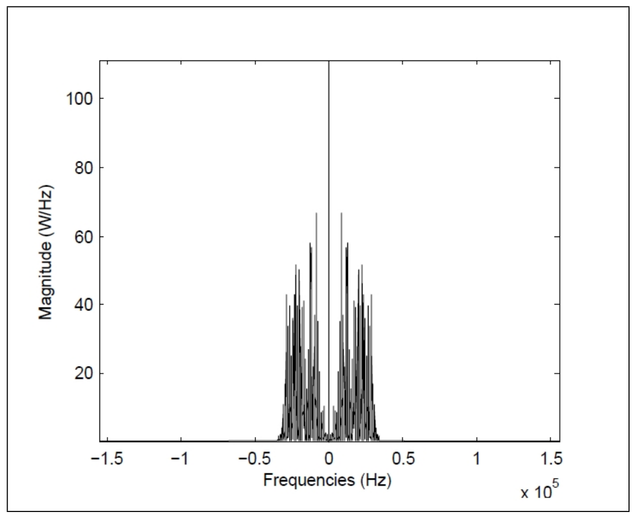

Fig. 7. *Noise Power Spectral Density in a $0\,dB$ SNR case*

The first experiment is presented on figure 8 and corresponds to a $0\,dB$ signal to noise ratio case.

The stochastic matched filtering method requires the *a priori* knowledge of the signal and of the noise reduced covariances (see relation 10). Concerning the noise one, we have estimated it directly from signals coming from the same IFREMER campaign. Applying a sliding sub-window processing, we have extracted several realizations of the noise. For each realization, we have computed its autocorrelation; averaging these autocorrelations conduces to the estimation of the noise autocorrelation, which permits the reduced noise covariance construction. The signal covariance has been computed considering an analytical model to describe the pulse. Solving the generalized eigenvalue problem (10) conduces to the determination of the eigen-elements (λ_m, $\boldsymbol{\Phi_m}$). Applying the algorithm described in section V allows to realize the pulse-compression of the received data. The result obtained using the SMF based pulse-compression scheme is presented on figure 8 in red lines, in linear magnitude ($\boldsymbol{Z}^{\text{SMF-PC}}$) and in dB-magnitude ($10\log_{10}\boldsymbol{Z}^{\text{SMF-PC}}$). In order to quantify the relevance of our process compared to the classical approach, the pulse-compressed data obtained by the use of the matched filter theory are presented on the same graphs.

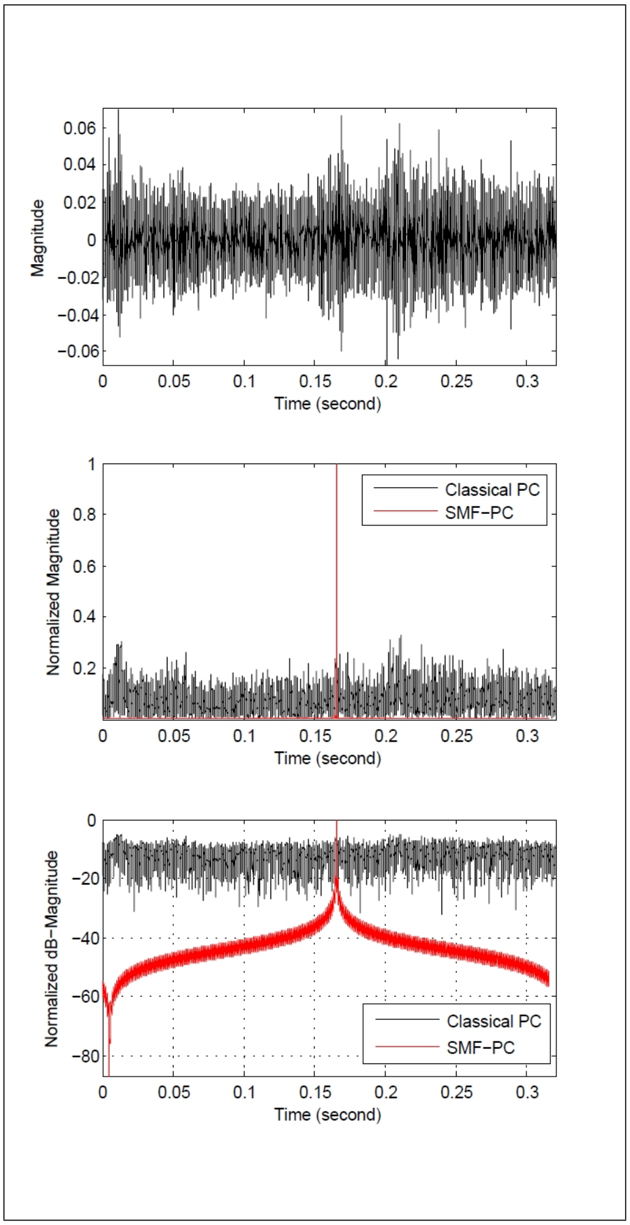

Fig. 8. *Pulse-compression in a $0\,dB$ SNR case. Top: Simulated received signal sampled at $75\,kHz$, the noise comes from an Ifremer campaign and the useful signal corresponds to a linear chirp (duration: $10.5\,ms$, bandwidth: $8.6\,kHz$, central frequency: $26\,kHz$) which happens near the 0.16^{th} second; the signal power has been adjusted in order to obtain a SNR equal to $0\,dB$. Middle: Normalized pulse-compressed data obtained applying the classical pulse-compression (classical PC) in black lines and using the new SMF based pulse-compression scheme (SMF-PC) in red lines; in both cases, a peak near the 0.16^{th} second reveals the useful signal presence, the main difference between the two methods being the noise level. Bottom: Normalized pulse-compressed data in dB magnitude; the SMF-PC method allows to obtain an average noise level $30\,dB$ lower than using the classical approach.*

These results show, on the one hand, that the new process allows to locate the received pulse the same way than using the classical approach and, on the other hand, a far better noise rejection with an average noise level $30\,dB$ lower than using the classical pulse-compression. Figure 9 shows the pulse-compressed main-lobes, extracted from figure 8. An analysis of these graphs reveals that the main-lobes have the same width, which means that the two methods reach the same resolution. So, this new method allows to far reduce the false alarm probability induced by the classical pulse-compression, while keeping all its advantages.

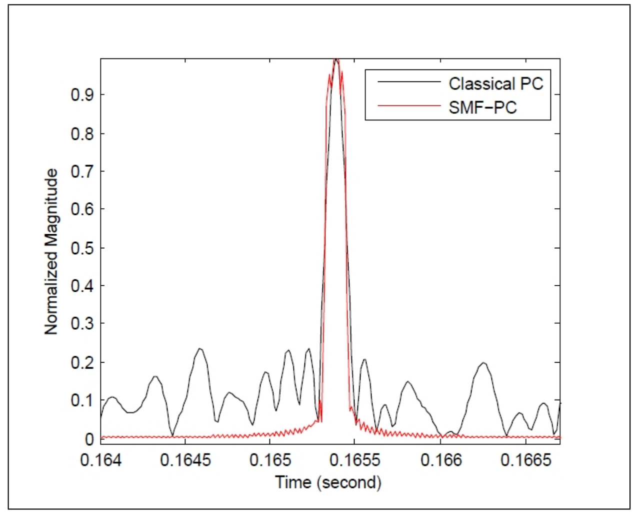

Fig. 9. *Detail of the figure 8 pointing out the similar characteristics of the pulse-compressed peak.*

The second experiment concerns a $5\,dB$ SNR case. Furthermore, in order to take into account some possible deteriorations of the pulse, linked to the Doppler effect, the transmission channel, the electronic accuracy ..., the useful signal has been modified according to a non-realistic Doppler shift of $5\,\%$ to evaluate the robustness of the proposed method. In these conditions, the useful signal characteristics are:

$$\left\{\begin{array}{l}\text{Duration: } T = 10.5e^{-3}\times(1+5\%) = 11.1\,ms\\ \text{Central frequency: } f_0 = 26e^{3}\times(1-5\%) = 24.7\,kHz\\ \text{Bandwidth: } \Delta f = 8.6e^{3}\times(1-5\%) = 8.17\,kHz\end{array}\right. .$$

As for the previous experiment, the results show the good performances of the SMF-PC: the pulse-compressed useful signal presents the same main-lobe width (see figure 11) and the average noise level is $40\,dB$ lower than using the classical pulse-compression approach.

In order to quantify the robustness of the SMF based pulse-compression scheme for several SNR and for different Doppler shift cases, we have built several realizations of the received signal with the SNR taking values between $-5\,dB$ and $10\,dB$ (with an increment of $0.1\,dB$) and with a Doppler shift taking values between $-5\,\%$ and $5\,\%$. For each realization, the classical pulse-compression and the SMF based pulse-compression method have been applied. Next for each result, the main-lobe maximum amplitude value (corresponding to the useful signal) has been collected. Let $\sigma_S^2\big|_{\text{SMF-PC}}$ and $\sigma_S^2\big|_{\text{PC}}$ be the values obtained for the SMF based pulse-compression scheme and the classical pulse-compression respectively. We have proceed the same way for noise level maximum amplitude value. Let $\sigma_N^2|_{SMF\text{-}PC}$ and $\sigma_N^2|_{PC}$ be the values obtained.

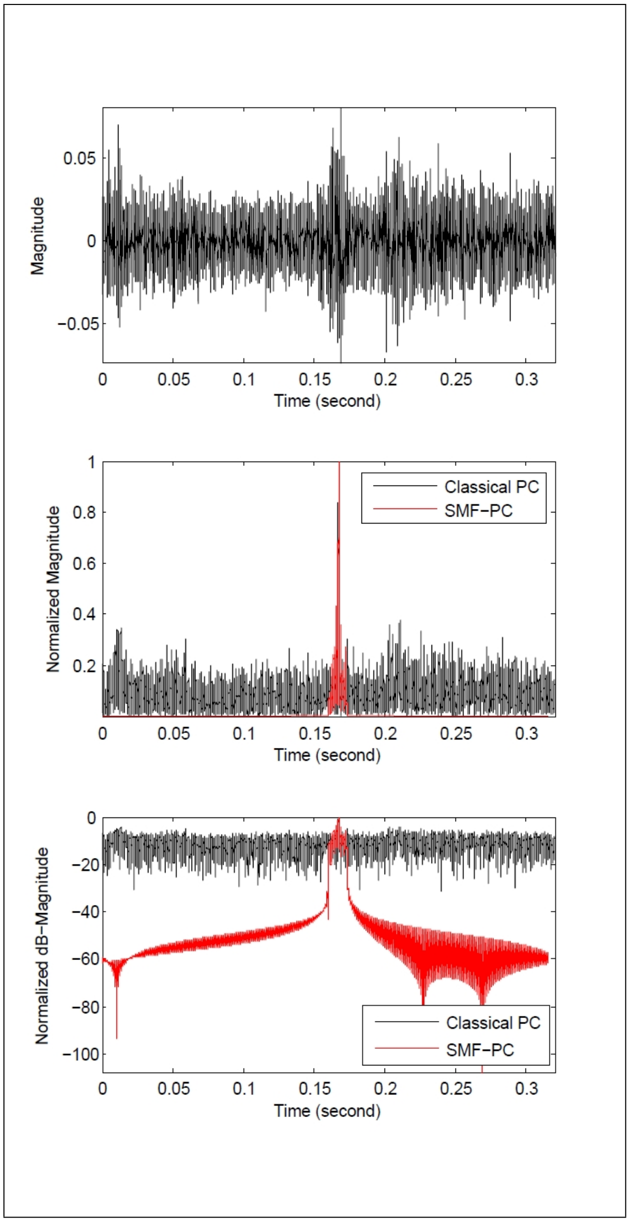

Fig. 10. *Pulse-compression in a $5\,dB$ SNR case with a non-realistic Doppler shift equal to $-5\,\%$. Top: Simulated received signal sampled at $75\,kHz$, the noise is the same than for figure 8 and the useful signal corresponds to a linear chirp (duration: $11.1\,ms$, modulation on $8.17\,kHz$, carrier frequency: $24.7\,kHz$) which happens near the 0.16^{th} second; the signal power has been adjusted in order to obtain a SNR equal to $5\,dB$. Middle: Normalized pulse-compressed data obtained applying the classical PC in black lines and using the SMF-PC in red lines; in both cases, a peak near the 0.16^{th} second reveals the useful signal presence, the main difference between the two methods being the noise level. Bottom: Normalized pulse-compressed data in dB magnitude; the SMF-PC method allows to obtain an average noise level $40\,dB$ lower than using the classical approach.*

Using these values, we have been able to compute the SNR gain on the pulse-compressed data using the SMF-CP in comparison with the classical CP for several Doppler shifts and for different received data SNR cases:

$$\text{SNR gain} = 10\log_{10}\left(\frac{\sigma_S^2|_{SMF\text{-}PC}}{\sigma_N^2|_{SMF\text{-}PC}}\right) - 10\log_{10}\left(\frac{\sigma_S^2|_{PC}}{\sigma_N^2|_{PC}}\right).$$

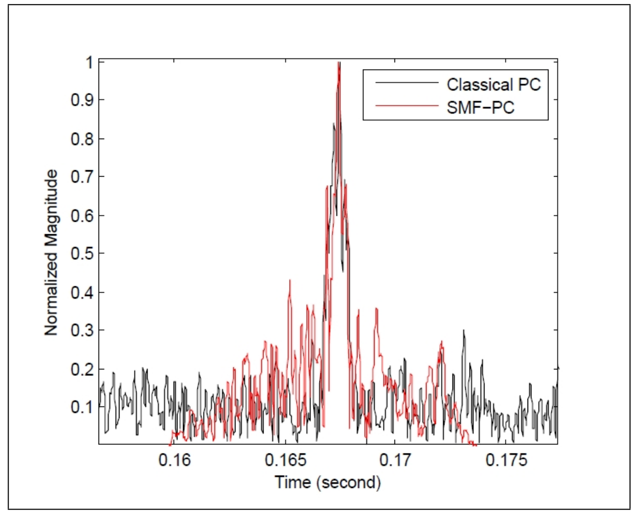

Fig. 11. *Detail of the figure 10 pointing out the similar characteristics of the pulse-compressed peak.*

The results are proposed on figure 12. They reveal that when the SNR is lower than $0\,dB$ the proposed method seems to be ineffective and when it is greater than this threshold the SMF-CP allows to obtain an average SNR gain equal to $40\,dB$.

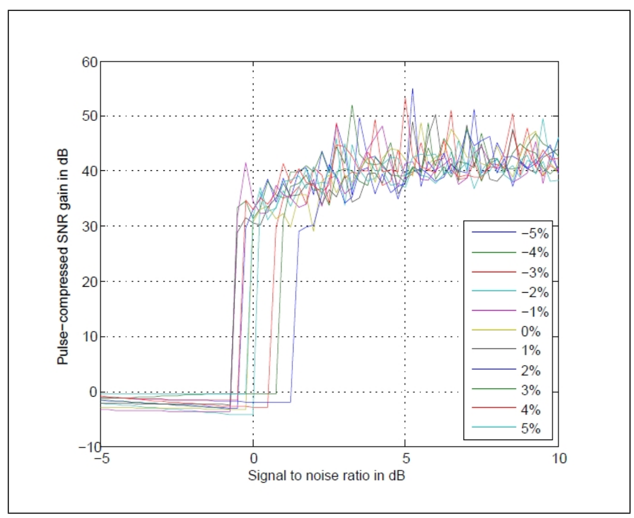

Fig. 12. *SNR gain on the pulse-compressed data using the SMF-CP in comparison with the classical CP for several Doppler shifts and for different received data SNR cases*

This "*Go-No go*" functioning can be simply explain considering the SMF-CP algorithm. Indeed, the number of eigenvalues retained to process the studied window is conditioned by the sub-window signal to noise ratio; if this SNR is too low, only one eigenvector will be retained to process the data, which corresponds to a strong smoothing of the observation and so a cancellation of the received pulse. For the SNR lower than $0\,dB$ the received pulse power is not sufficiently significant to lead to a high number of eigenvalues.

B. Real data

Let consider now two signals coming from the DIVACOU campaign realized in September 2009.

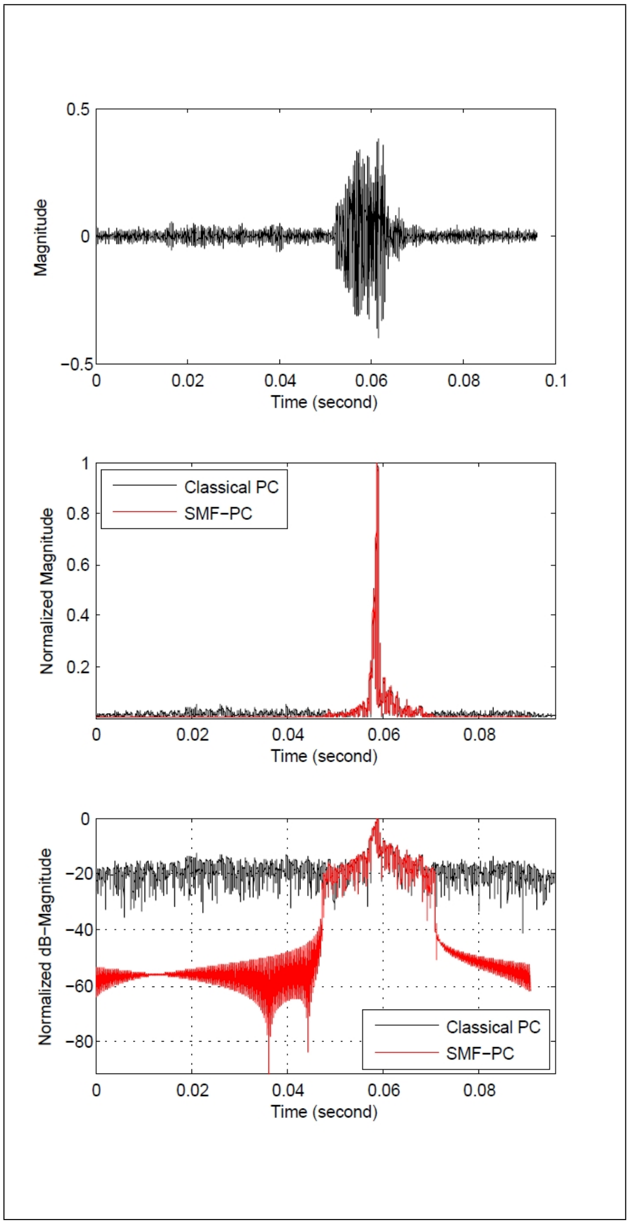

Fig. 13. *Pulse-compression for the first signal coming from the DIVACOU campaign. Top: Received signal sampled at $75\,kHz$, the pulse presented at figure 5 happens near the 0.06^{th} second; the SNR is approximatively equal to $20\,dB$. Middle: Normalized pulse-compressed data obtained applying the classical PC in black lines and using the SMF-PC in red lines; in both cases, a peak near the 0.06^{th} second reveals the useful signal presence, the main difference between the two methods being the noise level. Bottom: Normalized pulse-compressed data in dB magnitude; the SMF-PC method allows to obtain an average noise level $30\,dB$ lower than using the classical approach.*

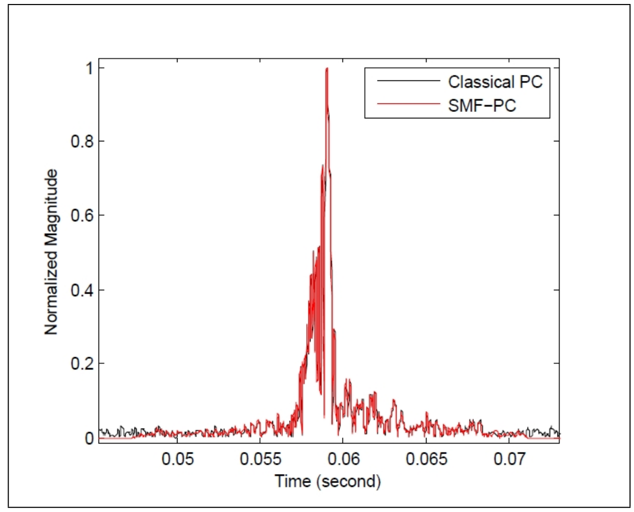

Fig. 14. *Detail of the figure 13 pointing out the similar characteristics of the pulse-compressed peak.*

These signals correspond to a measurement of a communication between an underwater beacon and a ship, in order to give to the ship the possibility to locate the beacon. The distance between the ship and the beacon in the horizontal plane was $300\,m$, the beacon immersion was $490\,m$. The beacon emits the pulse presented on figure 4 with a power of $190\,dB$. At reception, this pulse appears near the 0.06^{th} second for the first signal (see figure 13) and around 0.25 second for the second one (see figure 15), with for the first signal a more favorable SNR than for the second (near $13\,dB$ for the second, while it is near $20\,dB$ for the first one).

Remark: the signals presented here correspond to a small part of the measured signals; in reality the first pulses happen with a more important delay.

The use of our pulse-compression scheme on those signals gives the results presented on figures 13 and 15. The same observations than for synthetic data can be done: a strong SNR gain while keeping the pulse-compressed main lobe characteristics (see figure 14).

Furthermore, more than the SNR level, the main difference between the two received signals rests on the received pulse deformation compared to the theoretical pulse. Indeed, the two receivers used to collect these signals are not located on the same place and as the second one is quite near the ship hull, some reverberating signals should have corrupted the received pulse. This explains the SNR gain more significant for the first signal than for the second.

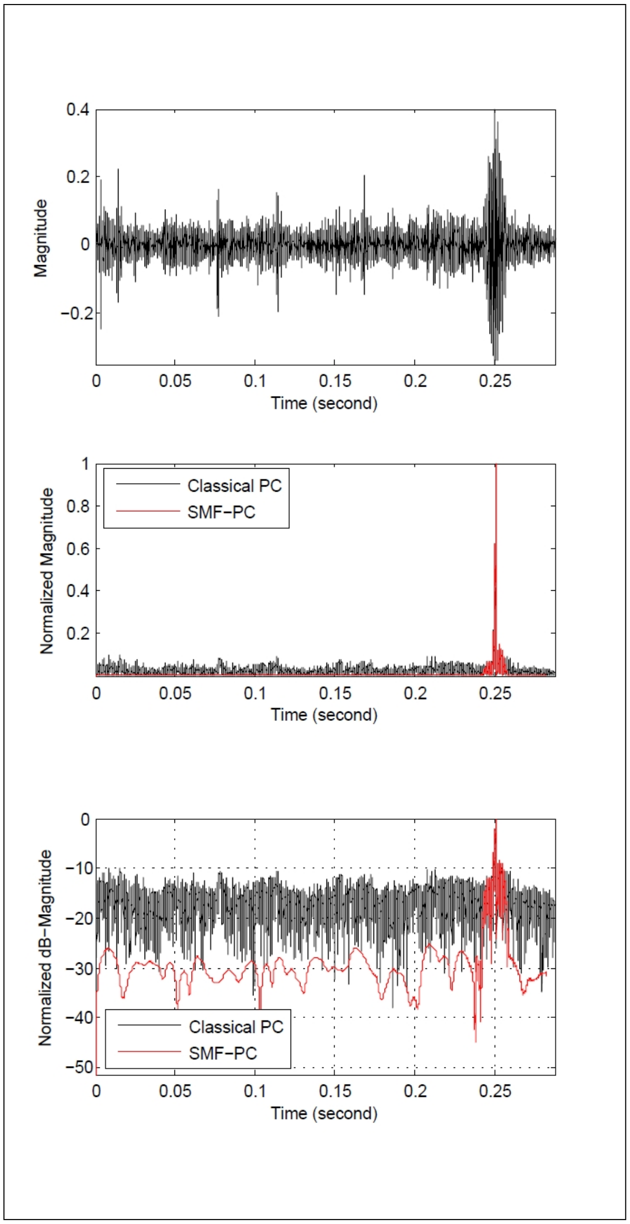

Fig. 15. *Pulse-compression for the second signal coming from the DIVACOU campaign. Top: Received signal sampled at* $75\,kHz$*, the pulse happens near the* 0.25^{th} *second; the SNR is approximatively equal to* $13\,dB$*. Middle: Normalized pulse-compressed data obtained applying the classical PC in black lines and using the SMF-PC in red lines; in both cases, a peak near the* 0.25^{th} *second reveals the useful signal presence, the main difference between the two methods being the noise level. Bottom: Normalized pulse-compressed data in* dB *magnitude; the SMF-PC method allows to obtain an average noise level* $20\,dB$ *lower than using the classical approach.*

VII. Conclusion

In this article, we have proposed a new pulse-compression scheme taking into account the random nature of the transmitted pulse and the coloration of the disturbing signal. This new method is based on the use of the stochastic matched filter, which corresponds to the noisy data projection onto a basis ensuring a maximization of the signal to noise ratio. This way and at the same moment, the received data is de-noised and compressed with the most adequate pulse, constructed considering the same basis than for the data. Results obtained on real and synthetic data reveal the effectiveness of such a process. Indeed, on the one hand, the signal to noise ratio of the stochastic matched filter based pulse-compressed data appears far lower than the one obtained using the classical method and, on the other hand, the range resolution remains the same, the main lobe width being the same than using the classical approach.

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