A Sub-Region Priority Reaching Control Scheme with a Fuzzy-Logic Algorithm for an Underwater Vehicle subject to Uncertain Restoring Forces

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Abstract-**This paper presents an adaptive sub-region priority reaching controller with a fuzzy logic algorithm for an underwater vehicle. In this method, the sub-region priority reaching control law is merged with a fuzzy technique to manage the generalization of the underwater vehicle set-point control problem. The fuzzy technique is used to handle multiple sub-region criteria. These criteria are: depth constraint, avoiding the region with an obstacle inside it, ensuring visibility of the feature during visual servoing and region shape constraint. Due to the unknown gravitational and buoyancy forces, an adaptive term is adopted in the proposed controller. Moreover, the unit quaternion representation is used in order to achieve a singularity-free attitude representation. The stability analysis is carried out using the Lyapunov type approach. Simulation results are presented to demonstrate the effectiveness of the proposed control technique.**

I. INTRODUCTION

Underwater vehicles have been widely used in underwater applications such as inspection of pipelines, oil and gas exploration and naval operations. In most of these applications, the underwater vehicle is controlled by using a conventional set-point controller. However, the control objective of an underwater vehicle could alternatively be defined as a region, instead of a point [1,2]. For example, maintaining the underwater vehicle within a minimum and maximum depth in water; underwater vehicle traveling inside a pipeline for a specific task; avoiding an obstacle located at a specific region or an underwater region constraint to ensure visibility during motion.

A region reaching control scheme with motion constraint was recently proposed by Cheah in [3] for a robot manipulator. In this concept, the control objective is available to perform at least one region objective. When the robot is subject to additional region objectives, the controller executes it as a robot motion constraint. Moreover, it allows the specification of a primary region objective which is fulfilled with a higher priority with respect to a secondary region. Note that, for the non-redundant robot the primary and secondary region objectives are overlapping each other as presented in [3]. There are multiple region objectives that can be defined as an additional motion constraint for the robot, for instance a repulsive region, depth constraint, visibility constraint, region complexity and singularity avoidance. In other words, there is an infinite number of region criteria for a given generalized set point control problem. This concept is also applicable for an underwater vehicle as illustrated in Fig. 1, where multiple region criteria can be defined in order to achieve faster motion and reduction of the energy consumption.

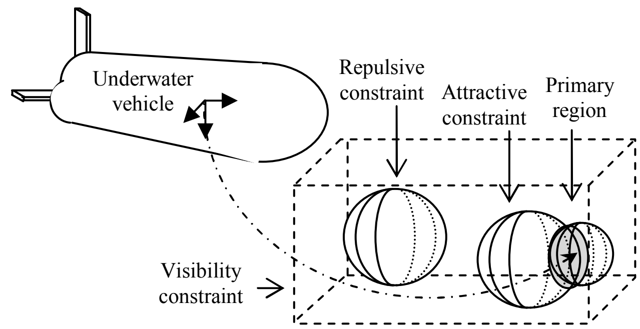

Fig. 1. Sub-region control for underwater vehicle with multiple criteria

To manage the multiple region criteria effectively, the perspective of an experienced human operator can play an important role in the successful fulfillment of a given reaching task. In an effort to include the skilled human operator into the region resolution, fuzzy logic can be used. Fuzzy logic incorporates human-like reasoning using if-then type fuzzy rules, reflecting a human pilot's expert knowledge. The proposed scheme is considered at the higher level with respect to the fuzzy logic based artificial pilot. In the past, many researchers have studied fuzzy approaches for marine system applications [4-11]. The autopilot fuzzy system, which had automatic adaptation and tuning ability, was initially proposed for ship control by Sutton and Jess [4]. Polkinghorne et al. [5] designed an industrial autopilot with a fuzzy controller that was adaptive to environment changes. In [6], Omerdic et al. presented a fuzzy autopilot for tracking control of a ship steering system. The design of a sliding mode fuzzy controller for an autonomous underwater vehicle (AUV) was proposed in [7] and Guo et al. presented the experimental results of an AUV using a sliding mode fuzzy control law [8]. Recently, a sliding mode control strategy with an adaptive fuzzy logic algorithm for the depth control of a remotely operated underwater vehicle was proposed by Bessa [9]. Fuzzy logic techniques were also used for obstacle avoidance in autonomous underwater vehicles control described in [10,11,12,13].

In this paper, the region priority reaching control law is merged with a fuzzy-logic approach that has been proposed for an underwater vehicle subject to the restoring forces uncertainties. While operating in an underwater environment, it is impossible to obtain an exact knowledge of gravitational and buoyancy forces of an underwater vehicle so that a controller

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can compensate due to the variation of ocean water density. Therefore, the gravity regressor which is composed of known and unknown gravitational parameters can be utilized to overcome the uncertain restoring forces [14]. In addition of the proposed scheme, each sub-region criterion is defined within the framework of a sub-region priority technique. The hierarchy of the secondary sub-region tasks is established by a low-level artificial pilot that determines a weighting factor for each region criterion based on if-then type fuzzy rules that reflect an expert human pilot's knowledge. To interpret the fuzzy rules, a Mamdani type fuzzy inference is utilized [15].

The rest of the paper is organized as follows. Section II describes the unit quaternion representation. Section III presents the kinematic and dynamic properties of underwater vehicle. Section IV states the sub-region priority control with restoring forces compensation and the stability analysis in term of Lyapunov technique. The use of multiple sub-region criteria for possible performance enhancement is present in Section V. Fuzzy logic for a decision-making process is presented in Section VI. Simulation results are given in Section VII. Finally, Section VIII contains concluding remarks.

II. UNIT QUATERNION REPRESENTATION

Consider two orthonormal right-handed coordinate frames: the inertial fixed frame, $\sum_i$ and body fixed frame, $\sum_v$. Define the matrix, R as 3×3 rotational matrix from the body fixed frame to the inertial fixed frame. The unit quaternion representation of the rotational matrix, R can be defined by

$$\epsilon = [\epsilon_0 \quad \epsilon_1 \quad \epsilon_2 \quad \epsilon_3]^T = [\epsilon_0 \quad \epsilon_\varepsilon^T]^T \in \mathbb{R}^4 \tag{1}$$

with

$$\epsilon_0 = \cos(\emptyset/2);\ \epsilon_\varepsilon \triangleq k\sin(\emptyset/2) \tag{2}$$

where $\emptyset$ is the angle and $k(\mathrm{t}) \in \mathbb{R}^3$ are the Euler angle/axis parameters subject to the constraint $\epsilon^T\epsilon = 1$. The rotational matrix can be determined through

$$R = ({\epsilon_0}^2 - {\epsilon_\varepsilon}^T\epsilon_\varepsilon)I_3 + 2\epsilon_\varepsilon{\epsilon_\varepsilon}^T - 2\epsilon_0{\epsilon_\varepsilon}^\times \tag{3}$$

where for any vector $a = [a_1 \quad a_2 \quad a_3]^T$, the notation $a^\times$ denotes the skew-symmetric matrix of the form

$$a^\times \triangleq \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{4}$$

where the product $a^T a^\times$ satisfies the following property

$$a^T a^\times = [0 \quad 0 \quad 0]^T \tag{5}$$

Generally, the relationship between unit quaternion and angular velocity is in the body fixed frame, ω can be obtained using the quaternion propagation equation

$$\dot{\epsilon} = \frac{1}{2}E(\epsilon)\omega \tag{6}$$

$$E(\epsilon) = \begin{bmatrix} -\epsilon_\varepsilon^T \\ \epsilon_0 I_3 + {\epsilon_\varepsilon}^\times \end{bmatrix} \tag{7}$$

where the Jacobian $E(\epsilon)$ satisfy the following important properties:

$$E(\epsilon)^T E(\epsilon) = I_{3\times3} \qquad \text{and} \qquad E(\epsilon)^T\epsilon = 0 \tag{8}$$

Consequently, from (6) and (7), the inverse kinematics can be computed as

$$\omega = 2E^T(\epsilon)\dot{\epsilon} \tag{9}$$

III. KINEMATICS AND DYNAMICS

A. Kinematic Model

The relationship between inertial and body fixed vehicle velocity can be described using the Jacobian matrix $J_v(e_v)$ in the following form

$$\begin{bmatrix} \dot{p}_v \\ \dot{e}_v \end{bmatrix} = \begin{bmatrix} R_v(e_v) & 0_{3\times3} \\ 0_{3\times3} & \frac{1}{2}E(e_v) \end{bmatrix} \begin{bmatrix} v_v \\ \omega_v \end{bmatrix} \Leftrightarrow \dot{\eta} = J_v(e_v)v \tag{10}$$

where $p_v \in \mathbb{R}^3$ and $e_v \in \mathbb{R}^4$ denote the position and the unit quaternion representation of the vehicle, respectively, expressed in the inertial fixed frame. R_v and E are defined as in (3) and (7), respectively. The linear and angular velocities of the vehicle, $v_v \in \mathbb{R}^3$ and $\omega_v \in \mathbb{R}^3$, respectively, are described in terms of the body fixed frame.

B. Dynamic Model

The dynamic equation of motion for an underwater vehicle has been previously investigated in detail [16]. Let the velocity state vector with respect to the body fixed frame be defined by $v \in \mathbb{R}^6$, the underwater vehicle dynamic equation can be expressed in closed form as

$$M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau \tag{11}$$

where M is the inertia matrix including the added mass term, $C(v)$ represents the matrix of the Coriolis and centripetal forces including the added mass term, $D(v)$ denotes the hydrodynamic damping and lift force, and $g(\eta)$ is the restoring force. The dynamic equation in (11) preserves the following properties [16,18]:

Property 1: The inertia matrix M is symmetric and positive definite such that $M = M^T > 0$

Property 2: $\dot{M} - 2C(v)$ is the skew-symmetric matrix.

Property 3: The hydrodynamic damping matrix $D(v)$ is positive definite, i.e.: $D(v) = D^T(v) > 0$

The restoring force on left-hand side of (11) can be written as

$$g(\eta) = Z(\eta)\Phi \tag{12}$$

where $\Phi \in \mathbb{R}^{n_p}$ is a set of parameters and $Z(\eta) \in \mathbb{R}^{n \times n_p}$ is the gravity regression matrix which represents the known part of $g(\eta)$; n_p is the total number of physical parameters.

IV. Sub-Region Priority Reaching Control with Uncertain Restoring Forces

To achieve an effective point to point motion of an underwater vehicle, a sub-region priority reaching control with restoring force compensation technique is considered. Within this framework, an operational space can be broken down into several sub-regions with priority order using the region-decomposition approach. For a redundant robot, the reaching task into each sub-region is performed using the degrees-of-freedom that remain after all the sub-regions with higher priority have been implemented. In other words, the region reaching control problem of redundant robot can be approached via the sub-region method by regarding a target region to be reached by a redundant system as the sub-region with first priority and using the redundancy to perform motion constraints defined by an additional sub-region as the sub-region with second priority.

In the case of a non-redundant system i.e. an underwater vehicle, the primary sub-region overlaps with the secondary sub-regions to form the intersection of the two sub-regions. Before the vehicle converges to an intersection of the sub-region, the vehicle must first execute the secondary sub-region objectives which are assigned along the pathway. At the final position, the vehicle converges into a region where both primary and secondary sub-region objectives are achieved simultaneously. Fig. 2 depicts the underwater vehicle being kept within a particular region in order to maintain its depth prior to the convergence to the primary sub-region.

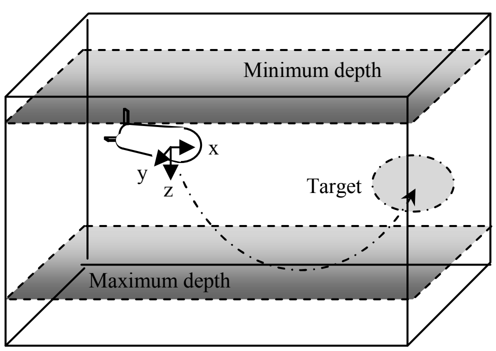


Fig. 2. Sub-region priority reaching control for an underwater vehicle

Now, define the desired primary sub-region for the underwater vehicle as a single region in the following equation

$$f(\delta p_v) \leq 0 \tag{13}$$

where $\delta p_v = p_v - p_{vd} \in \mathbb{R}^3$ is the continuous first partial derivatives of the primary region; p_{vd} is the stationary reference point inside the region such that $\dot{p}_{vd}(t) = 0$. In order to incorporate the multiple secondary sub-regions, the following equation can be used

$$\begin{aligned} &g(\delta p_{vs}) \\ &= \left[g_1(\delta p_{vs_1}) \;\; g_2(\delta p_{vs_2}) \;\; \cdot\cdot \;\; g_N(\delta p_{vs_N})\right]^T \leq 0 \end{aligned} \tag{14}$$

where N is the total number of secondary sub-regions and $\delta p_{vs} = p_{vs} - p_{vsd_i} \in \mathbb{R}^3$ is the continuous first partial derivatives of the secondary sub-region; ; p_{vsd_i} is the stationary reference point inside the i^{th} secondary sub-region such that $\dot{p}_{vsd_i}(t) = 0$. Since both objectives are accomplished concurrently, the position vector p_{vs} can be chosen as p_v and the body fixed velocity vector for the secondary sub-region can be defined as follows

$$\dot{p}_{vs} = \dot{p}_v = R_v(e_v)v_v \tag{15}$$

where the primary and secondary Jacobian matrix are identical. During the region reaching task, the secondary sub-region is initially fulfilled before the fulfillment of both primary and secondary sub-regions simultaneously. The corresponding potential energy function for the desired sub-region describes in (13) can be specified as

$$\begin{aligned} P_p(\delta p_v) &= \frac{k_{p1}}{2}\left[max\left(0, f(\delta p_v)\right)\right]^2 \\ &\triangleq \begin{cases} 0, & f(\delta p_v) \leq 0 \\ \dfrac{k_{p1}}{2} f^2(\delta p_v), & f(\delta p_v) > 0 \end{cases} \end{aligned} \tag{16}$$

where $k_{p1} \in \mathbb{R}$ is positive scalar. Similarly, the potential energy function for the multiple sub-regions in (14) can defined as follows

$$\begin{aligned} P_s(\delta p_{vs}) &= \sum_{i=1}^{N} \frac{\alpha_i}{2}\left[max\left(0, g_i(\delta p_{vs_i})\right)\right]^2 \\ &\triangleq \begin{cases} \displaystyle\sum_i^N 0, & f(\delta p_{vs_i}) \leq 0 \\ \displaystyle\sum_i^N \frac{\alpha_i}{2} g^2(\delta p_{vs_i}), & f(\delta p_{vs_i}) > 0 \end{cases} \end{aligned} \tag{17}$$

Differentiating (16) with respect to δp_v and rewriting the equation in an advantageous form leads to

$$\left(\frac{\partial P_p(\delta p_v)}{\partial(\delta p_v)}\right)^T = k_{p1}\, max\left(0, f(\delta p_v)\right)\left(\frac{\partial f(\delta p_v)}{\partial p_v}\right)^T \tag{18}$$

Since multiple secondary sub-regions are specified, the normalized function can be utilized in order to prevent one single region criterion from excessively dominating the solution and this allows the particular secondary sub-region to be assigned with the pre-defined priority level. Therefore, the following function can be obtained using the normalized gradient of the scalar potential energy in (17)

$$\left(\frac{\partial P_s(\delta p_{vs})}{\partial(\delta p_{vs})}\right)^T = \sum_{i=1}^{N} \alpha_i \left[max\left(0, g_i(\delta p_{vs_i})\right)\right] \frac{\left(\frac{\partial g_i(\delta p_{vs_i})}{\partial p_{vs}}\right)^T}{\left\|\partial g_i(\delta p_{vs_i})/\partial p_{vs}\right\|_2} \quad (19)$$

where α_i is a weight that determines the extent of how much emphasis is placed on each secondary region criterion with respect to the other and $\|\cdot\|_2$ is the Euclidean norm. Note that, when the norm of the function is equal to zero, the corresponding criterion is set to zero which leads the system to reach the primary region only. Now, let (18) and (19) be represented as the primary region error $\tilde{e}_p$ and secondary region error $\tilde{e}_s$ respectively in the following form

$$\tilde{e}_p = max\left(0, f(\delta p_v)\right)\left(\frac{\partial f(\delta p_v)}{\partial p_v}\right)^T \quad (20)$$

$$\tilde{e}_s = \sum_{i=1}^{N} \tilde{e}_{si} \quad (21)$$

That is,

$$\tilde{e}_{si} = \sum_{i=1}^{N} \beta_i \, max\left(0, g_i(\delta p_{vs})\right) \frac{\left(\frac{\partial g_i(\delta p_{vs_i})}{\partial p_{vs}}\right)^T}{\left\|\partial g_i(\delta p_{vs_i})/\partial p_{vs}\right\|_2} \quad (22)$$

where $\beta_i = \alpha_i / k_q$; k_q is a positive constant.

At this point, the error between the actual and desired orientation for the underwater vehicle can be formulated as follows [18]

$$\tilde{R} \triangleq RR_d^T = \left(\tilde{e}_0{}^2 - \tilde{e}_\varepsilon{}^T \tilde{e}_\varepsilon\right) I_3 + 2\tilde{e}_\varepsilon \tilde{e}_\varepsilon{}^T - 2\tilde{e}_0 \tilde{e}_\varepsilon{}^\times \quad (23)$$

where R is defined in (3) and R_d is the rotational matrix of R expressing the desired orientation which also is described by the quaternion $\epsilon \triangleq \left[\epsilon_{0,d} \quad \epsilon_{\varepsilon,d}^T\right]^T$. The corresponding unit quaternion representation is denoted by $\tilde{e}_v \triangleq [\tilde{e}_0 \quad \tilde{e}_\varepsilon^T]^T$. Thus, the quaternion propagation equation can be considered as

$$\dot{\tilde{e}}_v = \begin{bmatrix} -\frac{1}{2}\tilde{e}_\varepsilon^T \\ \frac{1}{2}\left(\tilde{e}_0 I_3 + \tilde{e}_\varepsilon{}^\times\right) \end{bmatrix} \widetilde{\omega}_v \Leftrightarrow \dot{\tilde{e}}_v = \frac{1}{2} E(\tilde{e}_v)\widetilde{\omega}_v \quad (24)$$

where $\widetilde{\omega}_v = \omega_v - \omega_{vd}$; ω_v is defined in (10) and the desired angular velocity of the vehicle ω_{vd} is always zero $\omega_{vd} = 0$, hence $\widetilde{\omega}_v = \omega_v$. The region reaching controller presented in [1] contains the Jacobian transpose in its update law and not its inverse which results in an inexact mapping and a coupling among the error directions [18]. Consequently, the Jacobian transpose based method is not suitable for design of the controller with restoring force compensation. As opposed to [1], the sub-region priority reaching controller with uncertain restoring force compensation for an underwater vehicle is proposed as follows

$$\tau = -K_0 \tilde{e}_T - K_s \tilde{e}_s - K_v v + Z(\eta)\hat{\varphi} \quad (25)$$

with

$$\begin{gathered} K_0 = diag\left\{R_v^T k_{p1}, \quad k_{p2} I_{3\times3}\right\}; \\ K_s = \left[R_v^T k_q \quad 0_{3\times3}\right]^T; \\ K_v = diag\{k_{v1} I_{3\times3}, \quad k_{v2} I_{3\times3}\}; \end{gathered} \quad (26)$$

where k_{p1} and k_q are previously defined in (16) and (22), respectively. k_{p2}, k_{v1} and k_{v2} are positive constants. $I_{3\times3}$ and $0_{3\times3}$ are the 3×3 identity and zero matrices, respectively. The proposed controller adopts the approach of the pseudo-inverse matrix in the parameters update law such that the drawback of the restoring compensation using the Jacobian transpose method is eliminated. Accordingly, the estimate vector $\widehat{\varphi}$ can be updated online by

$$\dot{\hat{\varphi}} = -\Gamma^{-1} \mathrm{Z}^{\mathrm{T}}(\eta)\left(v + \sigma J_v^\dagger(e_v)\tilde{e}_{T_s}\right) \quad (27)$$

with

$$\tilde{e}_{T_s} = \tilde{e}_T + (k_q/k_{p1}) \begin{bmatrix} \tilde{e}_s \\ 0_{3\times1} \end{bmatrix} \quad (28)$$

where Γ and σ denote the positive definite matrices and $J_v^\dagger(e_v)$ is the pseudo-inverse of matrix $J_v(e_v)$. The error $\tilde{e}_T$ is defined as

$$\tilde{e}_T = \left[\tilde{e}_p^T \quad \tilde{e}_\varepsilon^T\right]^T \quad (29)$$

while the quaternion vector $\tilde{e}_\varepsilon$ is part of $\tilde{e}_v$. Substituting (25) into (11) the closed-loop equation is obtained as

$$\begin{gathered} M\dot{v} + C(v)v + D(v)v + K_0 \tilde{e}_T + K_s \tilde{e}_s + K_v v \\ + Z(\eta)\tilde{\varphi} = 0 \end{gathered} \quad (30)$$

where $\tilde{\varphi} = \varphi - \hat{\varphi}$. Next, the following Lyapunov function is considered

$$\begin{aligned} V = {} & \frac{1}{2} v^T M v + \sigma \tilde{e}_{T_s}^T J_v^{\dagger T}(e_v) M v \\ & + \frac{1}{2}\tilde{\varphi}^T \Gamma \tilde{\varphi} + \left(k_{p2} + \sigma k_{v2}\right)[(1 - \tilde{e}_0)^2 + \tilde{e}_\varepsilon^T \tilde{e}_\varepsilon] \\ & + \frac{1}{2}\left(k_{p1} + \sigma k_{v1}\right)\left(\tilde{e}_p^T \tilde{e}_p / \left\|\frac{\partial f(\delta p_v)}{\partial p_v}\right\|^2\right) \\ & + \frac{1}{2}\left(k_q + \sigma k_w\right) \sum_{i=1}^{N} \frac{1}{\beta_i}\left(\tilde{e}_s^T \tilde{e}_s / \left\|\left(\frac{\partial g_i(\delta p_{vs_i})}{\partial p_{vs}}\right)\right\|^2\right) \end{aligned} \quad (31)$$

where $k_w = k_q k_{v1}/k_{p1}$. The last two terms of (31) also can be written separately as

$$\frac{1}{2}\left(k_{p1} + \sigma k_{v1}\right)\left(\tilde{e}_p^T \tilde{e}_p \Big/ \left\|\frac{\partial f(\delta p_v)}{\partial p_v}\right\|^2\right) = \frac{1}{2}\left(k_{p1} + \sigma k_{v1}\right)\left(max\left(0, f(\delta p_v)\right)\right)^2 \tag{32}$$

$$\frac{1}{2}\left(k_q + \sigma k_w\right)\sum_{i=1}^{N}\frac{1}{\beta_i}\left(\tilde{e}_s^T \tilde{e}_s \Big/ \left\|\left(\frac{\partial g_i(\delta p_{vs_i})}{\partial p_{vs}}\right)\right\|^2\right) = \frac{1}{2}\left(k_q + \sigma k_w\right)\sum_{i=1}^{N}\beta_i\left(max\left(0, g_i(\delta p_{vs})\right)\right)^2 \tag{33}$$

which gives a similar expression as (16) and (17), respectively. Note that, a part of the terms in (31) can be written in the following form

$$\begin{aligned}
&\frac{1}{4}v^T M v + \sigma \tilde{e}_{T_s}^T {J_v^{\dagger}}^T M v \\
&+ \frac{1}{2}\left(k_{p1} + \sigma k_{v1}\right)\left(\tilde{e}_p^T \tilde{e}_p \Big/ \left\|\frac{\partial f(\delta p_v)}{\partial p_v}\right\|^2\right) \\
&+ \left(k_{p2} + \sigma k_{v2}\right)[(1 - \tilde{e}_0)^2 + \tilde{e}_\varepsilon^T \tilde{e}_\varepsilon] \\
&+ \frac{1}{2}\left(k_q + \sigma k_w\right)\sum_{i=1}^{N}\frac{1}{\beta_i}\left(\tilde{e}_s^T \tilde{e}_s \Big/ \left\|\left(\frac{\partial g_i(\delta p_{vs_i})}{\partial p_{vs}}\right)\right\|^2\right) \\
&= \frac{1}{4}\left(v + 2\sigma J_v^{\dagger}\tilde{e}_{T_s}\right)^T M\left(v + 2\sigma J_v^{\dagger}\tilde{e}_{T_s}\right) \\
&- \sigma^2 \tilde{e}_{T_s}^T {J_v^{\dagger}}^T M J_v^{\dagger}\tilde{e}_{T_s} \\
&+ \frac{1}{2}\left(k_{p1} + \sigma k_{v1}\right)\left(\tilde{e}_p^T \tilde{e}_p \Big/ \left\|\frac{\partial f(\delta p_v)}{\partial p_v}\right\|^2\right) \\
&+ \left(k_{p2} + \sigma k_{v2}\right)[(1 - \tilde{e}_0)^2 + \tilde{e}_\varepsilon^T \tilde{e}_\varepsilon] \\
&+ \frac{1}{2}\left(k_q + \sigma k_w\right)\sum_{i=1}^{N}\frac{1}{\beta_i}\left(\tilde{e}_s^T \tilde{e}_s \Big/ \left\|\left(\frac{\partial g_i(\delta p_{vs_i})}{\partial p_{vs}}\right)\right\|^2\right) \\
&\geq \frac{1}{2}\left(k_{p1} + \sigma k_{v1}\right)\left(\tilde{e}_p^2 \Big/ \left\|\frac{\partial f(\delta p_v)}{\partial p_v}\right\|^2\right) - \sigma^2 \lambda_m \tilde{e}_p^2 \\
&+ \left(k_{p2} + \sigma k_{v2}\right)[(1 - \tilde{e}_0)^2 + \tilde{e}_\varepsilon^2] \\
&- \sigma^2 \lambda_m [(1 - \tilde{e}_0)^2 + \tilde{e}_\varepsilon^2] \\
&+ \frac{1}{2}\left(k_q + \sigma k_w\right) a_{min} b_{min}\sum_{i=1}^{N}\|\tilde{e}_{si}\|^2 \\
&- \sigma^2 \lambda_m N \sum_{i=1}^{N}\|\tilde{e}_{si}\|^2
\end{aligned} \tag{34}$$

where (34) uses the norm of (21) which was defined as follows

$$\|\tilde{e}_s\| \leq \|\tilde{e}_{s1}\| + \|\tilde{e}_{s2}\| + \cdots + \|\tilde{e}_{sN}\| \tag{35}$$

That is,

$$\|\tilde{e}_s\|^2 \leq N \sum_{i=1}^{N}\|\tilde{e}_{si}\|^2 \tag{36}$$

In (34), notice that

$$a_{min} \triangleq \min_i\left(\frac{1}{a_i}\right),$$

$$b_{min} \triangleq \min_i\left(1 \Big/ \left\|\left(\frac{\partial g_i(\delta p_{vs_i})}{\partial p_{vs}}\right)\right\|^2\right); i = 1,2 \ldots N$$

$$\lambda_m \triangleq \lambda_{max}\left[{J_v^{\dagger}}^T(e_v) M J_v^{\dagger}(e_v)\right]$$

where $\lambda_{max}[A]$ represents the maximum eigenvalue of matrix A. Substituting the inequality of (34) into (31), yields

$$\begin{aligned}
V \geq &\left[\left[\frac{1}{2}\left(k_{p1} + \sigma k_{v1}\right)\left(\tilde{e}_p^2 \Big/ \left\|\frac{\partial f(\delta p_v)}{\partial p_v}\right\|^2\right)\right.\right. \\
&- \sigma^2 \lambda_m \tilde{e}_p^2 \\
&+ \left(k_{p2} + \sigma k_{v2}\right)[(1 - \tilde{e}_0)^2 + \tilde{e}_\varepsilon^2] - \sigma^2 \lambda_m [(1 - \tilde{e}_0)^2 + \tilde{e}_\varepsilon^2] \\
&+ \frac{1}{2}\left(k_q + \sigma k_w\right) a_{min} b_{min}\sum_{i=1}^{N}\|\tilde{e}_{si}\|^2 \\
&\left.\left.- \sigma^2 \lambda_m N \sum_{i=1}^{N}\|\tilde{e}_{si}\|^2\right]\right] \\
&+ \frac{1}{4}v^T M v + \frac{1}{2}\tilde{\varphi}^T \Gamma \tilde{\varphi}
\end{aligned} \tag{37}$$

To ensure V is valid locally around $\tilde{e}_p$, $\tilde{e}_0$, $\tilde{e}_\varepsilon$, $\tilde{e}_{si}$, v and $\tilde{\varphi}$, σ can be chosen sufficiently small while k_{p1}, k_{v1}, k_{p2}, k_{v2} and k_q be chosen sufficiently large so that the double-bracketed term satisfies

$$\begin{gathered}
\frac{1}{2}\left(k_{p1} + \sigma k_{v1}\right) \Big/ \left\|\frac{\partial f(\delta p_v)}{\partial p_v}\right\|^2 - \sigma^2 \lambda_m > 0 \\
\left(k_{p2} + \sigma k_{v2}\right) - \sigma^2 \lambda_m > 0 \\
\frac{1}{2}\left(k_q + \sigma\left(\frac{k_q k_{v1}}{k_{p1}}\right)\right) a_{min} b_{min} - \sigma^2 \lambda_m N > 0
\end{gathered} \tag{38}$$

Differentiating V with respect to time gives

$$\begin{aligned}
&\dot{V} \\
&= v^T M \dot{v} + \sigma \tilde{e}_{T_s}^T {J_v^{\dagger}}^T(e_v) M \dot{v} + \sigma \dot{\tilde{e}}_{T_s}^T {J_v^{\dagger}}^T(e_v) M v \\
&+ \sigma \tilde{e}_{T_s}^T {\dot{J}_v^{\dagger}}^T(e_v) M v - \tilde{\varphi}^T \Gamma \dot{\hat{\varphi}} + \left(k_{p2} + \sigma k_{v2}\right)\tilde{e}_\varepsilon^T \dot{\tilde{e}}_\varepsilon \\
&+ \left(k_{p1} + \sigma k_{v1}\right)\left(max\left(0, f(\delta p_v)\right)\right)\frac{\partial f(\delta p_v)}{\partial p_v}\dot{p}_v \\
&+ \left(k_q + \sigma k_w\right)\sum_{i=1}^{N}\beta_i\left(max\left(0, g_i(\delta p_{vs_i})\right)\right)\frac{\partial g_i(\delta p_{vs_i})}{\partial p_{vs}}\dot{p}_{vs}
\end{aligned} \tag{39}$$

Substituting (27) and (30) into (39) and cancelling common terms, yields

$$\dot{V} = -W \tag{40}$$

with

$$\begin{aligned} W = {} & v^T D(v)v + v^T K_v v + \sigma \tilde{e}_{T_s}^T J_v^{\dagger^T}(e_v) K_0 \tilde{e}_T \\ & + \sigma \tilde{e}_{T_s}^T J_v^{\dagger^T}(e_v) K_s \tilde{e}_s \\ & + \Big\{ \sigma \tilde{e}_{T_s}^T J_v^{\dagger^T}(e_v) C(v) v \\ & + \sigma \tilde{e}_{T_s}^T J_v^{\dagger^T}(e_v) D(v) v \\ & - \sigma \dot{\tilde{e}}_{T_s}^T J_v^{\dagger^T}(e_v) M v \\ & - \sigma \tilde{e}_{T_s}^T \dot{J}_v^{\dagger^T}(e_v) M v \Big\} \end{aligned} \tag{41}$$

where the Property 2 and the following equation were considered

$$\begin{aligned} & v^T K_0 \tilde{e}_T + v^T K_s \tilde{e}_s + \sigma \tilde{e}_{T_s}^T J_v^{\dagger^T}(e_v) K_v v \\ & = (k_{p2} + \sigma k_{v2}) \tilde{e}_\varepsilon^T \omega_v \\ & + (k_{p1} + \sigma k_{v1}) \big(max(0, f(\delta p_v))\big) \frac{\partial f(\delta p_v)}{\partial p_v} \dot{p}_v \\ & + (k_q + \sigma k_w) \sum_{i=1}^{N} \beta_i \left(max\left(0, g_i(\delta p_{vs_i})\right)\right) \frac{\partial g_i(\delta p_{vs_i})}{\partial p_{vs}} \dot{p}_{vs} \end{aligned} \tag{42}$$

Note that, the bracketed terms of (41) can be manipulated such that

$$\begin{aligned} \sigma \Big| & \tilde{e}_{T_s}^T J_v^{\dagger^T}(e_v) C(v) v + \tilde{e}_{T_s}^T J_v^{\dagger^T}(e_v) D(v) v \\ & - \dot{\tilde{e}}_{T_s}^T J_v^{\dagger^T}(e_v) M v \\ & - \tilde{e}_{T_s}^T \dot{J}_v^{\dagger^T}(e_v) M v \Big| \geq -\sigma c_0 \|v\|^2 \end{aligned} \tag{43}$$

where there exists a constant $c_0 > 0$ locally. $\dot{\tilde{e}}_T$ is a function of η and $\dot{\eta} = J_v(e_v)v$. Consequently, (41) can be bounded as follows

$$\begin{aligned} W & \geq v^T (D(v) + K_v - \sigma c_0 I) v + \sigma \tilde{e}_{T_s}^T J_v(e_v) K_0 \tilde{e}_{T_s} \\ & \geq (\lambda_{min}[D(v)] + \lambda_{min}[K_v] - \sigma c_0) \|v\|^2 \\ & \qquad + \sigma \lambda_{min}[J_v(e_v) K_0] \left\|\tilde{e}_{T_s}\right\|^2 \end{aligned} \tag{44}$$

Taking into account (28) and (44), K_0, K_v, k_q, and σ can be chosen so that

$$\begin{gathered} \lambda_{min}[D(v)] + \lambda_{min}[K_v] - \sigma c_0 > 0, \\ \lambda_{min}[J_v(e_v) K_0] > 0, \\ k_q - \lambda_{min}[K_0] > 0 \end{gathered} \tag{45}$$

which leads to the positive definiteness of W in v, $\tilde{e}_T$ and $\tilde{e}_s$. $\lambda_{min}[A]$ denotes the minimum eigenvalue of matrix A. Now, the theorem can be stated as:

Theorem 1: The region priority control law in (25) guarantees the asymptotic convergence of v, $\tilde{e}_p$, $\tilde{e}_s$, and $\tilde{e}_\varepsilon$ for an underwater vehicle given by (11) provided that (26), (27) and (28) are fulfilled and the feedback gains are chosen to satisfy conditions (38) and (45).

Proof: Using the fact that V in (31) and W in (41) are positive definite, then $\dot{V}$ is negative semidefinite as described in (40). The asymptotic convergence follows from the LaSalle Invariance Principle; $\dot{V} = 0$ implies that $v = 0$ and $\tilde{e}_p = \tilde{e}_s = \tilde{e}_\varepsilon = 0$. Hence, $v \to 0$ and $\tilde{e}_p$, $\tilde{e}_s$, $\tilde{e}_\varepsilon \to 0$ as $t \to \infty$ which finally leads to the convergence of $f(\delta p_v) \leq 0$, $g(\delta p_{vs}) = \left[g_1(\delta p_{vs_1}) \;\; g_2(\delta p_{vs_2}) \;\; \cdots \;\; g_N(\delta p_{vs_N})\right]^T \leq 0$ and $\tilde{e}_\varepsilon = 0$ as $t \to \infty$.

V. Regional Constraints

A. Obstacle avoidance

In order to take into account the obstacle avoidance goal, it is useful to define a specified sub-region containing an obstruction. When the obstacle is within the vehicle path or close to the primary target, then a repulsive sub-region must be specified to avoid the collision. A repulsive spherical shape sub-region can be specified as

$$\begin{aligned} & g_1(\delta p_{vs_1}) = \\ & r_{s_1}^2 - (x - x_{o1})^2 - (y - y_{o1})^2 - (z - z_{o1})^2 \leq 0 \end{aligned} \tag{46}$$

where r_{s_1} denotes the tolerance of the repulsive sub-region. The repulsive spherical region for an underwater vehicle was illustrated previously in Fig. 1.

B. Shape Constraint

Unlike a point or a spot, a region can be designed with various shapes such as a circle or rectangle for a planar plane or a sphere or cubic for 3D since they are associated with length, width, radius or diameter. To define a simple shape like a circle or a sphere is very straightforward as less boundary constraints are required. However, some regions (i.e.: pentagon shape or hexagonal prism) are difficult to determine even though the robot can reach it. These regions are defined with abundant and complex boundary constraints. Therefore, it is undesirable not only for a robot to fall into these regions but also for it to even to come too close.

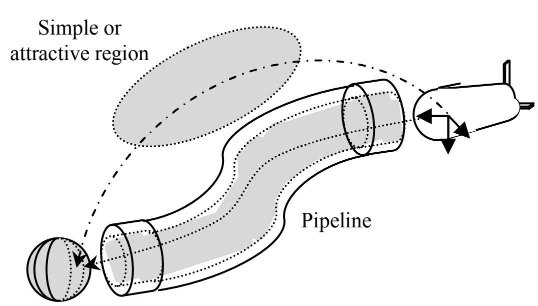


Fig. 3. Illustration of simple and complex sub-region constraints

In the case of an underwater application where a region is classified into two types; a simple region and a complex region, the complex region shape is applied when the vehicle is required to navigate through a bend pipeline while the non-complex region is the region where the vehicle maneuvers outside the pipeline as illustrated in Fig. 3. Note that, it is difficult to determine the form of a complex region especially for the region inside a structure or the region that is very close to the seabed. In this work, it is proposed that the underwater vehicle is required to be kept inside a simple or attractive region shape instead of navigating through a complex region.

C. Depth Constraint

The underwater vehicle exhibits different static forces depending on its depth. If the vehicle is positive buoyant, a large force is required for the vehicle to maintain a certain depth far below the water surface. Therefore, by defining the depth limit, less energy is required for the system. Moreover, it certainly assists the operator to monitor the vehicle navigation visually. As shown in Fig. 2, it is proposed that the depth boundary for vehicle navigation can be defined as follows

$$g_2(\delta p_{vs_2}) = z - z_{o1} \leq r_{s_2} \quad (47)$$

$$g_2(\delta p_{vs_2}) = z - z_{o1} \geq -r_{s_2} \quad (48)$$

where r_{s_2} is the depth tolerance value while (47) and (48) represent the maximum and minimum boundary, respectively. Note that, these boundaries can be changed to the other way round depending on the initial and final position of the vehicle.

VI. FUZZY INFERENCE SYSTEM

In section V, the regional motion constraints were introduced. However it remains that a strategy must be presented that determines the importance of each defined region objectives with respect to the others. As mentioned in Section IV, the weight factor $\alpha_i \in [0,1]$ can be used as a tool to define the relative importance of each objective within the set of objective functions. Greater values indicate a higher demand, whereas smaller values indicate a lower demand for that particular region objective. Note that, only a highest priority control gain is needed to be activated at a time due to the summation factor in the control law defined previously. Accordingly, it leads to less oscillatory movement of the underwater vehicle when approaching the primary sub-region.

Fuzzy logic is known as a decision-making process by mimicking the human pilot's term. A Mamdani fuzzy inference method which involves the phases of fuzzifying the inputs, applying a fuzzy operator, applying an implication method, aggregating the output and defuzzifying the output fuzzy set can be used for a decision-making procedure. In this work, the input linguistic variables and the corresponding fuzzy sets are: $obstacle = \{exist, not\ exist\}$, $shape = \{simple, complex\}$, $depth = \{maintain, not\ maintain\}$. Regarding the output linguistic variables and the corresponding fuzzy sets, they are set to be $\alpha_k = \{low, high\}$; $k = 1 \ldots 3$. The following set of rules has been implemented:

- If the obstacle exists then α_1 is high;
- If the obstacle does not exist then α_1 is low;
- If the obstacle exists then α_2 is low;
- If the shape is simple then α_2 is low;
- If the obstacle does not exist and the shape is complex then α_2 is high;
- If the obstacle exists or the shape is complex then α_3 is low;
- If the depth is maintained then α_3 is low;
- If the obstacle does not exist and the shape is simple and the depth is not maintained then α_3 is high;

A complete and consistent set of fuzzy rules with three linguistic variables defined with two fuzzy sets for each linguistic variable requires a total of 24 rules. To reduce the number of rules, a hierarchical structure that gives higher priority to the obstacle avoidance and the lowest priority to the depth keeping in the form of $obstacle \rightarrow shape \rightarrow depth$. This leads to only 8 rules in total instead of 24, and hence provides a significant reduction in the number of rules. Given that only one sub-region is activated when there are multiple sub-regions overlapping, a prevention of any possible conflict between the regions is solved in favor of the higher priority sub-region. In addition, the higher gain can be avoided since only a single α_i is allowed to be high at a time, reducing the oscillatory movement of the vehicle when reaching its target.

The $and - or$ logic operations have been calculated based on the $min - max$ operations, respectively. In addition, the implication-aggregation operations have been carried out by implementing the $min - max$ operations, respectively. The values of $\alpha_k \in [0,1]$ are obtained by the defuzzification phase using the centroid technique with normalization [15].

VII. SIMULATION RESULTS

In this section, simulations are carried out to assess the effectiveness of the proposed fuzzy-logic based sub-region priority control law for an underwater vehicle. An ODIN AUV is chosen for the numerical simulation and the parameter models can be found in [19].

The main objective is to ensure the underwater vehicle reaches the primary sub-region while fulfilling the secondary objectives (obstacle avoidance, shape and depth constraint) defined by the secondary sub-regions as much as possible. The vehicle is initialized to position $p_v(0) = [0 \quad 0 \quad 1]^T$m and orientation $e_v(0) = [0 \quad 0 \quad 0 \quad 1]^T$. Note that, only the position vector is specified as a region since the operator is incapable of observing the sub-region reaching of the orientation in the Cartesian space. In the simulation, the primary sub-region is specified as

$$f(\delta p_v) = (x_v - x_{vd})^2 + (y_v - y_{vd})^2 + (z_v - z_{vd})^2 - r_v^2 \le 0 \tag{49}$$

where $[x_{vd} \quad y_{vd} \quad z_{vd}]^T = [8 \quad 0 \quad 5]^T$m and the orientation in the unit quaternion is kept constant. Similarly, the objective functions for the secondary sub-regions with constraints can be specified as follows

$$g_1(\delta p_{vs_1}) = (x_v - x_{vsd_1})^2 + (y_v - y_{vsd_1})^2 + (z_v - z_{vsd_1})^2 - r_{vs_1}^2 \ge 0 \tag{50}$$

$$g_2(\delta p_{vs_2}) = (x_v - x_{vsd_2})^2 + (y_v - y_{vsd_2})^2 + (z_v - z_{vsd_2})^2 - r_{vs_2}^2 \ge 0 \tag{51}$$

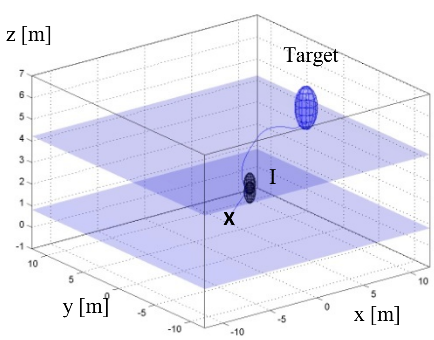
$$g_3(\delta p_{vs_3}) = (z_v - z_{vsd_3})^2 - r_{vs_3}^2 \ge 0 \tag{52}$$

where (50), (51) and (52) are assigned for obstacles avoidance, shape and depth constraints, respectively. The reference points for each secondary sub-region are set to $[x_{vsd_1} \quad y_{vsd_1} \quad z_{vsd_1}]^T = [2 \quad 0 \quad 1.8]^T$ m ; $[x_{vsd_2} \quad y_{vsd_2} \quad z_{vsd_2}]^T = [2 \quad 0 \quad 2.2]^T$m ; $z_{vsd_3} = 2.5$ while the tolerances are chosen as $r_{vs_1} = 0.5$m ; $r_{vs_2} = 0.5$m ; $r_{vs_3} = 1.8$m. For the best performance of the region priority reaching controller with restoring forces compensation, the gains are set to the following

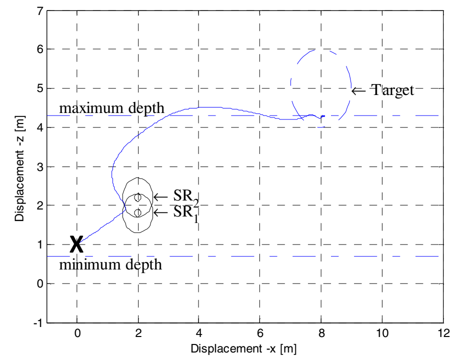
$$k_{p1} = 18.8; k_{p2} = 36; k_q = 100;$$
$$K_v = diag([90\ 90\ 90\ 40\ 40\ 40]); \sigma = diag([0.08 I_3\ 0.9 I_3])$$
$$\alpha_i = [100\ 100\ 0.5]^T; \Gamma = 2I_4;$$

where $i = 1, 2, 3$ and I is an identity matrix.

Fig. 5 shows the convergence of an underwater vehicle into the primary sub-region while fulfilling the secondary sub-regions constraints. At a secondary region intersection where it contains the obstacle, shape and depth constraints, the fuzzy approach is utilized in order to select that only one sub-region is activated at a time. Consequently, the control effort leads the vehicle to move out from this undesired sub-region as can be clearly seen in Fig. 5(b).



(a)



(b)

Fig. 5. (a) 3D illustration of underwater vehicle position; "X" and "I" mark the initial position and the intersection of the secondary sub-regions, respectively, (b) A planar plane view; SR_1 and SR_2 denote the obstacle and complex sub-regions, respectively.

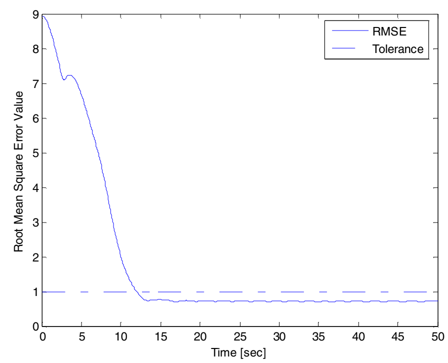


Fig. 6. Convergence of position error in terms of root mean square error

Fig. 6 shows that the controller error expressed in terms of the root mean square error converges to the specified tolerance value in parallel with the stability properties of the designed controller. With regards to the vehicle attitude, there are presented in Fig. 7. As the figure reveals, the vehicle attitude was kept constant with a slight change at the initial start time due to the control needing to adapt with the uncertain restoring forces. An intersection of the secondary sub-regions occurs at 2.65 s to 2.95 s and its normalized index is presented in Fig. 8. To avoid an excessive control effort and oscillatory vehicle movement that might happen due to the high gains resulting from the summation feature in the control law, a fuzzy approach is implemented during this period. Therefore, only the highest priority secondary sub-region is allowed to be activated at one time. In other words, a possible conflict among the sub-regions with constraints is prevented by solving in favor of the fuzzy inference system.

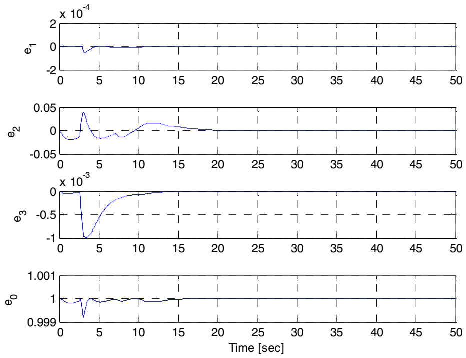


Fig. 7. The vehicle attitude

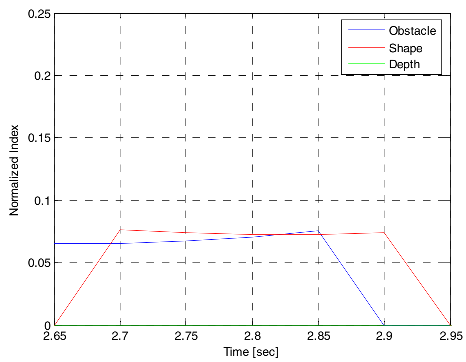


Fig. 8. Normalized index at the region intersection

VIII. CONCLUSION

In this paper, a new adaptive sub-region control concept with a fuzzy logic algorithm is presented for an underwater vehicle system. The controller was designed to allow the convergence of a vehicle into the primary and secondary sub-regions. When the multiple sub-regions with constraints are overlapping, a fuzzy approach is used to determine one sub-region with the highest priority as the active sub-region. Therefore, only the priority sub-region is allowed to be high at a particular time in order to avoid an excessive control effort and oscillatory motion. In addition, the unit quaternion representation is used in order to achieve a singularity-free attitude representation. With respect to uncertain gravitational and buoyancy forces, sufficient conditions are placed for choosing the proper feedback gains where the stability is guaranteed using the Lyapunov type approach. A simulation case study of an ODIN vehicle has been developed to demonstrate the validity of the proposed control law and the fuzzy logic technique.

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