Time Reversal Mirror: Temporal and Spatial Focusing Tool

Divya Pratap Singh Parihar, Ankit Agarwal and Monika Agrawal
Indian Institute of Technology, Delhi
Hauz Khas, New Delhi, India

***Abstract*-Time reversal is a technique to focus waves, both temporally and spatially, through an inhomogeneous medium. The incident pressure wave is time reversed and re-transmitted from an array of transducers to focus the pressure on a source or at a point target which may be reflective. This array of sensors is called time reversal mirror (TRM). In this paper, we have studied the spatial and the temporal focusing properties of different TRM array geometries. Two dimensional as well as three dimensional geometries for different configurations and environment have been studied. It has been found that there exists optimal array configuration for both temporal and spatial focusing.**

I. INTRODUCTION

The term, time reversal mirror (TRM), was originally introduced by M. Fink [1] in early 90's. They extended the concept of optical phase conjugate mirrors, valid for monochromatic signals to wideband signals followed by its experimental demonstration [2],[3]. M. Fink et al. demonstrated the effect of multiple scattering on both temporal and spatial focusing [8] and also suggested the time reversal cavities [9] for better focusing. Further continuous closed phase conjugated arrays (TRM are also known as phase conjugated arrays) were suggested by Jackson and Dowling [10]. After this, the TRM theory resulted in the development of concepts like passive phase conjugation (PPC) [12] and D.O.R.T [11], iterative time reversal etc which has wide applications in communication, non destructive testing, imaging, active sonar, selective focusing etc.

Focusing a wave on a target/source through an inhomogeneous medium raises the signal to noise ratio (SNR), therefore enhancing the performance. Time reversal techniques take advantage of high order multiple scattering, multi-paths, which are otherwise considered big hurdles in designing many systems.

In practice, the TRM is designed using limited number of transducers, arranged in some geometry. This limited dimension of array induces spread whose width is related to the angular aperture of the array. Another problem is of grating lobes [6], which is due to the spatial sampling of the transducers. Further the environment and configurations of array geometry also effect the focusing of TRM.

In this paper, we have studied the spatial and the temporal focusing properties of different TRM array geometries. Two dimensional (2D) as well as three dimensional (3D) array geometries for different configurations and environment have been studied. The focusing properties of linear (standard array used by M. Fink), circular arc - 2D arrays and cylindrical, cubical - 3D arrays, have been studied for various simulation parameters. Depths of the water column, separation between the source and the array, array aperture, sensor separation etc. are the parameters that have been varied to study both the temporal and the spatial focusing.

II. TIME REVERSAL MIRROR: BASIC CONCEPTS

An acoustic wave propagating in an inhomogeneous and lossless fluid medium with acoustic velocity $c(r)$, having pressure field $p(r,t)$ can be described as [4],

$$\Delta p(r,t) - \frac{1}{c^2(r)}\frac{\partial^2}{\partial t^2}p(r,t) = 0 \quad (1)$$

Where r and t are the space and time coordinates.

Looking at this propagation equation, we note that it is a second order equation in variable time variable t, therefore, if $p(r,t)$ is a solution of this differential equation then $p(r,-t)$ is also the solution [1],[2], i.e. the pressure filed is invariant under time reversal. This is the basic principle behind the time reversal (TR) techniques. If the medium has a frequency dependent attenuation [5], the propagation equation may contain odd order time derivative operator and therefore, this property is not valid in that case.

Practically, a probe signal is transmitted from the receiver position. The pressure field $p(r\ ,t)$ at each transducer of TRM array is stored during a time interval ΔT and is then time reversed and sent back. This time reversed signal focuses at the location of probe source both temporally and spatially.

The TR focusing is related to the matched filter technique. Assume that a probe source transmits an impulse $\delta(t)$. Let TRM consists of array of N transducers. This array records the corresponding signal, $h_i(t)$, i=1,2,3....N. Due to the reciprocity, the propagation impulse response from the probe source to the i^{th} element of TRM aray is same as the impulse response from i^{th} element array to the loaction of probe source. Therefore the time reversed signal received at the probe source is given by,

$$r(t) = \sum_{i=1}^{N} h_i(t) * h_i(-t) = \sum_{i=1}^{N} c_i(t) \quad (2)$$

Where r(t) is the total time reversed signal received.

978-1-4244-5222-4/10/$26.00 ©2010 IEEE

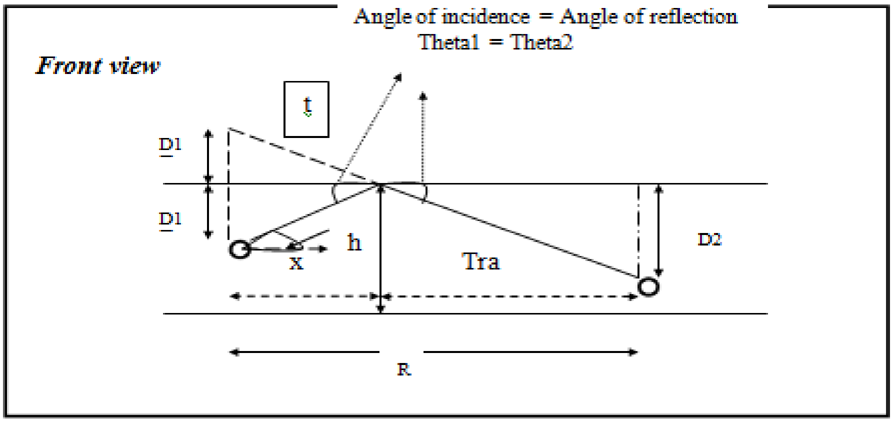


Figure 1. Path traced by a ray from transmitter to receiver

III. SYSTEM MODEL

The spatial and temporal focusing of TRM having different array geometries has been studied, in a simulated environment using MATLAB. An underwater channel model based upon ray propagation of acoustic waves has been developed, assuming that reflections are present only at surface and at sea bottom, reflections at the surface are perfect, sound speed is constant and rays obey inverse square law. These assumptions are being made to obtain a simple and realizable channel.

To calculate the path length traversed by the wave in multipath environment, the source/target and the transducer are assumed at some arbitrary depth from the surface.

As shown in Figure 1, Let $D1$, $D2$ be the depth of the transmitter and the receiver respectively from the water surface. Let h be the depth of water column and R be distance between the transmitter and receiver. Using square law it can be shown that the length of direct path is $Sqrt(R^2 + (D2 - D1)^2$. Similarly the path length after n (odd) reflections with the last reflection from the surface is $Sqrt\{(D1 + D2 + (n-1) * h)^2 + R^2\}$ and with last reflection on the bottom is $Sqrt\{(2 * h - D1 - D2 + (n-1) * h)^2 + R^2\}$. Path length of the signal reached after n (even) reflections, with last reflection from the surface is $Sqrt\{(h + D1 + D2 + (n-1) * h)^2 + R^2\}$ and the last reflection from the bottom is $Sqrt\{(h - D1 + D2 + (n-1) * h)^2 + R^2\}$.

These path lengths have been used to calculate the time taken by all the multipath to reach at each transducer from probe source and vice-versa. The total signal received at the TRM array is obtained using principle of superposition, by adding all these signals from different multipath and transducers . In the simulation, whenever the TRM is 1D or a 2D array, channel is assumed to be 2D channel. i.e., the multipath propagation is assumed to be planar, or all waves reaching the array from the probe source are in same plane. For 3D, TRM array 3D channel has been simulated.

IV. SIMULATION MODEL

Using the above channel model, the signal at TRM and at source/target has been simulated in Matlab for different channel and array configurations - linear, circular-arc, cubical and cylindrical array geometries with different dimensions. The three parameters that have been used as variables to study both the temporal and spatial focusing properties of all four arrays are – the number of transducers in the array, the height of the water column and the separation between the source and the transducer array.

Focusing has been studied using focusing metrics. These metrics have been utilised in deciding the performance of different arrays considering their temporal and spatial focusing.

A_focusing has been defined as the ratio of amplitude of the main lobe to amplitude of first side lobe of signal received at source,

$$A\_focusing = \frac{Amplitude\ of\ mainlobe}{Amplitude\ of\ first\ sidelobe}$$

Second is *S_focusing*, defined as 3dB bandwidth of the signal received at the source location.

$$S\_focusing = 3dB\ Bandwidth\ of\ mainlobe$$

Focusing properties of four types of array geometries have been studied. We have used some sample configurations of these arrays to study their focusing properties. Before explaining the results of simulation studies, array geometries and the different design parameters have been discussed.

Length of each array has been taken as 1 unit for any depth of the water column and the separation between the source and the array. Length of the array remains constant unless the aperture is used as parameters to study the focusing properties. Linear array geometry is same as the standard array used by M. Fink [1] in which transducers has been arranged in a linear fashion. For the circular arc array, arc length has been taken as 1 unit. Also we have used a constant sector angle with a constant radius Rd which has been taken as 10 units in most of the simulations, unless the radius is a variable in the simulation.

For cylindrical array, we have taken the radius as 0.3 units and height as 1 unit. Cylindrical array is consisted of circular rings arranged one top of another, with 5 transducers in a single ring and the separation between the two adjacent rings determined by total number of transducers used in that simulation. For cubical array, the dimension of each side is 1 unit. Transducers have been put on the sides of the cube in a similar fashion to the linear arrays. Here the separation between the array and the source has been taken from the centroid of that array.

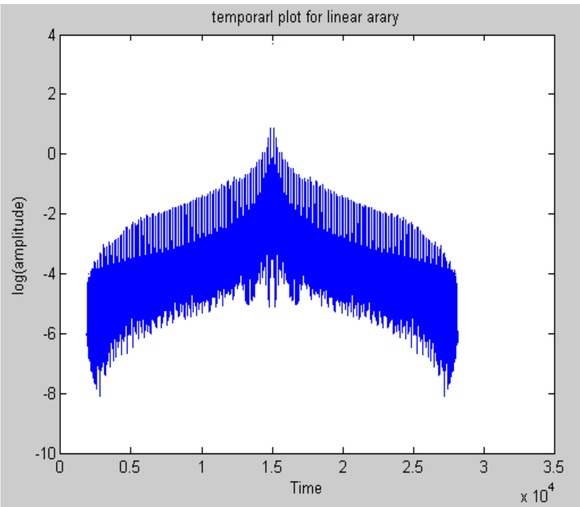
A unit impulse is being sent from the probe source location to the array. This impulse function generates a sharp peak in the signal received after TR by TRM at the probe source location, at a particular instant, due to temporal focusing. The spatial spectrum at the source has been taken along the line parallel to the array.

V. Simulation Results

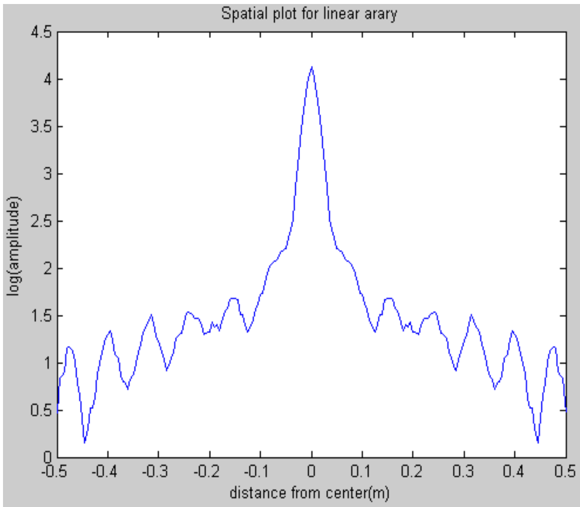
M. Fink[1] [2] introduced the concept of a linear TRM in which the receiver array transducers were arranged in a linear fashion. This linear array results not only in temporal focusing at the target but also the strong spatial focusing. Fig.2(a) and Fig.2(b) show temporal and spatial focusing for the linear array. Amplitude of signal received at the probe source as function of time is plotted in fig. 2(a). It can be easily noted that most of the signal energy is concentrated for some time instant, which is much smaller than impulse signal transmitted from probe source. It can be further noted that unlike other systems, in TRM multipath are not spreading the signal energy but focusing it to probe source location. Fig 2(b) plots the amplitude of signal received in neighborhood of probe source, validates the spatial focusing.

Temporal focusing properties of different array geometries have been studied using *A_focusing* and *S_focusing.*

First for constant aperture, the temporal focusing is measured in terms of *A_focusing* by increasing the number of transducers for all array geometries. Depth of the water column – H, has been taken as 4 units which is precisely the separation between the upper and the lower reflecting surfaces. Separation between the array and the source has also been taken to be 4 units. All four array geometries, of 1 unit dimension as described earlier have been studied. It has been observed that *A_focusing* increases with the increase in number of transducer, but it reaches to a saturation limit after which increase in the transducer will not improve the performance as shown in Fig 3. The above observation is because the aperture of the array is fixed. In Fig. 3, y-axis is basically *A_focusing* metric which has been plotted for each array against the number of transducers in that array. Similar focusing characteristics have been observed for all four array geometries. The *A_focusing* metric has similar behaviour for other values of the H the depth of water column and R the range between source and TRM array.



2(a).



2(b)

Figure 2(a) and 2(b). Temporal and Spatial spectrum for the linear array (standard array used by M. Fink)

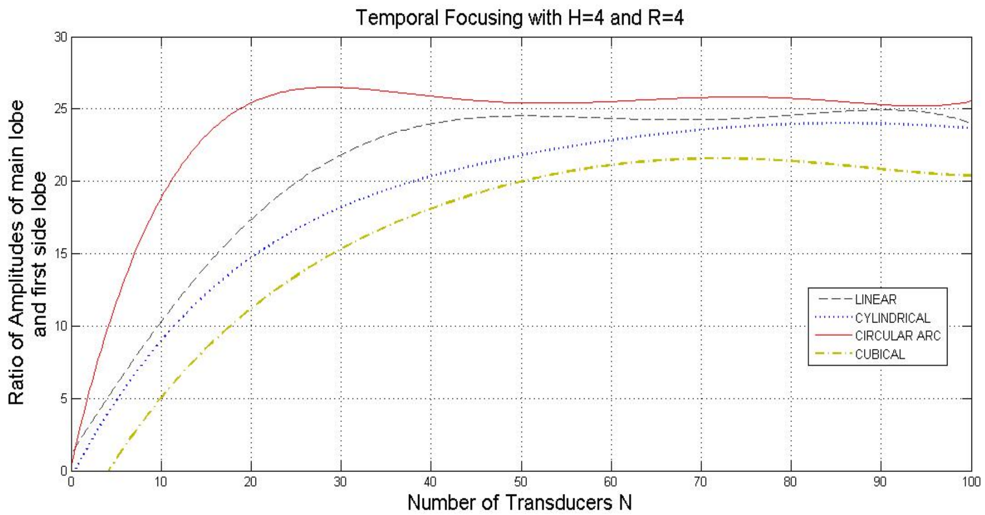


Figure 3. *A_focusing* as a function of number of transducers in the array

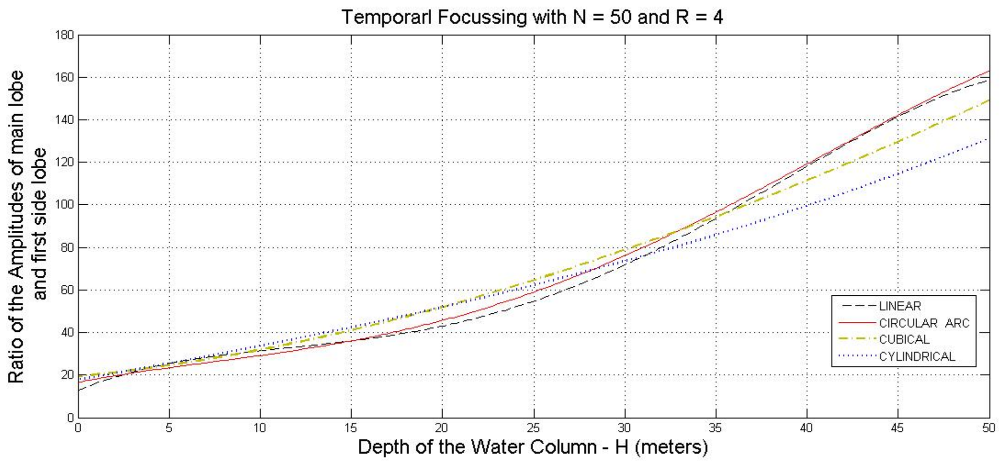
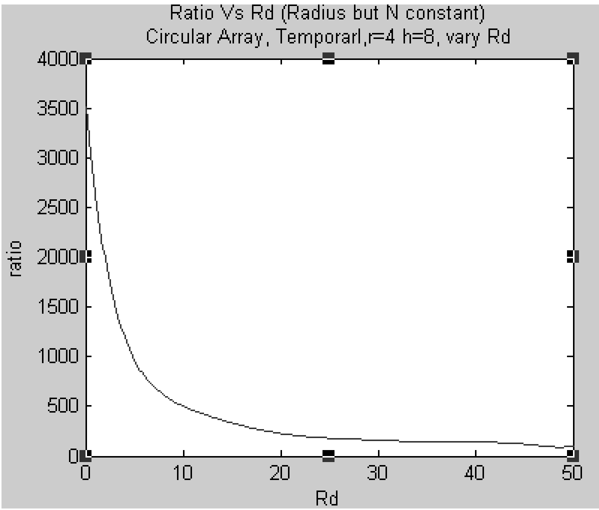


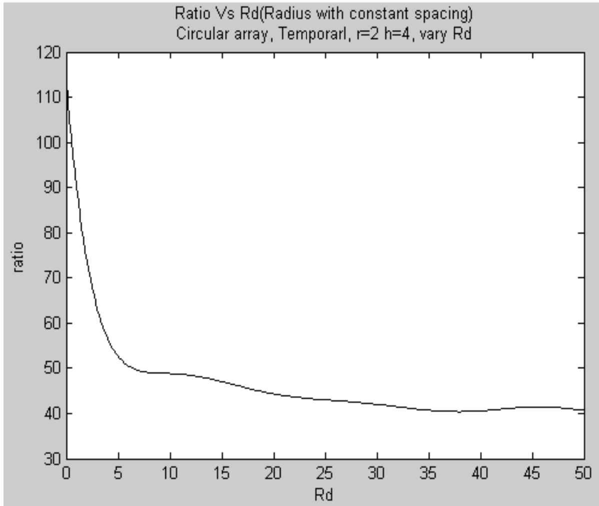
Figure 4. *A_focusing* as a function of depth of water column

Further the focusing has been studied as the depth of water column, for each array, by varying the depth of the water column, while keeping other channel parameters, i.e. number of transducers in each array and the separation R, fixed. The temporal focusing, measured in terms of *A_focusing* increases as shown in Fig.4. Similar observations have been made while increasing the separation between array and the source.

Further, comparing the performance curves of different array geometries reveals that circular arc array provides the best focusing among the geometries studied. Optimal focusing can be achieved by 30% lesser number of transducers when compared with other geometries as can be seen in Fig.3. Linear arrays can be seen as special case of circular arc array having infinite radius. To compare circular arc and linear array, the focusing has been studied as the function of the radius of circular arc array. . For fixed bounded medium as the radius of the arc is increased, focusing decreases even if the number of transducers is increased with the radius to maintain the constant sector angle. The focusing decreases by almost 1.5dB as shown in Fig. 5(b). And if the separation between the transducers is increased with radius, the focusing decreases by almost 3dB as shown in Fig. 5(a). Therefore in both cases circular arc focuses better than linear array. There is performance improvement of almost 3dB. Here, *A_focusing* i.e. the ratio of the amplitudes of primary and the first side lobe has been plotted against the increasing radius of the circular-arc.



5(a)



5(b)

Figure 5. A_focusing as a function of Radius (in meters) of the circular arc. (a) separation between the transducers is increasing with radius. (b) Number of transducers are increasing to maintain the constant sector angle and spacing

Spatial focusing has been studied using *S_focusing* as well as *A_focusing*. In a bounded medium, for fixed aperture, *S_focusing* i.e. the 3db band-width remains constant even by increasing the number of transducers for all the array geometries as in Fig. 6, though the amplitude of the focused signal at the source increases with the increase in the number of transducers in array. In Fig 6 distance of the 3db point, in the spatial spectrum of that array, from the source has been plotted against the number of transducers with channel dimensions as H=4 and R=4 units. As the magnitude of the focused signal at the source increases on increasing the no. of transducers and 3db bandwidth remains constant, this together implies increase in spatial focusing.

As the depth of the water column is increased, *A_focusing* remains constant and *S_focusing* metric increases and becomes stagnant after some distance as in Fig. 7 and Fig.8.

A significant difference between the 2D and the 3D array properties is evident from fig.7. The *S_focusing* of 3D arrays is much less as compared to 2D arrays with increase in water column depth. Therefore it can be said that spatial focusing achieved using 3D arrays is better than that of 2D arrays.

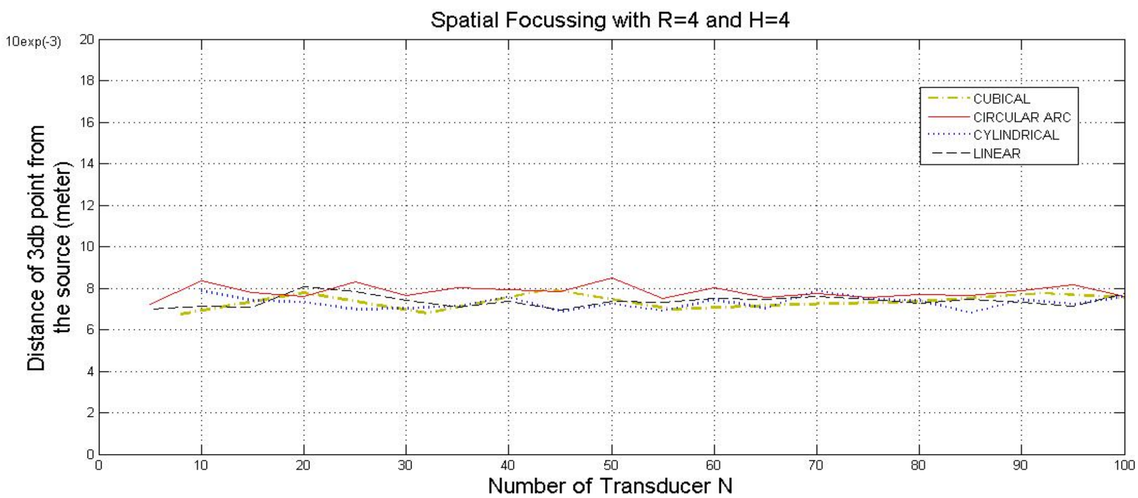


Figure 6. *S_focusing* as a function of number of transducers in an array

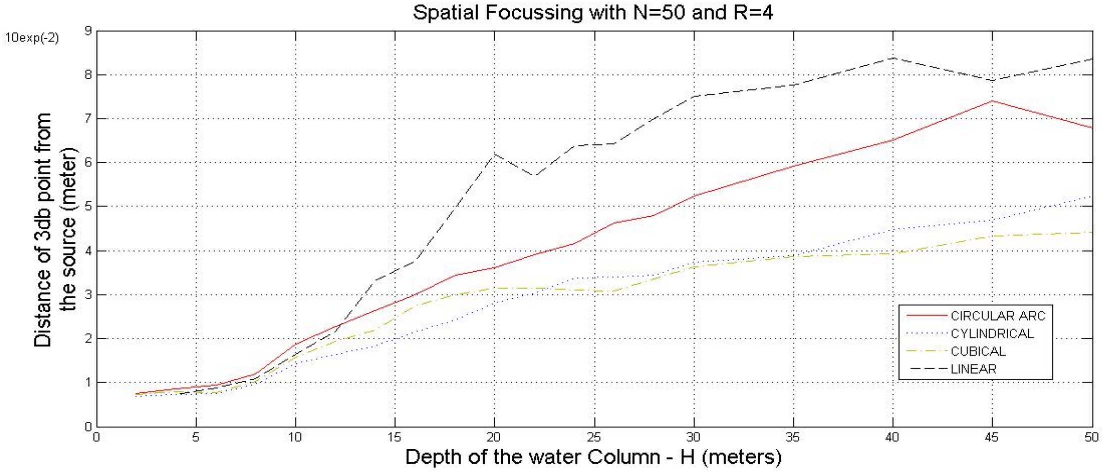


Figure 7. *S_focusing* as a function of depth of the water column

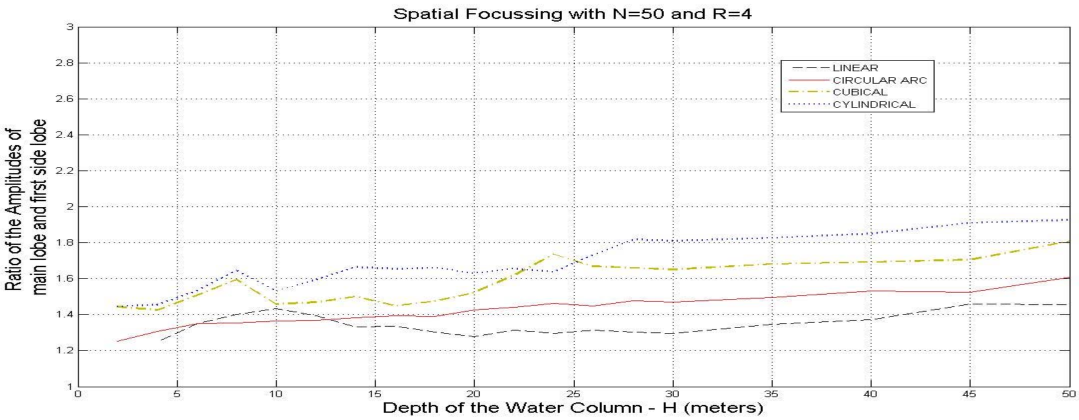
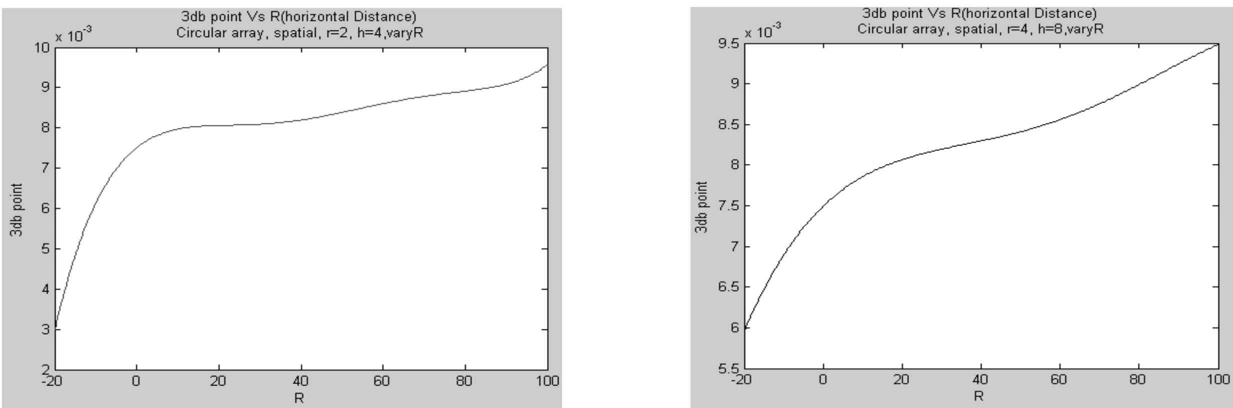


Figure 8. *A_focusing* as a function of depth of the water column

Similar observations have been made while increasing the horizontal distance between the source and the array. Fig. 9(a) and Fig. 9(b) show a case of circular arc arrays where the position of 3db point has been plotted against the separation between the source and the array for two different arc radius and channel dimensions. As in the previous case, distance of the 3db point increases and tends towards saturation after some time.



(a) (b)

Figure 9. *S_focusing (meters)* as a function of R(meters). (a) radius of the arc = 2 meters, depth=4. (b) radius =4 and depth= 8 meters

This shows us that channel parameters such as depth of water column H and distance between the source and the array R, play different roles in temporal and spatial focusing. Increase in depth and range increases the quality of temporal focusing whereas deteriorates the spatial focusing. There is a trade-off between spatial and temporal focusing with the range and water-column depth.

VI. CONCLUSION

Though the simulation model used here is a simplified one but it can be concluded that geometries do affect the focusing of TRM and there exists an optimal number for transducers which give the maximum focusing depending upon the geometry. Circular arc array proves to be the best of the lot in this regard. Moreover, the channel dimensions such as the depth of the water column and the separation between the array and the source/target do have certain effects on the focusing. Increase in depth and range improves the temporal focusing but deteriorates the spatial focusing. Further 3D arrays seems to have better spatial focusing. One needs to handle these parameters carefully while deciding about the required temporal and spatial focusing at the target.

REFERENCES

[1] Mathias Fink, "Time Reversal of Ultrasonic Fields – Part 1 : Basic Principles" *IEEE Transactions on ultrasonics, Ferroelectricals and Frequency control,Vol. 39*, September 1992

[2] Francois Wu, Jean- Louis Thomas and Mathias Fink, " Time Reversal of Ultrasonic Fields – Part 2: Experimental results," *IEEE Transactions on ultrasonics, Ferroelectricals and Frequency control*,Vol. 39, September 1992

[3] M. Fink, C. Prada, F. Wu and D. Cssereau, "Self focusing with time reversal mirror in inhomogeneous media," *in Proc. IEEE Ultrasonic. Symp. 1989*

[4] P. M. Morse and K. U. Ingard, *Theoritical Acoustics*. New York: McGraw Hill, 1968

[5] A. Nachman, J. Smith, R. Waag, "An equation for acoustic praopagation in inhomogeneous media with relaxation losses," *J. Acoust. Soc. Amer.*, vol. 88, no. 3, pp. 1584-1595 1990.

[6] G. S. Kino, Acoustic Waves, *Signal Processing Series. Englewood Cliffs*, NJ: Prentice Hall, 1987.

[7] A. Popoulis, *Signal analysis*, New York McGraw Hill, 1984.

[8] A. Derode, A. Tourin, and M. Fink, "Limits of time reversal focusing through multiple scattering: Long range correlation," *J. Acoust. Soc. Amer*. 107 (6), June 2000.

[9] Didier Cassereau and M. Fink, "Time reversal of ultrasonic fields Part3: Theory of the Closed Time Reversal Cavity" *IEEE Trans. on Ultrasonic, Ferroelectrics and Frequency Control*, Vol. 39, no.5 , September 1992

[10] D. R. Jackson, D. R. Dowling, "Phase conjugation in underwater acoustics", *J. Acoust. Soc. Am. 89*, 171–181 (1991)

[11] C. Prada, J.-L. Thomas, M. Fink, "The iterative time-reversal process: analysis of the convergence", *J. Acoust. Soc. Am.* 97, 62 (1995)

[12] Rouseff, D. Jackson, D.R. Fox, W.L.J. Jones, C.D. Ritcey, J.A. Dowling, D.R. "Underwater acoustic communication by passive-phase conjugation: theory and experimental results*", IEEE Journal of Oceanic Engineering*, Issue: 4, Volume: 26, 821-831(2001).