# Passive Localization of an Autonomous Underwater Vehicle with Periodic Sonar Signaling

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*Abstract-* **As Autonomous Underwater Vehicles (AUVs) play an increasingly important role in ocean community, localization of AUVs is attracting more and more attention in various applications. This paper presents a simple iterative solution to passive localization of an AUV which has a periodic sonar transmission. The method exploits the time difference of sonar transmissions and receiving interval measurements using a single linear array. Simulations show that the method yields an effective source location estimator without using a complex receiving array configuration; in the meantime, it can be readily implemented in a real-time system with moderate computational complexity.**

## I. INTRODUCTION

The problem of localizing an autonomous underwater vehicle (AUV), both actively and passively, is encountered in many diverse applications. In a passive mode, the receivers receive signals emitted by the vehicle in order to determine its location. For such scenarios, an accurate range measurement is typically more difficult than the azimuth one, since time synchronization is rarely available.

Many methods have been developed to measure locations of a maneuvering target, e.g. the methods based on the angle of arrival (AOA), the time of arrival (TOA), and the time difference of arrival (TDOA) [1]. The AOA method originates from classical radio direction finding techniques. Implementation of the AOA requires complex receiving antenna arrays and proper angle of arrival estimation algorithms. Three or more angle estimates are needed because by combining only two AOA estimates there would be a large uncertainty associated with any measurements of an object lying close or on the line that connects the two arrays performing the measurements.

The TOA algorithm assumes that the signal sent by the target takes an absolute time to reach the array. Specifically the measurement of absolute TOAs of a known burst transmitted by target to array is employed by TOA algorithms, which requires that all the participating targets and stations' clocks are synchronized.

The TDOA algorithms are based on measuring the difference in the time of reception of signals at different stations, without requiring synchronization across all the participating stations and targets. However the stations involved in the location estimation process must be tightly synchronized. The target position can be identified as the interception point of the two hyperbolae [1]. A number of algorithms for TDOA are available; some of them uses iterative methods [2]-[3] that require good initial guesses of the solution.

The method combining TDOA and AOA estimates the target position more precisely than the conventional AOA-only or TDOA-only methods, however it also needs complex receiving arrays as the AOA method does [4]. Another method combining TOA, DOA and the associated changing rate is proposed in Ref. [5], but it also needs to precisely know TOA and usually requires a good initial guess to reduce the convergence time of an optimal solution. All methods mentioned above are based on either multiple stations or a complicated array.

In this paper, we present a simple iterative solution to passive localization of an autonomous underwater vehicle which has a periodic sonar transmission. The method exploits the time difference of sonar transmissions and receiving interval measurements using a single linear array. The rest of the paper is organized as follows. The description of the scenario, the AUV trajectory model, and the features of the proposed method are presented in Section II. Section III presents the distance measurement error analysis of the method. Simulation results are given in Section IV, which are used to demonstrate the effectiveness of the developed approach. Finally, Section V concludes the paper.

## II. PROBLEM AND METHODOLOGIES

### A. Localization Scenario

Consider the localization scenario shown in Fig. 1. An autonomous underwater vehicle (AUV) is assumed to move with a constant velocity along a straight line, which is appropriate for AUV moving in a short period. The AUV sonar emits a pulse signal periodically with a period of $T$. The signals radiated from the AUV are received by a linear array of $M$ stationary sensors, which is located way from the vehicle. The center of the array is defined as the coordinate origin $O$, and the array elongation direction defines one of the coordinate axes. Suppose that within the data processing period $\Delta t = KT$ ( $K$ is an integer), the vehicle moves from point $A(r_0, \theta_0)$ to point $B(r_1, \theta_1)$, where $r_0$ denotes the range and $\theta_0$ denotes the azimuth, and that time interval of two pulse signals arriving the center of the array is $\Delta\tau$.

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For the stationary case, $\Delta t$ and $\Delta\tau$ can be the same. When the vehicle is moving, those two quantities would be different except for a circular motion mode centered about the receiving array. It is this difference that is exploited by the method in this paper.

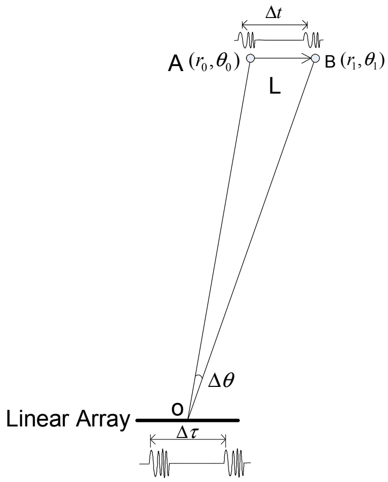


Figure 1. AUV localization scenarios.

### B. *Azimuth and range computation*

For simplification, the linear array and the vehicle are assumed to locate in the same horizontal plane. The azimuth of each AUV location would be computed by beamforming process of the received signal. After obtaining the azimuths of points $A$ and $B$, we can compute the azimuth difference of two positions by

$$\Delta\theta = \theta_1 - \theta_0 \tag{1}$$

Suppose that the velocity of the vehicle, $v$, is known. Thus the distance between $A$ and $B$ can be calculated by

$$L = \Delta t \times v\,. \tag{2}$$

As mentioned above, the time interval of two adjacent pulse signals arriving the center of array is different. The difference of $r_1$ and $r_0$ can be evaluated by

$$\Delta r_1 = r_1 - r_0 = (\Delta\tau_1 - \Delta t)\times c\,. \tag{3}$$

Here $c$ is the speed of sound. When the vehicle is moving in a circle around the array, $\Delta\tau_n$ is equal to $\Delta t$ and thus $\Delta r_n$ is equal to zero. Combining with the azimuth changing rate, this situation can be distinguished from the stationary case, for which the azimuth difference between $\Delta t$ would be zero.

We first derive a method of computing the AUV initial distance, $r_0$, based on the triangulation geometry. From Fig. 1, we see that $\Delta\theta_1$, $L$ and $r_1$ are mutually related and this relation can be expressed in terms of $r_0$ and $\Delta r_1$, i.e.,

$$L^2 = r_1^{\,2} + r_0^{\,2} - 2r_1 r_0 \cos\Delta\theta_1\,. \tag{4}$$

Substituting (3) into (4) yields

$$r_0 = \frac{\Delta r_1}{2} + \sqrt{\frac{L^2 - \Delta r_1^{\,2}}{2\times(1-\cos\Delta\theta_1)} + \frac{\Delta r_1^{\,2}}{4}}\,. \tag{5}$$

Applying the well-known Maclaurin series expansion

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots \tag{6}$$

to (5), we obtain an approximation for calculating the initial vehicle distance, $r_0$:

$$r_0 \approx \frac{\Delta r_1}{2} + \sqrt{\frac{L^2 - \Delta r_1^{\,2}}{\Delta\theta_1^{\,2}} + \frac{\Delta r_1^{\,2}}{4}}\,. \tag{7}$$

Clearly error in the approximate solution is on the order of $\mathrm{O}(\Delta\theta_1^{\,4})$.

In the localization scenario, after the working parameters of the vehicle are set, $\Delta t$ is known. In order to calculate the range difference $\Delta r_1$ use (3), we need to know $\Delta\tau_1$. A number of methods have been developed to calculate the time delay of two signals [6]-[8]. In this paper, we adopt a classical method based on generalized cross-correlation of the two signal arrivals. Specifically the received signal of each sensor is divided into time slices of duration $\Delta t$. Two adjacent slices are transformed into the frequency domain to form the conjugate product. After appropriate weighting, an inverse DFT is performed and the peak of the resulting generalized cross-correlation function is located in the time domain, from which $\Delta\tau_n$ at sequential steps ($n = 1, 2, \ldots$) is estimated. The range difference, $\Delta r_n$ of each step can be calculated in an iterative manner with above calculated $\Delta\tau_n$, i.e.,

$$\begin{aligned} r_n &= r_{n-1} + \Delta r_n \\ &= r_{n-1} + (\Delta\tau_n - \Delta t)\times c \end{aligned}\,. \tag{8}$$

Combining the azimuth and range measured above, the vehicle coordinates $(r_n, \theta_n)$ can be determined sequentially.

## III. MEASUREMENT ERROR ANALYSIS

With some simple algebra, we can have from (4)

$$\frac{L^2 - \Delta r_1^{\,2}}{1-\cos\Delta\theta_1} = 2r_0(r_0 + \Delta r_1)\,. \tag{9}$$

Taking a derivation of $r_0$ with respect to $\Delta\theta_1$ yields

$$\frac{\partial\left(\dfrac{L^2 - \Delta r_1^{\,2}}{1-\cos\Delta\theta_1}\right)}{\partial\Delta\theta_1} = \left(4r_0 + 2\Delta r_1\right)\frac{\partial r_0}{\partial\Delta\theta_1}\,. \tag{10}$$

Simplifying (10) gives

$$\frac{\partial r_0}{\partial\Delta\theta_1} = \frac{(L^2 - \Delta r_1^{\,2})\sin\Delta\theta_1}{(1-\cos\Delta\theta_1)^2\left(4r_0 + 2\Delta r_1\right)} \tag{11}$$

Substituting (5) into (11) gives

$$\begin{aligned} \frac{\partial r_0}{\partial\Delta\theta_1} &= \frac{(L^2 - \Delta r_1^{\,2})\sin\Delta\theta_1}{(1-\cos\Delta\theta_1)^2\left(4r_0 + 2\Delta r_1\right)} \\ &= \frac{(L^2 - \Delta r_1^{\,2})\sin\Delta\theta_1}{(1-\cos\Delta\theta_1)^2\left(4\Delta r_1 + 2\sqrt{\dfrac{L^2 - \Delta r_1^{\,2}}{2\times(1-\cos\Delta\theta_1)} + \dfrac{\Delta r_1^{\,2}}{4}}\right)} \end{aligned} \tag{12}$$

This equation represents the sensitivity of $r_0$ to $\Delta\theta_1$. Given $L$ in (2), Eq. (12) is evaluated as a function of $\Delta\theta_1$ and the results are shown in Fig. 2.

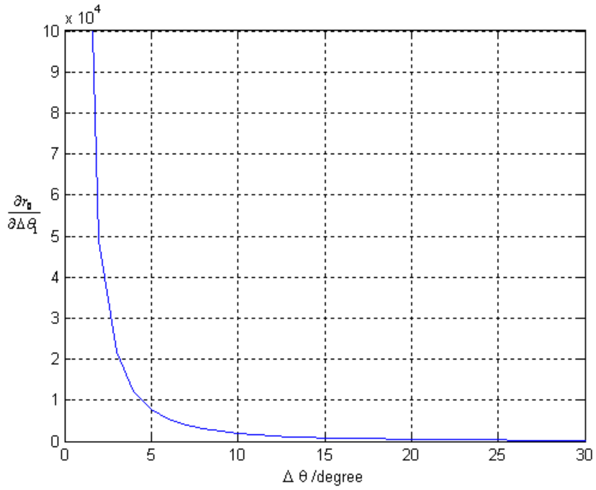


Figure 2. The sensitivity of $r_0$ to $\Delta\theta_1$

As shown in Fig. 2, the initial range, $r_0$ is sensitive to $\Delta\theta_1$ when $\Delta\theta_1$ is small. The sensitivity decreases as $\Delta\theta_1$ increases. The accuracy of azimuth is determined by the aperture length of the linear array. Thus $\Delta\theta_1$ should be as large as possible in order to improve the accuracy of $r_0$ estimation. According to (8), the range $r_n$ ($n \neq 0$) estimation accuracy is only related to estimation of $\Delta\tau_n$, which is related to the movement of vehicle and $\Delta t$. In section II, we mentioned that $\Delta t$ is the time interval of data processing. It should be larger than or equal to the period of AUV sonar signal $T$. If the accuracy of the linear array is high enough to measure $\Delta\theta_n$, $\Delta t$ should be as short as possible except for the calculation of $r_0$. This makes the assumption of AUV moving along a straight line within $\Delta t$ more accurate.

## IV. SIMULATION RESULTS

For clarification, we would define the steps of the proposed method more precisely:

1. Calculate the initial azimuth difference $\Delta\theta_1$ by beamforming.
2. Use the method to calculate the initial time delay $\Delta\tau_1$.
3. Calculate the initial range, $r_0$, with $\Delta\theta_1$, $\Delta\tau_1$ and the parameters given.
4. Calculate azimuth $\theta_n$ and time delay $\Delta\tau_n$ for each received pulse signal; then update $\Delta r_n$.
5. Update $r_n$.
6. Combining the azimuth and range information, update the vehicle position $(r_n, \theta_n)$.
7. Repeat step 4-6.

Under the geometry shown in Fig. 1, four conditions listed in Table I are simulated. To compare the measurement error, we choose time interval of the second condition equal to five times of the first one. Here $c$ is $1500\, m/s$; the sonar signal is LFM whose central frequency $F_c$ is $12kHz$ and bandwidth is $10\, kHz$; the signal is received by a uniform linear array with 100 elements and $\lambda_c/2$ spacing, where $\lambda_c = c/F_c$ is the wavelength associated with the center frequency.

TABLE I
SIMULATION CONDITIONS

| AUV trajectory / Simulation parameters | Straight line 1 | Straight line 2 | Circle | Irregular trajectory |
|---|---|---|---|---|
| $v$ | $1.5\, m/s$ | $1.5\, m/s$ | $1.5\, m/s$ | $1.5\, m/s$ |
| $\theta_0$ | $60^\circ$ | $60^\circ$ | $60^\circ$ | $60^\circ$ |
| $r_0$ | $700\, m$ | $700\, m$ | $700\, m$ | $700\, m$ |
| $\Delta t$ | $T$ | $5T$ | $T$ | $5T$ |

The simulation results of straight lines are shown in Figs. 3 and 4. The Calculated $r_0$ is $660.9\, m$ for the first condition and it is $687.7\, m$ for the second condition. Fig. 5 is the result of AUV moving in a circle. Because the AUV trajectory is curved, $\Delta t$ should be as short as possible. This makes the assumption of vehicle moving along a straight line in $\Delta t$ more accurate. The calculated $r_0$ is $661.1\, m$. That is close to the straight line condition when $\Delta t$ is equal to $T$. Fig. 6 is the result of AUV moving in an irregular trajectory. The calculated $r_0$ is $693.1\, m$.

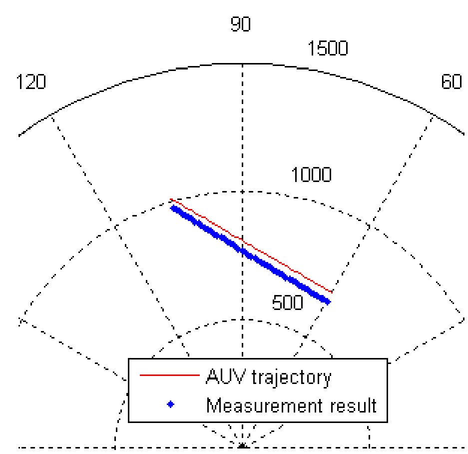


Figure 3. AUV moving along line with $\Delta t = T$.

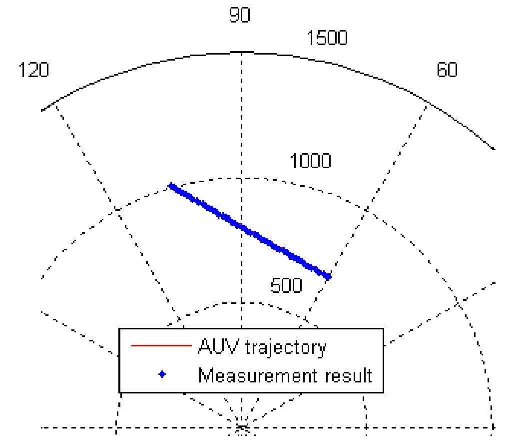


Figure 4. AUV moving along line with $\Delta t = 5T$ .

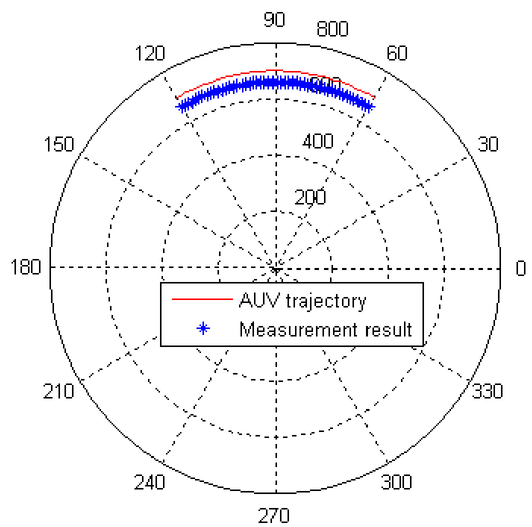


Figure 5. AUV moving in a circle with $\Delta t = T$ .

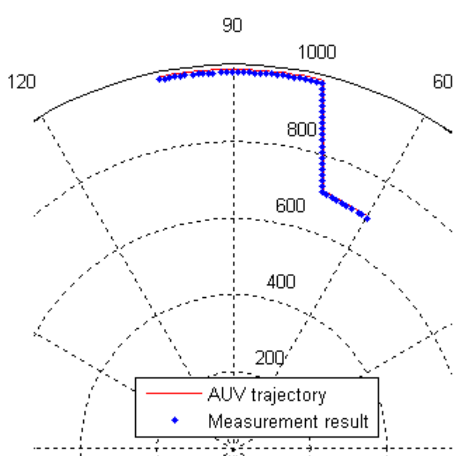


Figure 6. AUV moving in an irregular trajectory with $\Delta t = 5T$ .

## V. CONCLUSION

A simple passive vehicle localization method has been developed based on the time difference of sonar transmissions and receiving interval measurements using a single linear array. This method can realize localization of a vehicle with a periodic sonar transmission and a few vehicle operation parameters. The simple iterative processing to update the vehicle location makes the algorithm more readily implemented in a real-time system. Measurement error analysis shows that the method is sensitive to azimuths difference error caused by azimuth measurement error.

Simulations show that the method yields an effective source location estimator without using a complex receiving array configuration. Besides, the method can be applied to calculating the initial range of a vehicle moving in an arbitrary trajectory.

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