Simple Karman Street model

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Abstract— **Karman gait is a fish locomotion mode that is observed in fish swimming between vortices shed from a cylinder (the so-called Karman Street). During Karman Gait fish shows a reduced muscle activity, revealing that sometimes fish can passively move against turbulent flow.**

A controller adapted to generate this kind of behavior in a robot fish is being developed, but the main problem by now is the complexity of the robot-environment interaction model in order to design the gait controller for the robot.

This article describes a simplified Karman Street model that is supported with some CFD simulations. This model is currently being used to train robot fishes' controllers before throwing them into water.

I. Introduction

Karman Vortex Street is a well patterned sequence of alternated vortices shed in a laminar flow that is disturbed by a bluff body. This phenomenon is observed in nature in clouds and water for example, and is used by fish in different ways, like schooling or just holding station behind a fixed body.

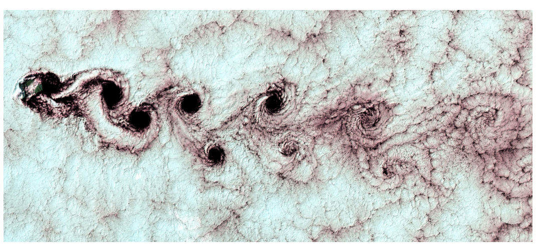

Fig. 1. Vortex Street that appeared over Alexander Selkirk Island. Picture obtained from: NASA Goddard Space Flight Center (NASA-GSFC)

Let us imagine a robot targeting the same goals. Such robot requires an adaptive and robust control to support adequate swimming. From conventional nonlinear control perspective, such controller is in need of the full detailed model for the local fluid mechanics. From computational point of view, this is unrealistic because it is too complex: a full numerical method handling active movements of elastic bodies in a fluid has not yet been developed [1].

In our work we focus on the holding station behavior inside the Kaman Street. In this situation, the fish is energetically efficient in combining movements known as the Karman gait to keep a quasi stationary position. For a fish robot, such a behavior, e.g., keeping a quasi stationary position with minimizing energy, can be translated into a traditional control problem as follows: generate motion patterns to keep a position and with minimizing the energetic cost.

The previous formulation looks simple but the reality is more complex: it is still not clear how fish exploit vortices in underwater locomotion. Consequently, nature cannot help us to build directly controllable underwater bio-inspired and unnatural mechanisms. This mainly due to the very low knowledge we have about locomotion in liquid environments, including fluids in interaction with bodies. Some models based on Computational Fluid Dynamics have been proposed, but the complexity of this tool makes such models useless in developing robust and adaptive controllers. All this drives sometimes to a very tricky approach that is biomimicry [2].

We follow the approach of George Cayley around 1809 that explained himself saying that the subject of body's drag is so complex that it would be more useful to investigate it by experiment than by reasoning. In this work, we developed a minimal model that approaches enough the reality to allow us developing and experiencing new controllers able to handle the turbulent conditions we presented before. In other words, we are not dealing anymore with lowering body's drag but with exploiting energy that carries turbulence in water. More specifically, we worked on a special region of the Karman Street. Namely, the region where fish have been observed to hold station performing the Karman gait. In this paper we do not address the control problem even we use this model to "train" fish robots controllers to achieve the Karman gait.

Before going further, we have to mention that this Karman Street simulator [1] has been obtained using the most important kinematic characteristics of water flow behind a circular cylinder for Reynolds numbers in the range $50 < Re < 2x10^5$ and at distance from the cylinder where the sequence of vortex shedding becomes highly patterned.

II. Karman Vortex Street Description

A Karman Vortex Street can be described as a fluid phenomenon where a sequence of vortices is shed on the sides of a body that is perturbing a laminar flow. These vortices alternate clockwise and counterclockwise leaving a nearly laminar flow in between, as seen in the schema of Figure 2.

Karman Street occurs for some specific fluid kinematic conditions. These conditions can be described by some standard measures of flow like Reynolds and Strouhal number. These characteristics and the regions where Karman Street appears are described below.

[1]Built on Webots [3].

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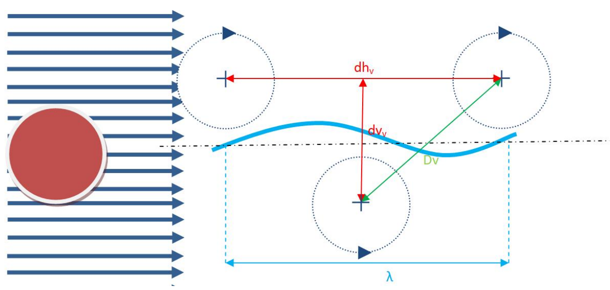


Fig. 2. Scheme of a Karman Vortex Street showing the vortices direction of rotation regarding the flow direction (in blue)

A. Reynolds and Strouhal number

Reynolds number is a dimensionless number. It gives the ratio of inertial forces to viscous forces.

For a cylinder inside a water laminar flow, the Reynolds number is expressed as follow:

$$Re = \frac{Ud}{\gamma} \tag{1}$$

Where:

- γ is the kinematic viscosity of the fluid. In this case, for water at 20^oC this is $1.004 \cdot 10^{-6}[m/s^2]$.
- U is the flow velocity in the laminar region.
- d is the diameter of the cylinder.

Strouhal number is another dimensionless number. It describes oscillating flow mechanisms.

In terms of a cylinder inside a laminar flow, Strouhal number will be expressed as:

$$St = f\frac{d}{U} \tag{2}$$

With f being the vortex shedding frequency in Hz.

Strouhal number, for a range of Reynolds number between $250 < Re < 2 \cdot 10^5$, can be expressed as in Equation 3 (calculated by G. I. Taylor (1886-1975)):

$$St = 0.198\left(1 - \frac{19.7}{Re}\right) \tag{3}$$

Where Re is the Reynolds number calculated with Equation 1.

This equation is especially useful to get the vortex shedding frequency of the Karman Street for the previous Reynolds number range.

B. Reynolds, Strouhal and Karman vortex Street occurrence

Having in mind that the main objective of this model is to build an environment for a bio-inspired robot, we had a look at the swimming frequency characteristics of fish. Strouhal number in terms of fish swimming characteristics is observed to be in a range of $0.2 < St < 0.4$ ([4], [5]). This latter is the result of a tail beat frequency ranging from 1 to 4 Hz in normal conditions.

By looking to the relationship between Reynolds number and the generation of a Karman Street, it can be observed [2] that a well patterned succession of vortices can be obtained only in some ranges of Reynolds numbers:

$Re < 10$	No vortex
$10 < Re < 40$	Attached vortices
$40 < Re < 150$	Karman vortex Street with smoothly increasing Strouhal number
$150 < Re < 2x10^5$	Karman vortex Street almost constant St
$2x10^5 < Re < 3x10^6$	Fully turbulent wake
$3x10^6 < Re < 10^{10}$	Karman Vortex Street with increasing Strouhal number

Based on the previous information, we choose to work in a limited and specific region of Reynolds and Strouhal numbers. The limits have been chosen for Reynolds number between $150 < Re < 2x10^5$. Within this interval, the presence of a Karman Street and an almost constant Strouhal number has been observed. The fluid behavior in this region can be seen in Figure 3. It shows the Karman Street behavior function of respectively, the Reynolds and Strouhal numbers.

The chosen range coincides with living fish experiments (Re around 20.000) where the Karman Gait has been observed [6], [7], [8], [9]

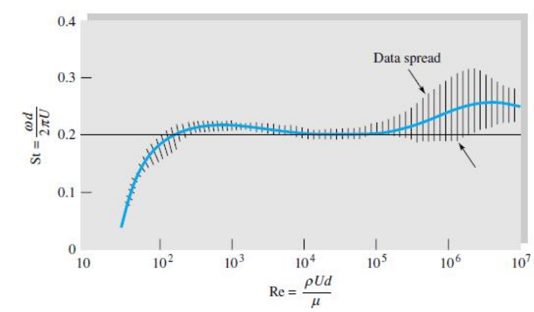


Fig. 3. Behavior of Karman Street in function of Reynolds number and Strouhal number: The horizontal lines show the regularity of the vortices shed. Extracted from [10]

C. Karman Gait and Karman Street

Fish performs Karman gait in a region where Karman Street is in a sort of steady state, as observed by Liao et al. in [6]. This distance, measured from the cylinder that is generating the Karman Street, was observed to be bigger than the length of the fish body. As well, within this region, well patterned vortices are traveling at an approximately constant speed. Moreover, in this region, vortices keep a stable configuration (more or less a constant triangle) like in Figure 2.

The methodology used to model this region is detailed in the following part.

D. Methodology to create an environment where fish-robots can learn to swim

In order to define the kinematic characteristics of the Karman Street that will be generated, we do some basic calculations:

1) Define Reynolds number for the case.
2) Define the desired vortex shedding frequency.

3) Calculate Strouhal number: from Equation 3 or from literature: for example [11].
4) Having defined Reynolds and Strouhal number and having the kinematic viscosity of water we can relate and obtain the cylinder diameter and the flow velocity as follows:

$$Re\gamma = C_1 = Ud \tag{4}$$

$$\frac{St}{f} = C_2 = \frac{d}{U} \tag{5}$$

$$d^2 = C_1 C_2 \tag{6}$$

5) Once we have the flow velocity and the diameter we get the wavelength λ of the Karman Street:

$$\lambda = \frac{U}{f} \tag{7}$$

6) Using λ and assuming that we are in a stable regime, we use the ratio of $3.56 : 1$ of distance between the centers of two rows of vortices [2] to get the vertical distance from the centers. This relationship is shown in Figure 2 as $dh_v : dv_v$.
7) We assume that the core of the vortex has a linear velocity equal to U.

III. Continuous turbulence - CFD information for the Karman Street

Computational fluid dynamics is one of the branches of fluid mechanics that uses numerical methods and algorithms to solve and to analyze problems that involve fluid flows. Computers are used to perform the millions of calculations required to simulate the interaction of liquids and gases with surfaces defined by boundary conditions.

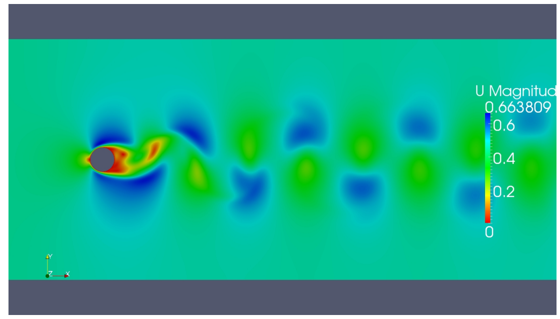
For Karman Street as defined in Section II-D, a CFD simulation is performed. We generate a mesh defining a cylinder of diameter d as obtained before. Then we simulate a constant speed flow of water perturbed by this cylinder and we obtain the velocity (See Figure 4(a)) and pressure information (See Figure 4(b)) for our Karman Street.

From these simulations, one extracts the vortex's external layer flow velocity that will be applied as a part of the drag forces on the bodies of the fish.

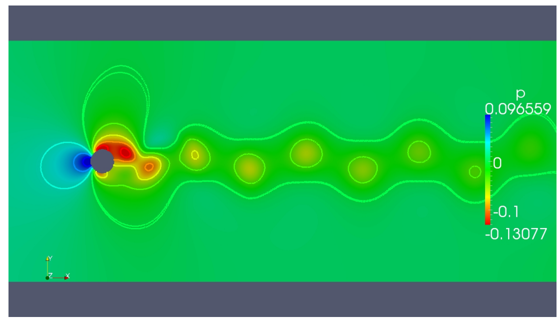
This is made by drawing an isobar line limiting the external layer of the vortices in a steady state, and by using the velocity of the internal region of the Karman Street that interact directly with the fish.

In Figure 4 it is shown a simulation case of a fluid at $Re = 150$ for a graphic explanation purpose, where in Figure 4(a) it is shown the velocity of the flow, in Figure 4(b) it is shown the Pressure at the same simulation time of Figure 4(a) and some isobar lines have been identified to be superimposed on the velocity graphic as shown in Figure 4(c).

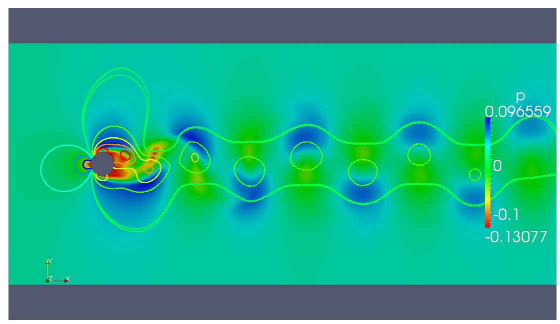
The internal region of Karman Street velocity obtained from these simulations is added as a parameter for each vortex. The Physics plug-in made for still water and modified for laminar



(a) Velocity magnitude



(b) Pressure magnitude and Isobar lines



(c) Velocity magnitude and Isobar lines

Fig. 4. Example of OpenFoam CFD simulation of a Karman Street at $Re = 150$

flow, has been modified once more in order to compute the distance between the vortex and the body of the fish, so when the vortex approaches the fish the drag on the fish body is influenced by the oncoming vortex.

These simulations support the generation of a synthetic model giving several cases within the range of characteristics chosen. This variety of cases allows the controller to get prepared for a more realistic situation.

IV. Physics on the robot

At this point, we have obtained the basic information to simulate a synthetic environment for the robot. The robots and controllers are then simulated in a software called Webots [3] together with the synthetic Karman Street that has been integrated using this same software.

A. Construction

We generate in Webots a Karman Street object defined as a robot. This robot is made of rigid bodies representing the core

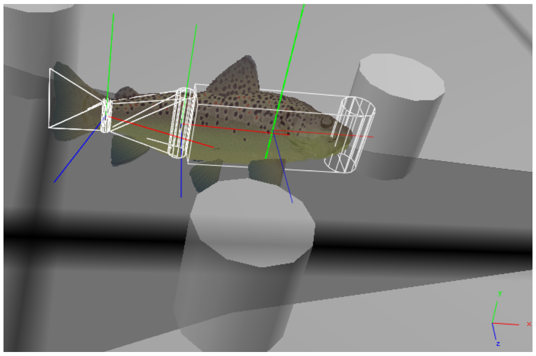


Fig. 5. Trout robot swimming in Karman Street

of a Rankine Vortex turning around its own axis and being towed by a bigger cylinder as seen in figure 6.

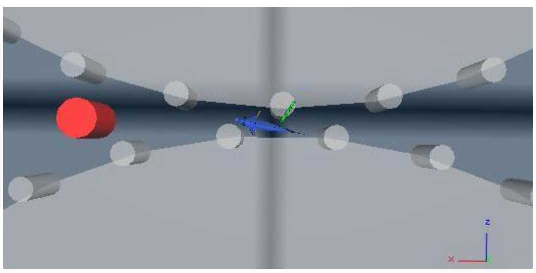


Fig. 6. Arrangement of rotating cylinders creating the Karman Street

The instantaneous linear velocity of each cylinder representing the vortex is calculated as shown in the previous section and the size and angular velocity of the main cylinders are calculated according to the distance between vortices also as in the previous section and taking in account the number of cylinders each cylinder will take in charge.

As an example, the case of figure 6 consists of two main mechanisms one turning clockwise and the other one counterclockwise. Each of them is carrying 36 cylinders, each one turning on its own axis in the contrary direction of the main cylinder carrying them.

The described mechanism is used to train a 1 DOF robot controller. This controller adapts to the frequency of the vortex shedding process. In this case, optical sensors (depth IR-based sensors) have been used and the signal perceived by the robot is shown in figure 7.

This basic Karman Street gives to the robot the kinematic clues to adapt to the oncoming vortices in a contact free approach. That is to say, the robot relies only on visual information to train the controller. In terms of frequency adaptation, this information is sufficient. However, it is not enough to support dynamic aspects.

One step ahead from this visual-only fish-robot world, we add dynamic information by introducing forces on the body of the robot. This is done by means of physics plug-ins, as detailed below.

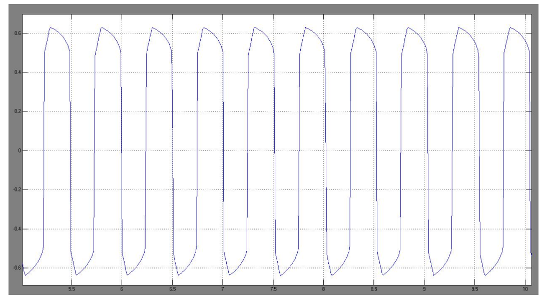


Fig. 7. Signal obtained from the visual sensors of the robot of figure 6 of a Karman Street with a Vortex shedding frequency of 2 Hz.

B. Buoyancy and Archimedes force

We assume that our fish robot is neutrally buoyant. This means that it will stay submerged in a steady level. To do this, the body should have approximately the same density of water: the weight of the water displaced by the body is the same as the body itself and gravity compensates buoyancy force.

Forces equilibrium is expressed as follows:

$$F_b = \rho V_b g \tag{8}$$

Where:

V_b is the volume of the fish body that is displacing an equal volume of water (we assumed that the robot is fully submerged).

g is gravity acceleration (9.81 m/s^2.)

This force is applied directly on the center of mass of the body keeping the robot in a stable position in the y axis. For the moment we haven't made any attempt to control the vertical motion of the robots we are training. In fact, this approach keeps the robot in a 2 dimensions dynamic world.

C. Other forces in still water

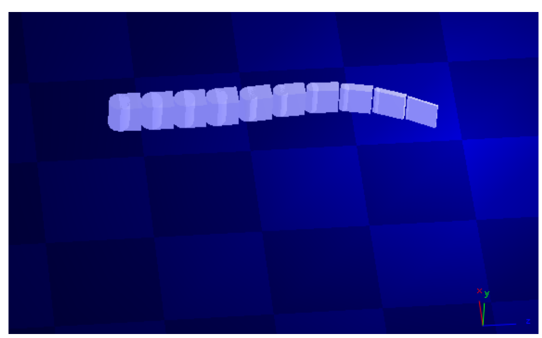


Fig. 8. Eel like robot swimming in still water

Based on the plug-in created by [12] to add forces to their robotic salamander, we created our own Physics plug-in to allow to our robots to swim in still water first. This step calculates and applies drag over the fish body converting the relative speed of the fish in drag:

$$F_d = \frac{1}{2}\rho U^2 A_p C_d \tag{9}$$

Where:

ρ	is the density of water (997.0479 kg/m^3 at 25^oC).
A_p	Is the projected area perpendicular to the direction of U.
U	Flow velocity.
C_d	Drag coefficient depends on Reynolds number and the projection of the area perpendicular to the flow or relative flow velocity.

Drag coefficients for these simulations have been obtained from [2] function of shape, area and an approximate Reynolds number and from [13] for the eel-like robot shown in Figure 8.

The fixed frame of reference is oriented with gravity parallel to y axis in negative direction as shown in Figure 9.

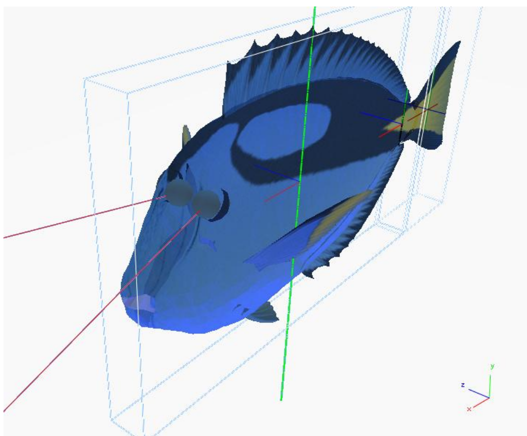


Fig. 9. Frame of reference. The green line shows the y axis

D. Integrating the Karman Street

As it has been already defined, Karman Street is created when a bluff body perturbs a laminar flow. We start by considering the laminar flow in the simulation and finally we add the forces generated by the vortices of the Karman Street.

1) Inside laminar flow : Laminar flow is represented as a flow with constant velocity in a fixed direction, as in Figure 10, and the drag force can be derived from equation 9, using U as the relative velocity of the body regarding the flow.

Drag forces obtained before are calculated and applied in the robot's body reference frame. This is valid because the drag coefficient is totally depending from the characteristics of the face of the concerned body. In addition, we assume that these forces are applied on the center of mass of the robot which coincides with its geometric center.

Fig. 10. Laminar flow against fish swimming direction

2) The vortices: To take into account the vortex influence on the robot's body, we use an approximated velocity of the flow in a selected Isobar line taken from the CFD simulations (See Figure 4(c) for a graphical description). This velocity changes the drag force applied on the body's center of mass. But, this force is only applied when the body is inside an influence region of the vortex. As seen in Figure 11.

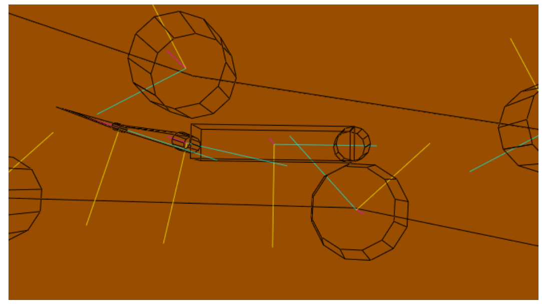


Fig. 11. Scheme of fish robot inside the simulated region of a Karman Street

Robots inside this Karman Street region get the influence of the vortices on their bodies, they have optical information and velocity and proprioceptive cues to train its controllers.

V. Results and Conclusions

In this paper, we presented a simplified model of the Karman Street. We describes the methodology we followed to achieve this goal. Namely, we abstracted the main characteristics of the Karman Street. We derived a compact description of a specific region (corresponding to the area where the fish performs a Karman Gait). This compact model has been used to train different robots controlling their tail beat frequency with neural based controllers, where the feedback information is essential.

Robots simulations used till now include: 1 DOF Blue fin fish robot, 2 DOF brown trout and a 9 DOF eel-like robot with different controllers each of them.

Robots have successfully matched their tail beat frequency with the Karman Street but the right phase to attack it to perform the Karman gait is still being matched manually and studied in the robot control part of the project.

Acknowledgment

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