Biomimetic Mechanical Design for Soft-Bodied Underwater Vehicles

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***Abstract*— This paper describes a biomimetic underwater fish robot prototype and its design methodology. The key question directing our design is the transfer of functionality from fish to a fish robot with respect to efficient mobility. We want to minimize mechanical complexity and achieve a low-cost fabrication. We argue for the case of morphological computation, i.e. achieving high mobility and efficiency by duplicating fish physical body structure. In this way, a possibly large part of the fish motion ability is outsourced to the embodiment, i.e. achieved by the interaction of the fish body parts and the water flow. This approach makes us focus on the material properties of a compliant tail propulsion mechanism. The tail is actuated by a single motor and we want to make it efficient by exploiting the energy propagation from the body to the surrounding fluid. We explain our design constraints, material choices and describe the design process. We draw conclusions about the relevance of our design parameters and design choices.**

I. Motivation and Scope

It has been known for at least half a century that autonomy would become indispensable for successful underwater robotics. So far, this challenge has not been met in full. In order to develop effective underwater autonomous vehicles, further progress with respect to mobility is needed in the following key areas: i) agility in motion; ii) variety in information types that can be acquired and possibly also used during operation; iii) low cost and fast production line and iv) effective context-driven control strategies.

In order to successfully realize such features, the following aspects need to be synergistically addressed: i) mechanical design of the robot must be adequate to support the functionalities; ii) the sensor system must be sufficiently rich to carry significant information on the external environment; iii) signals should be processed so as to recognize the hydrodynamic patterns in the environment, identify their salient elements and allow for the proper control action to be selected.

We focus on a bio-inspired design for underwater robots. Key features of our approach are: soft-bodied design; innovative sensing technologies mimicking the functionalities of the lateral line in fish and employed both as a support for motion and for information gathering; effective exploitation of common materials, so as to realize low-cost design; bio-inspired sensorimotor integration, to guarantee contextually-driven control. We treat mechanical design as the development of a platform capable of supporting all such aspects.

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In this paper, we present our progress on mechanical design; we show innovative design solutions for bio-inspired soft-bodied vehicles that have the potential of being refined into the underwater search vehicle of the future. Our design is suited for complex task environments, such as in-shipwreck search, underwater marine cave exploration or shallow-water near-coast inspection.

We also take a stance on the key question in biomimetics: to what extent does biomimesis entail replication? We would like to achieve a technological transfer of functionality, while keeping to simple design. With one actuator only, instead of distributed actuation, we can still achieve an adequate locomotor functionality. As a general guideline, we will try to copy embodiment shape and wave propagation mechanics, to gain the benefits of morphological computation found in fish.

Therefore, we address the following questions:

1) Partially vs. fully compliant design: are there any sections that we can safely treat as rigid?
2) How can we identify an appropriate elasticity distribution? How can such a distribution be realized in the embodiment?
3) How can we reconcile the concept of compliant body with the use of rigid electromechanical components?
4) Is it truly advantageous to envision a non-uniform elasticity distribution and a tuning mechanism?
5) How can we actuate compliant bodies?

We answer these questions by showing how we have progressed from a simple, single-material design to multi-layered prototypes; we summarize the experimental test data that has oriented us towards our current prototype.

II. From biological fish to robotic fish

We first turn to fish for inspiration concerning the global concept. We are particularly interested in subcarangiform swimming (trout-like), as it embodies a good balance between speed and maneuverability. Subcarangiform swimmers are generalists and can be found in almost every ecological niche. Their minimum turning radius is very contained (on average 20% of the bodylength) – a tenfold improvement with respect to best maneuvering ships, and without having to slow down ([3]). In addition to their agility, these fish are also efficient at sustained swimming. This natural design possesses all the qualities of an ideal underwater device for exploration in complex environments.

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A. Three-component approximation

In first approximation, we consider three macrosections – head, body and tail. We initially keep a rigid head; fish do have a rigid part – the skull. In fish, the head constitutes less than 20% of the body length, but the propulsive role of the anterior part of the body is rather contained. In steady swimming, the anterior musculature is recruited only at high swimming speeds; the preferred muscle groups for subcarangiform swimming are located in the posterior region; progressively more anterior musculature is additionally recruited at higher swimming speeds ([4], [9]). On the other hand, the tail is flexible to exploit the mechanics of tail-water interaction developed by Nature. The central section accommodates the transmission mechanism, so that actuation effectively occurs within the compliant tail. We think that the key point lies in ensuring smooth transitions between sections, rather than focusing on life-like, all-compliant design.

We have designed the central section to adhere to the organization of biological agents, even when accommodating some rigid mechanical parts. It is quite unnatural to implant a rigid mechanism in an embodiment that is meant to be soft and compliant. To reduce the impact of rigid components on the operation of the flexible parts, we have thought of accommodating sensing-related electronics in volumes that in fish contain the inner organs and are thus not involved in propulsion. The actuator is placed within the rigid head and motion is simply transmitted through the middle part; we have preferred this solution to directly embedding a motor in soft regions, thus altering their concept.

B. Operation

The key idea is operation through bending wave propagation. We do not generate the wave through distributed actuation – we generate it in a single location and then tune the propagation. Tuning occurs both at the design level and through control, in such a manner as to induce thrust via non-stationary waves.

Under different swimming conditions (flow regimens and flow speeds) and depending on swimming mode and intended action (steady state, turning, fast start), the bending curves in fish can look markedly different. It is important for our autonomous device to be able to adapt to different hydrodynamic circumstances and support different control actions. However, this kind of flexibility needs to be explicitly designed for. Designing an embodiment that can support a range of bending styles is not a trivial accomplishment. It involves taking simultaneously into account both the properties of the embodiment and the actuation (type of actuation, localization and actual signal).

Ebbs et al. ([6]) have shown that localized actuation of a compliant body can mimic the bending behavior of fish. Alvarado ([5]) posits a single operational frequency and realizes embodiment/actuation design to yield a desired kinematics at the operational frequency – the resonant frequency of the resulting embodiment. In a recent paper ([6]), testing of the prototype so designed shows that the prototype can operate well in a range of frequencies, although it does not fully achieve its design objectives in terms of extent of bending (approximatively 25% of the target). Alvarado's work shows clearly that, with compliant body design and localized actuation, achieving different bending curves may require very different material distributions. Since we cannot change material distribution during operation, it is apparent that there is need to design for a baseline range of bending behaviors and make room for methods to change stiffness adaptively.

We believe that much can be accomplished by designing the embodiment to react in a qualitatively different manner as a function of the features of the actuation signal (waveform, frequency, amplitude). So our strategy consists of tuning the material properties in design.

C. Compliant body vs. multi-link design

Designs based on multiple actuated links are not suited for our purposes. Multi-link design is an engineering solution that is feasible and has given amazing results (Robotuna [3], Essex robot [8]), but that requires forcing the interaction with the flow: the body takes the lead and shapes the flow. The fish-flow boundary is governed with an a-priori strategy, in order to test hypotheses and elucidate the principles of fish behavior. It is, on the other hand, very difficult with this kind of design, to *observe* the effects of the environmental flows and react to them. In a sense, we wish to look at the fish-flow boundary (here the bending curve) as the result of the interaction between material and water, not as the result of pre-defined purposeful action. We are looking for a support to realize innovative improvements in efficiency through flow adaptation, not limited to efficient propulsive vortex generation.

III. Tail design: elastic properties

We initially concentrate on tail design. Elasticity and geometry are known to affect heavily the performance of hydrofoils, hence we consider them parameters worthy of careful tuning.

Analytical methods to estimate design parameters become very complicated when realistic shapes are considered. In [1], we report on a novel method to identify the elasticity profile in fish, using myometry for direct investigation. Results (in Figure 1) show that elasticity increases towards the tail and provide an average value to consider for soft-bodied design. We identified silicon-based materials that could capture the average (pink) and maximum (green) values.

We have casted a selection of compliant tails (in Fig. 2) to explore if adhering to the biological elasticity distribution is advantageous to accomplish fish-like passive kinematics. We actuated each prototype and a dead fish with a servomotor at 5 frequencies (1-5 Hz) in three laminar flow speeds (0, 0.25, 0.5 m/s). We recorded video data for each experiment and extracted the midbody line for comparison.

Prototype 6 shown in Figure 2 is hybrid and represents a 2-value approximation of the trend found in fish. Even though the approximation is very rough, P6 exhibits non-negligible improvement with respect to the other fish-shaped prototypes. We interpret this result as a confirmation that designing for

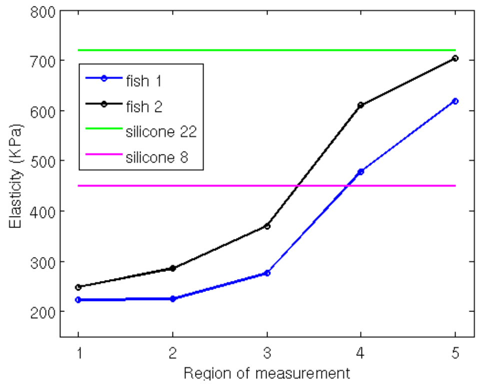

Fig. 1. Elasticity distribution as measured through myometry on two freshly killed trout *Oncorhyncus mykiss* (blue and black lines) and elasticity values for the pink and green materials used in the artificial tails of Figure 2. The pink materials is Elite Double 8 (E8, Shore A scale: hardness 8, elasticity $450KPa$) and the green material Elite Double 22 (E22, Shore A scale: hardness 22, elasticity $720KPa$). [1]

Fig. 2. Six compliant tail prototypes of varying elasticity (E8, E22) and shape (cylinder, truncated elliptical cone and elliptical cone with tail). The cylinder is taken as analytical reference and the cones serve as an intermediate step towards the more realistic fish-like shapes [1]

non-uniform elasticity is indeed beneficial in realizing fish-like kinematics.

We also evaluated the interplay of geometry and material. The initial results show that geometry and elasticity should not be considered independently. The mutual effects on bending performance are such that as one design parameter approaches the values found in fish, performance does not improve even if all other factors are kept constant. Average elasticity also does not affect all shapes in the same manner. In short, the searchscape of heuristic design methods is not regular.

IV. Tail design: wave properties

In our eyes, the difficulty in understanding the searchspace might be related to the fact that we have chosen mechanically relevant parameters that do not, however, encapsulate a cohesive picture of fish-flow interaction. In fact, we have focused on factors that are related to the *method* by which the fish outsources computation to the embodiment and governs such interaction; the method can be either hard-wired structurally (through evolution of a shape fit for the strategy) or in real time (through stiffness control, occurring through differential muscle contraction).

In order to orient design, we need to understand, on the other hand, the fundamental traits of the *strategy* governing the fish-flow interaction, as opposed to the mechanics ruling the interaction's features. A clear indication that we have understood something fundamental about how compliant bodies are successfully employed in water lies in the ability to identify a unifying factor able to relate to performance in adhering to the biological bending profile.

We change approach and look at bending in terms of a propulsive wave that travels along the body – as opposed to the spatial localization of the body during operation. This approach seems sound because the main descriptors of the bending wave have been extensively employed to convert the kinematics into estimates for forces, power and Froude efficiency ([12], [13], [14], [15], [16]). With the advent of DPIV technology, it has been possible to validate such relations experimentally; in particular, Tytell shows that estimates for power and Froude efficiency are reasonably accurate, whereas force can be underestimated by as much as 50% ([10], [11]).

Design parameters such as shape and elasticity have an immediate translation in terms of the wave picture: shape affects the proportion of the propulsive wave that is reflected into the embodiment at the posterior boundary, whereas elasticity has a very natural interpretation in terms of wave speed. Interestingly, this perspective allows capturing also some characteristics of the environment.

A. Bending vs speed of the propulsive wave

Further analysis of the bending performance for the prototypes in Figure 2 shows that higher laminar flow speeds result in higher speeds of wave propagation and in a more contained bending – as if the flow provided an "added stiffness" to the embodiment. This effect should be considered when designing a prototype tail for deployment in environments where noticeable flow is expected.

Figure 3 shows the wavespeed profiles and amplitude envelopes of the three fish-shaped prototypes, compared to a dead trout. The hybrid prototype is closest to the biological reference also according to wavspeed, taken as an additional criterion of kinematic similarity. None of the prototypes achieves a bending extent comparable to the trout, even though the wave speed for the hybrid tail matches very closely.

There seems to be a tradeoff between amplitude and wavespeed, if elasticity is the only design parameter. Interestingly, a similar result can be deduced from [6], where the propulsive wave speed closely matches the target speed, but bending is only 25% of the target. Fish seem to escape this tradeoff, as they possess the ability to transmit the propulsive wave at a high speed, while still achieving high body bending.

One possible explanation is that the traveling wave that can be observed on the fish-flow boundary is a resultant wave – and that the effective wave traveling down the fish body is much faster. This hypothesis seems to be corroborated by the fact that the muscle actuation wave also runs much faster than the bending wave. Having taken elasticity to be uniform within the cross-section may account for the artificial prototypes' failure to reach the performance of fish. If we allow for elasticity to

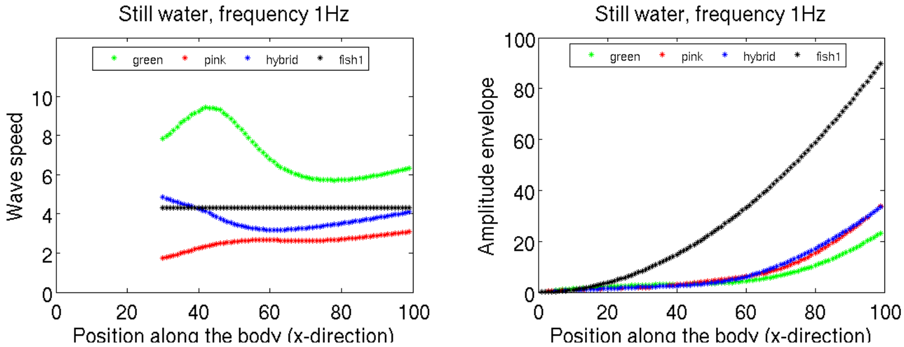

Fig. 3. The wave speed profile and amplitude envelope along the body for the fish-shaped prototypes in Fig. 2 (hybrid, pink and green), actuated at 1Hz in still water. The values for wave speed in the anterior part of the body are not considered reliable due to considerable numerical noise and are therefore not represented. In black, we show values for a dead trout actuated in the same conditions; we report only the average value of wave speed, because calculations were carried out with a different method.

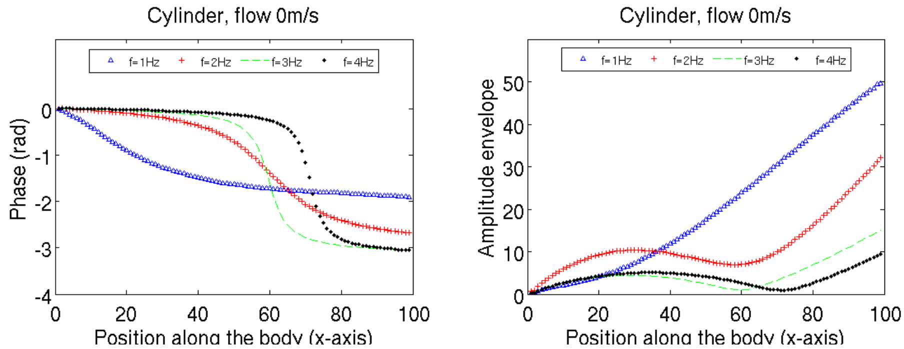

Fig. 4. Phase ϕ and amplitude envelope for the cylinder-shaped prototype of Figure 2, when fitted with the model $h(t,x) = A(x)\cos(\omega t + \phi(x))$ (left). Data refers to actuation frequencies 1-4Hz in still water.

vary within the cross-section, a simple two-value approximation yields a salient feature of the biological organization: a region of markedly higher stiffness – a backbone. Under this perspective, fish transmit the actual traveling wave along the backbone – the stiffest part of the body; on the other hand, the softer surrounding tissues allow bending to greater extents.

B. Standing wave on cylinder prototype

In this section we highlight the importance of tail shape in motion efficiency. In Fig. 4 we show the behavior of the pink cylindrical prototype in still water and under a sinusoidal actuation of different frequencies. The presence of a propulsive traveling wave translates in terms of the phase picture as a monotonically decreasing trend. This trend is apparent at 1Hz, but disrupted at higher operational frequencies. In fact, with increasing frequency the phase graphs look progressively closer to a sigmoid function, which captures a transition between two constant values. Inspection of the bending behavior confirms that the cylinder exhibits a standing wave during most of the body. The situation is reflected in the amplitude envelope, where we see positions approximating the behavior of nodes, located in the transitional region.

The reason for the appearance of standing waves lies in the posterior body-flow boundary: there is a flat surface that allows the reflection of the traveling wave. In such circumstances, the cylinder can only transfer a minor part of the energy in an orderly fashion, resulting in very poor propulsive efficiency. The phenomenon, albeit less severe, is present also in the cone-shaped prototypes. The experimental data shows that, although it is very convenient to reduce analytical complexity by resorting to simple shapes during design, the impact of having reflecting surfaces simply cannot be ignored.

V. Tails with Backbone

The previous discussions in Sections III and IV encourage us to design a tail with backbone to achieve more fish-like behaviour and therefore higher propulsion efficiency. We believe that such design permits achieving a high propagation wave speed together with a high bending amplitude. The stiffer backbone would permit faster wave propagation while softer material surrounding it would permit a higher bending amplitude. Moreover such design also allows us to get a increasing elasticity profile along the body.

The previous discussion also reveals that the geometry plays an important role in how the traveling wave is propagated to the surrounded medium (flow) or reflected back. Since the fish tail geometry appeared to be efficient with that respect we decided to fix the geometry of a subcarangiform swimmer and tune the elasticity profile based on the ratio between backbone and softer material which is determined by the shape of the tail.

We designed molds with fish tail geometry using the mathematical representation of the fish outer shape. We then checked

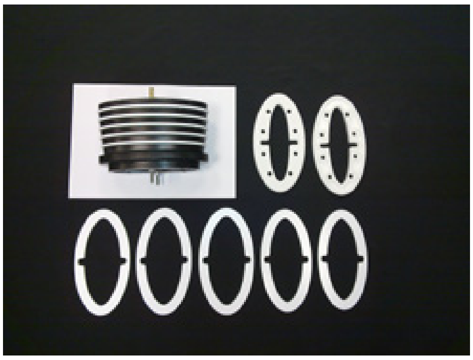

Fig. 5. The cross-section of Dragon Skin A10 tail with hollow mid-section (above). The ribs are used to support the skin as well as functioning as a backbone (below).

the availability of silicone materials—in terms of elasticity values— that can enable us to design a tail with backbone within the average elasticity range of biological fish (Figure 1). We cast a tail prototype using the Dragon Skin A 10 material which was the least stiff silicone available at that time. The initial testing with the casted tail revealed that even with the least stiff silicone, high torque was needed to bend the tail.

To overcome this problem, hollowness inside the tail was needed to reduce the bending stiffness. To maintain the stability we had to keep the material in the center; but remove some from the sides while keeping the outer surface untouched (skin) (Figure 5 above). To reduce the unpredictable bending of the skin, the rib structure was designed (Figure 5 below).

While testing the new prototype we however discovered that it has a problem of lateral buckling as shown in Figure 6. Therefore we looked for a solution how to reduce it. As a solution a new tail was casted from a foam (Soma foama 15 by Smooth on). In comparison to the Dragon Skin (or other silicone materials) of the previous prototype tail, foams have very different properties regarding energy storage, reactivity and compressibility. With this choice of materials, we can effectively exploit an additional element, which plays the structural role of a backbone and guides the propagation wave.

The foam prototype was realized using Dragon Skin A20 for the backbone (which is a stiffer version than Dragon Skin A10), the foam for the body and covering the tail with a thin layer of Dragon Skin A10 (Figure 7). The outer skin will prevent the foam from absorbing water and swallowing up. This prototype has also an advantage that its elasticity distribution is much closer to the biological fish and the foam is almost neutrally buoyant.

Fig. 6. The lateral buckling problem for the tail made of Dragon Skin A10 with hollow mid-section.

Fig. 7. The foam prototype with backbone design.

a) The preliminary experiments with foam prototype: Having designed the foam prototype, we are currently testing it in our flow tank. Figure 8 illustrates series of images showing motion of the foam prototype in one cycle when it is actuated at f=3Hz in still water. The evaluation of the prototype in terms of wave propagation and propulsion efficiency is currently under progress for different flow regimes, operational frequencies and different actuation waveforms.

VI. Component Integration and platform design

Besides having worked with the design of compliant tails, we also investigated how to integrate the tail prototypes to the rest of the robot platform. The design requirements we had for the fish robot prototype were the following: i) the head should be fish-shaped to reduce the drag but big enough to contain the onboard electronics, ii) the fish tail is actuated by a single-point actuator. However, we also wanted to investigate the performance of different actuators and therefore the design should permit changing them without making modifications in the rest of the prototype, iii) the tails should be easily changeable because we want to use the platform to investigate different propulsion mechanisms and their effect to swimming efficiency and iv) the outer shell should be waterproof because we did not want to be concerned about water-proofing every component individually.

Taking those requirements into account, the head is manufactured with rapid prototyping. However, we had to compromise and make the head bigger than the head of a biological fish to accommodate the electronics and motors. All the main components (head, tail and motor) are attached to the middle part. Choosing the appropriate design of the middle part was the most important issue to achieve proper sealing. The fastest solution was to make the middle part from aluminum using CNC milling. A later version was made of polyamide

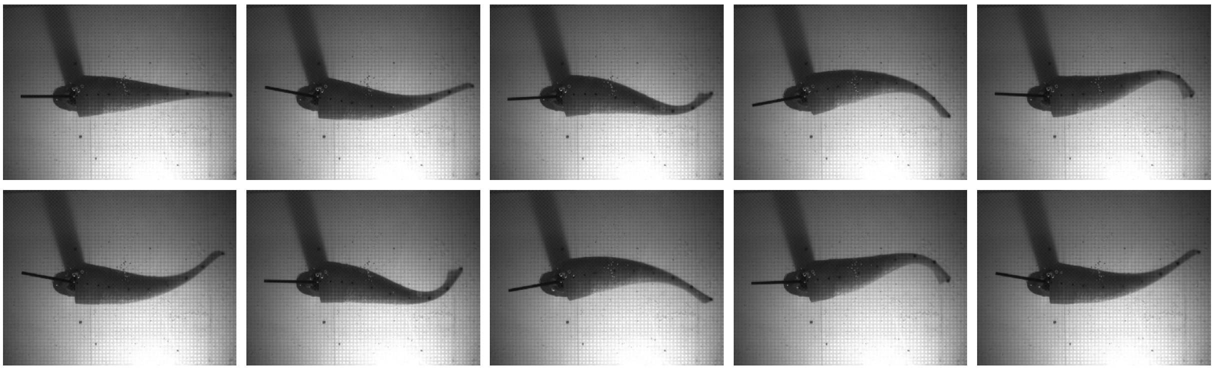

Fig. 8. The series of snapshots taken while foam prototype is being tested ($f = 3Hz$, in still water).

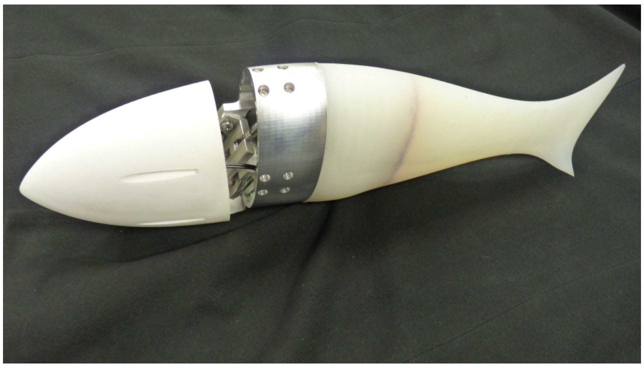

Fig. 9. Prototype in Dragon Skin. The rib structure is located in the central section and helps maintain shape around the hollow.

using rapid prototyping. Moving from external actuation of the tail to internal actuation, we realized two kinds of actuation mechanism.

A. *Actuation mechanisms*

In both cases the actuator itself is placed in the head of the fish and motion is transfered to the tail by actuating a plate casted in the silicon. In the first case the actuation is achieved with a single rigid link. In the second case it is achieved with a pair of rods pulling the plate (Figure 10).

The constructional complexity of both methods is rather similar, however, the second method is more interesting for several reasons. We expect that it transfers forces better along the tail and minimize the propagation along the normal of the surface.

Also the actuation mechanism pulls the strings and lets the silicon do the reverse work. Call-back mechanisms in biology can be purely passive and outsourced to the morphology – relying on the mechanical properties of the material and the environment for instance breathing out. Also the newest generation of hybrid passive walkers has purely passive swing phase and actuated lift phase. The comparison between the actuation mechanisms enables us to investigate the interplay between the forces and the material. We can also investigate and make conclusions on how much the efficiency of the propeller depends on the properties of the body and the actuation style.

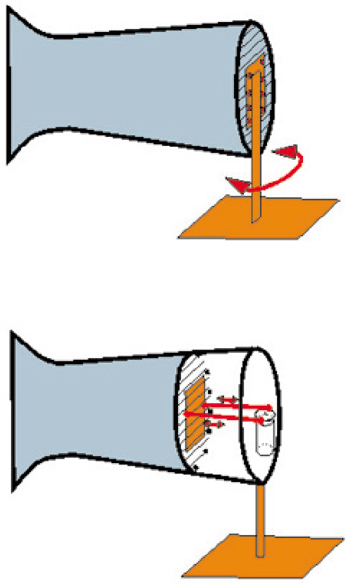

Fig. 10. Two actuation mechanisms; rotational actuation around fixed axis (above), actuating plate casted inside the tail and actuated by pully-strings (below).

One of the areas which we are planning to investigate with the described platform is to test the role of actuation mechanism in terms of waveforms, parameters of waveforms (frequency, phase and amplitude) and actuation styles. Such analysis may lead to interesting conclusions about embodiment-actuation interaction to design more robust underwater robots. The proposed form of work is under progress in our research-lab.

VII. Conclusions

In this paper we describe a case for biomimetic design of a fish robot and explain our design choices that led us to the current prototype. Our driving factor for the design is simplicity without conceding motion capabilities of biological fish. We therefore explore different solutions on prototype realization using a compliant tail actuated with a single actuator. We argue that undulatory motion of designed prototypes depends on the embodiment and in particular, we investigate how the stiffness, stiffness distribution and wave propagation can be used to orient the design. Based on these findings we propose a new tail design based on realizing the elasticity distribution through a composite tail with a backbone. We believe that such design should also permit optimizing both for the traveling wave speed and the bending. We also explore methods of actuating the tail as well as a waterproof robot platform to integrate the actuation with the embodiment.

VIII. Acknowledgements

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