Probabilistic Sonar Scan Matching SLAM for Underwater Environment

Angelos Mallios, Pere Ridao, David Ribas and Emili Hernández
Department of Computer Engineering
University of Girona
17071, Girona, Spain
Email: {amallios,pere,dribas,emilihb}@eia.udg.edu

Abstract— **This paper proposes a pose-based algorithm to solve the full Simultaneous Localization And Mapping (SLAM) problem for an Autonomous Underwater Vehicle (AUV), navigating in an unknown and possibly unstructured environment. A probabilistic scan matching technique using range scans gathered from a Mechanical Scanning Imaging Sonar (MSIS) is used together with the robot dead-reckoning displacements. The proposed method utilizes two Extended Kalman Filters (EKFs). The first, estimates the local path traveled by the robot while forming the scan as well as its uncertainty, providing position estimates for correcting the distortions that the vehicle motion produces in the acoustic images. The second is an augmented state EKF that estimates and keeps the registered scans poses. The raw data from the sensors are processed and fused in-line. No priory structural information or initial pose are considered. Also, a method of estimating the uncertainty of the scan matching estimation is provided. The algorithm has been tested on an AUV guided along a 600 m path within a marina environment and is compared against previous work from the authors, showing the viability of the proposed approach.**

I. Introduction

The last decades, a number of studies in mobile robotics had developed techniques to address the localization problem with very promising results. In particular, the so-called Simultaneous Localization And Mapping (SLAM) techniques have been broadly and successfully applied to indoor and outdoor environments. Hence, it is of interest to study how to adapt these techniques for their use in hostile underwater environments.

This paper is a contribution in this area and the future work of [1], proposing a pose-based algorithm to solve the full SLAM problem of an Autonomous Underwater Vehicle (AUV) navigating in an unknown and possibly unstructured environment. A Doppler Velocity Logger (DVL) and a low cost Motion Reference Unit (MRU) are used for dead reckoning while a Mechanical Scanning Imaging Sonar (MSIS) is used for sensing the environment. Due to the rapidly attenuation of the high frequencies in water, it makes impractical the use of high resolution devices like laser scanners, so low frequency sonars are utilized normally. Sonar frequencies can penetrate deep the water (e.g. 10 - 150 m for a forward looking sonar), they are not subject of the water visibility but provide limited information and medium to low resolution and refresh rate.

Scan matching is a technique that estimates the robot relative displacement between two configurations, by maximizing the overlap between the range scans normally gathered with a laser or a sonar sensor [2]. The Mechanical Scanning Imaging Sonar Probabilistic Iterative Correspondence (MSISpIC) algorithm proposed in [3], is dealing with data gathered by an AUV utilizing MSIS. It is based on a modified version of the Probabilistic Iterative Correspondence (pIC) algorithm [4] and an Extended Kalman Filter (EKF) using a constant velocity model with acceleration noise updated with velocity and attitude measurements, is used to estimate the trajectory followed by the robot along the scan. This trajectory is used to remove the motion induced distortion of the acoustic image as well as to predict the uncertainty of the range scans prior to register them through the modified pIC algorithm.

In this paper we extend the MSISpIC algorithm in the pose-based SLAM framework. Now, each new pose of a scan is maintained in a second Augmented State EKF (ASEKF) and is compared with previous scans that are in the nearby area. If there is enough data overlapping, a new scan match will put a constrain between the poses updating the ASEKF. These constrains help to identify and close the loops which correct the entire previously trajectory, bounding the drift. Also, we incorporate a method of estimating the uncertainty of the pIC algorithm based on the Hessian of the error function.

The proposed method has been tested with a real world dataset, including Differential Global Positioning System (DGPS) for ground truth, acquired during a survey mission in an abandoned marina located in the Girona coast. The results are compared against our previous work in [1], and substantial improvements in trajectory correction and map reconstruction are notable.

The paper is structured as follows. In section II the probabilistic scan matching algorithm is described. Section III details the MSISpIC following by its uncertainty estimation in section IV. Both are essential parts of the proposed SLAM algorithm which is described in section V. Section VI reports the experimental results before conclusions and future work.

II. Probabilistic Scan Matching

The goal of scan matching is to compute the relative displacement of a vehicle between two consecutive configurations by maximizing the overlap between the range measurements obtained from a laser or a sonar sensor. That means, that given a reference scan S_{ref}, a new scan S_{new} and an rough

978-1-4244-5222-4/10/$26.00 ©2010 IEEE

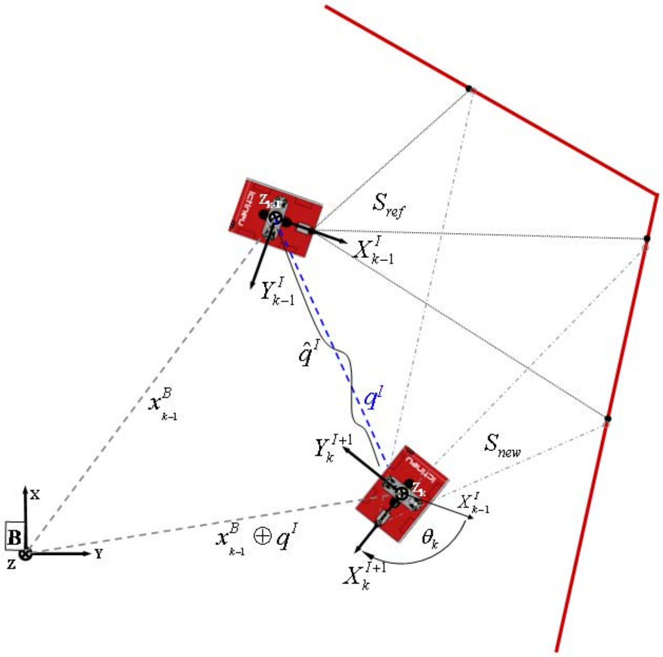


Fig. 1. Scan matching problem description.

Algorithm 1 The modified pIC algorithm

$\hat{\mathbf{q}}_{pIC} = pIC(S_{ref}, S_{new}, \hat{\mathbf{q}}, \mathbf{P}_q)$ {
  $k = 0$
  $\hat{\mathbf{q}}_k = \hat{\mathbf{q}}$
  do {
    for$(i = 0; size(S_{new}); i++)$ {
      $\hat{\mathbf{c}}_i = \hat{\mathbf{q}}_k \oplus \hat{\mathbf{n}}_i$
      $A = \{\mathbf{r}_i \in S_{ref} / D^2_M(\mathbf{r}_i, \mathbf{c}_i) \le \chi^2_{2,\alpha}\}$
      $\hat{\mathbf{a}}_i = arg\ min_{\mathbf{r}_i \in A}\{D^2_M(\mathbf{r}_i, \mathbf{c}_i)\}$
      $\hat{\mathbf{e}}_i = \hat{\mathbf{a}}_i - \hat{\mathbf{q}}_k \oplus \hat{\mathbf{n}}_i$
      $\mathbf{P}_{e_i} = \mathbf{P}_{a_i} + \mathbf{J}_q \mathbf{P}_q \mathbf{J}_q^T + \mathbf{J}_n \mathbf{P}_{n_i} \mathbf{J}_n^T$
    }
    $\hat{\mathbf{q}}_{min} = arg\ min_{\mathbf{q}} \left\{ \frac{1}{2} \sum_i^T \left(\hat{\mathbf{e}}_i^T \mathbf{P}_{e_i}^{-1} \hat{\mathbf{e}}_i \right) \right\}$
    if$(Convergence())$
      $\hat{\mathbf{q}}_{pIC} = \hat{\mathbf{q}}_{min}$
    else {
      $\hat{\mathbf{q}}_{k+1} = \hat{\mathbf{q}}_{min}$
      $k++$
    }
  }
  while$(!Convergence()$ **and** $k < maxIterations)$
}

displacement estimation $\mathbf{q}_0$ between them, the objective of scan matching methods is to obtain a better estimation of the real displacement $\mathbf{q} = (x, y, \theta)$ (Fig. 1).

Several scan matching algorithms exists with most of them being variations of the Iterative Closest Point (ICP) algorithm. The geometric representation of a scan in the conventional ICP algorithm does not model the uncertainty of the sensor measurements. Correspondences between two scans are chosen based on the closest-point rule normally using the Euclidean distance. As pointed out in [4], this distance does not take into account that the points in the new scan, which are far from the sensor, could be far from their correspondents in the previous scan. On the other hand, if the scan data are very noisy, two statistically compatible points could appear far enough, in terms of the Euclidean distance. Both situations might prevent a possible association or even generate a wrong one. Probabilistic scan matching algorithms are statistical extensions of the ICP algorithm where the relative displacement $\mathbf{q}_0$ as well as the observed points in both scans $\mathbf{r_i}$ and $\mathbf{n_i}$, are modeled as random Gaussian vectors (r.g.v.).

For a better understanding the modified algorithm is reproduced in Algorithm 1.

The inputs are the reference scan S_{ref} with points $\mathbf{r}_i$ ($i = 1..n$), the new scan S_{new} with points $\mathbf{n}_i$ ($i = 1..m$) and the initial relative displacement estimation $\hat{\mathbf{q}}$ with its covariance $\mathbf{P}_q$. The following procedure is iteratively executed until convergence. First, the points of the new scan ($\mathbf{n}_i$) are compounded with the robot displacement ($\mathbf{q}_k$). The result ($\mathbf{c}_i$), are the points of the new scan referenced to the reference frame. Then, for each point ($\mathbf{c}_i$), the set of statistically compatible points (A) is computed, using the *Mahalanovis* distance and a certain confidence level a. From that, the association point ($\mathbf{a}_i$) is computed using the Individual Compatibility Nearest Neighbor (ICNN) algorithm. Given that $\mathbf{q} \equiv N(\hat{\mathbf{q}}, \mathbf{P}_q)$, $\mathbf{n}_i \equiv N(\hat{\mathbf{n}}_i, \mathbf{P}_{n_i})$ and $\mathbf{r}_i \equiv N(\hat{\mathbf{r}}_i, \mathbf{P}_{r_i})$, are r.g.v. which describes the matching error of the $\{\mathbf{r}_i, \mathbf{c}_i\}$ pairing, it can be computed as:

$$\mathbf{e}_i = \mathbf{a}_i - \mathbf{q}_k \oplus \mathbf{n}_i \quad (1)$$

$$\mathbf{e}_i \cong N(\hat{\mathbf{a}}_i - \hat{\mathbf{q}}_k \oplus \hat{\mathbf{n}}_i, \mathbf{P}_{e_i}) \quad (2)$$

$$\mathbf{P}_{e_i} = \mathbf{P}_{a_i} + \mathbf{J}_q \mathbf{P}_q \mathbf{J}_q^T + \mathbf{J}_n \mathbf{P}_{n_i} \mathbf{J}_n^T \quad (3)$$

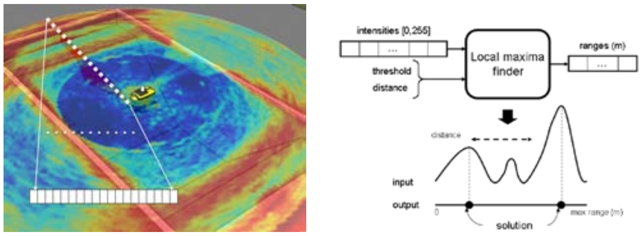
where

$$\mathbf{J}_q = \left. \frac{\partial\, \mathbf{a}_i - \mathbf{q} \oplus \mathbf{n}_i}{\partial\, \mathbf{q}} \right|_{\hat{\mathbf{q}}}, \mathbf{J}_n = \left. \frac{\partial\, \mathbf{a}_i - \mathbf{q} \oplus \mathbf{n}_i}{\partial\, \mathbf{n}_i} \right|_{\hat{\mathbf{n}}_i}$$

$\mathbf{P}_{e_i}$ is the uncertainty of the matching error $(\mathbf{a}_i - \mathbf{c}_i)$ which is used to estimate the displacement $\hat{\mathbf{q}}_{min}$ through the minimization of the square error of the *Mahalanobis* distance between $\mathbf{a}_i$ and $\mathbf{c}_i$. This is done using Non-Linear Least Squares (NLLS) minimization method. If there is convergence, the function returns, otherwise another iteration is required.

III. MSISpIC Algorithm

Scan matching techniques are conceived to accept as input parameters two range scans with a rough displacement estimation between them. Most of the approaches utilize laser technology sensors, which beside their high accuracy, they also have two more major advantages: they gather each scan almost instantaneously and the beams angle can be consider almost perfect. However, when using ultrasonic range finders these advantages are no longer valid because of their lower angular resolution and the sparsity of the readings. To overcome this problem the Sonar Probabilistic Iterative Correspondence (spIC) is proposed in [5], as an extension of the pIC. Further



a) b)

Fig. 2. a) MSIS beam acquisition; b) MSIS beam post-processing.

than mobile robotics, as the light propagation in the water is very poor, AUVs utilized acoustic sonars instead of laser sensors. In [3] the MSISpIC algorithm is proposed, modifying the pIC in order to deal with the strong deformations that, by its nature, are introduced in the scan whilst is formed in the underwater domain.

Commercially available underwater scan sensors are based on acoustics with a mechanical head that rotates at fixed angular steps. At each step, a beam is emitted and received, measuring ranges and intensities to the obstacles found across its trajectory. Acquiring a complete scan can take several seconds, therefor it can be distorted from the vehicle's motion. Thus, getting a complete scan that lasts few seconds while the vehicle is moving, generates deformed acoustic images. For this reason it is necessary to take into account the robot pose when the beam was grabbed.

One part of the MSISpIC algorithm forms a scan, corrected from the vehicle's motion distortion. MSISpIC uses an EKF with a constant velocity model and acceleration noise for the prediction step. Then, it updates with velocity and attitude measurements obtained from a DVL and a MRU respectively, in order to estimate the trajectory followed by the robot along the scan. This trajectory is used to remove the motion induced distortion of the acoustic image as well as to predict the uncertainty of the range scans prior to register them. After the corrected scan has formed, MSISpIC grabs two scans and registers them using a modified pIC algorithm. MSISpIC algorithm, consist of three major parts: Beam segmentation, Relative vehicle localization and Scan forming.

A. Beam Segmentation and Range Detection

The MSIS returns a polar acoustic image composed of beams. Each beam has a particular bearing angle value and a set of intensity measurements. The angle corresponds to the orientation of the sensor head when the beam was emitted. The acoustic linear image corresponding to one beam is returned as an array of acoustic intensities detected at a certain distance. The beam is then segmented using a predefined threshold to compute the intensity peaks. Due to the noisy nature of the acoustic data, a minimum distance between peaks criteria is also applied. Hence, positions finally considered are those

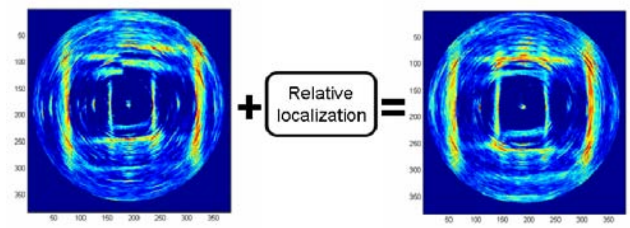


Fig. 3. The distortion produced by the displacement of the robot while acquiring data can be corrected with the relative displacement.

corresponding to high intensity values above the threshold with a minimum distance between each other (Fig. 2).

B. Relative Vehicle Localization

To maximize the probability for data overlapping, we collect a complete 360^o scan sector and register it with the previous one in order to estimate the robots' displacement. From now on we will refer this sector as a *scan*. Since MSIS needs a considerable period of time to obtain a complete scan ($\sim$14 sec), if the robot does not remain static (which is very common in water), the robot's motion induces a distortion in the acoustic image (Fig. 3). To deal with this problem it is necessary to know the robot's pose at the beam reception time. Hence, it is possible to define an initial coordinate system $\{I\}$ to reference all the range measurements belonging to the same scan. In order to reduce the influence of the motion uncertainties to the scan, as [5] suggested, we set this initial frame at the robot pose where the center beam of the current scan was read. The localization system used in this work is a slight modification of the navigation system described in [6]. In this system, a MRU provides heading measurements and a DVL unit is used to update the robot's velocity during the scan. MSIS beams are read at 30 Hz while DVL and MRU readings arrive asynchronously at a frequency of 1.5 Hz and 10 Hz respectively. An EKF is used to estimate the robot's 6 Degrees Of Freedom (DOF) pose whenever a sonar beam is read. DVL and MRU readings are used asynchronously to update the filter. To reduce the noise inherent to the DVL measurements, a simple 6 DOF constant velocity kinematics model is used. The model prediction is updated by the standard Kalman filter equations each time a new DVL or MRU measurement arrives.

C. Scan Forming

The navigation system presented above is able to estimate the robot's pose, but the uncertainty will grow without limit due to its dead-reckoning nature. Moreover, we are only interested in the robot's relative position and uncertainty with respect to the center of the scan ($\{I_c\}$ frame, Fig. 4). Hence, a slight modification to the filter is introduced making a reset in position (setting x, y, z to 0 in the vector state) whenever a new beam is emitted. Note that it is important to keep the yaw (ψ) value (it is not reset) because it represents an absolute angle with respect to the magnetic north and a reset would mean an unreal high rotation during the scan. The same thing happens with roll (ϕ) and pitch (θ). Since we are only interested in the uncertainty accumulated during the scan, the reset process also affects the x, y, and z terms of the covariance matrix ($\mathbf{P}$). Now, the modified filter provides the robot's relative position where the beams where gathered including their uncertainty. Hence, it is possible to reference all the ranges computed from the beams to the frame $\{I_c\}$, removing the distortion induced by the robot's motion.

It is very common in robotics that the dead reckoning system provides the relative angle and displacement between consecutive poses, together with their uncertainties. Then is straight forward to form a trajectory of several poses and to reference them in the desired frame by means of compounding or inverse transformations, as described in [5]. However, absolute compass readings are not in the same reference frame than the corresponding displacements, hence we propose a different approach to form the scan in respect to a common coordinate frame $\{I\}$ which is the first major contribution to our previous work [1].

Let

- $\mathbf{z} = \left\{\mathbf{z}_1^{I_1}, \ldots, \mathbf{z}_{end}^{I_{end}}\right\}$ be the measurement points of the scan to be formed represented in the $\{I_i\}$ frame, corresponding to the related robot pose at step i
- $\{B_i\}$ be the north aligned frame sharing the same origin as $\{I_i\}$.
- $\mathbf{r}_{I_i}^{B_i} = [0, 0, \psi_r]^T$ be the robot heading from the compass which actually is also the rotation transformation between the robot frame $\{I_i\}$ and the correspondent north align frame $\{B_i\}$, and
- $\mathbf{v}_{B_{i+1}}^{B_i} = [x_v, y_v, 0]^T$ be the robot displacement between time step $i-1$ and i, represented in the north aligned frame $\{B\}$

Then, we can form the scan with respect to the center frame $\{I_c\}$ (Fig. 4), as follows:

$$\mathbf{z}_i = \begin{cases} \mathbf{z}_i & i = c \\ \ominus\mathbf{r}_{I_c}^{B_c} \oplus \mathbf{V}^+ \oplus \mathbf{r}_{I_i}^{B_i} \oplus \mathbf{z}_i^{I_i} & i > c \\ \ominus\mathbf{r}_{I_c}^{B_c} \oplus \mathbf{V}^- \oplus \mathbf{r}_{I_i}^{B_i} \oplus \mathbf{z}_i^{I_i} & i < c \end{cases} \tag{4}$$

where $\mathbf{V}^+$ and $\mathbf{V}^-$ are computed from the linear operations:

$$\mathbf{V}^+ = \left(\mathbf{v}_c^{B_c} + \ldots + \mathbf{v}_{i-1}^{B_{i-1}}\right), \quad \mathbf{V}^- = \left(-\mathbf{v}_c^{B_c} - \ldots - \mathbf{v}_{i-1}^{B_{i-1}}\right)$$

IV. Scan Matching Covariance

Although the ICP-style algorithms lead to good estimations of relative displacements, they are missing the part of the uncertainty of their estimation. Calculating the covariance of a measurement is essential when it has to be fused with other measurements in a stochastic localization framework, like the SLAM. To the authors knowledge, very few works exists which address in depth this problem. Covariance estimation based on the environment and the uncertainties of the sensor model has been introduced in [7] and [8] but as has been shown in [9], that kind of estimation in a number of cases can be very optimistic. Moreover, in the same work [9], two methods are proposed: 1) the Hessian and 2) the Off-line sample-based method. The first one uses the Hessian of the error function and the variance of the sample to compute the uncertainty of the LS estimator. The second, builds a geometric map for the reference scan to generate a virtual scan and a sample of the initial guess of the robot pose. Then the new scan is registered against the virtual scan and the process is repeated several times allowing to compute the covariance for the different samples. It was concluded that the Hessian method is suitable for online estimation being able to capture the shape but not always the size of the covariance matrix. On the other hand, the second method while capturing the size and shape of the covariance matrix can only be applied offline due to the high computational cost. In a SLAM context, an approach similar to the offline method is used in [10], although in this case only the initial guess of the displacement is sampled.

Hereafter, the Hessian method is adapted and tested with the MSISpIC algorithm which is the second major contribution to our previous work [1].

Let us consider the nonlinear, point-to-point, matching equation:

$$\mathbf{A} = \mathbf{q} \oplus \mathbf{N} + \mathbf{E} \tag{5}$$

or more detailed:

$$\begin{bmatrix} a_1 \\ \vdots \\ a_m \end{bmatrix} = \begin{bmatrix} \mathbf{q} \oplus n_1 \\ \vdots \\ \mathbf{q} \oplus n_m \end{bmatrix} + \begin{bmatrix} e_1 \\ \vdots \\ e_m \end{bmatrix} \tag{6}$$

where $\{a_i, n_i\}$ denotes a matching pair. Using NLLS it is possible to solve for $\mathbf{q} = [\hat{\mathbf{q}}_{min}, \mathbf{P}_{q_{min}}]$ through the minimization of the point-to-point error function (7), as we saw in Section II

$$\begin{aligned} \mathbf{F} &= \frac{1}{2}\sum_i^k \left(\hat{\mathbf{e}}_i^T \mathbf{P}_{e_i}^{-1} \hat{\mathbf{e}}_i\right) = \\ &= \frac{1}{2}\sum_i^k \left(\left[\hat{\mathbf{a}}_i - \hat{\mathbf{q}}_k \oplus \hat{\mathbf{n}}_i\right]^T \mathbf{P}_{e_i}^{-1} \left[\hat{\mathbf{a}}_i - \hat{\mathbf{q}}_k \oplus \hat{\mathbf{n}}_i\right]\right) \end{aligned} \tag{7}$$

The NLLS estimator that minimizes (7) is obtained by setting the gradient with respect to $\mathbf{q}_k$ to zero

$$\nabla_{q_k}\mathbf{F} = -\mathbf{J}_q^T \mathbf{P}_e^{-1} \left[\mathbf{Y} - \mathbf{J}_q \hat{\mathbf{q}}_k\right] = 0 \tag{8}$$

which yields

$$\hat{\mathbf{q}}_{min} = \left[\mathbf{J}_q^T \mathbf{P}_e^{-1} \mathbf{J}_q\right]^{-1} \mathbf{J}_q^T \mathbf{P}_e^{-1} \mathbf{Y} \tag{9}$$

where

- $\mathbf{J}_q = \left.\dfrac{\partial\, \mathbf{q} \oplus \mathbf{N}}{\partial\, \mathbf{q}}\right|_{\hat{\mathbf{q}}} = \begin{bmatrix} \frac{\partial\, \mathbf{q} \oplus \mathbf{n}_1}{\partial\, \mathbf{q}} \\ \vdots \\ \frac{\partial\, \mathbf{q} \oplus \mathbf{n}_m}{\partial\, \mathbf{q}} \end{bmatrix}$
- $\mathbf{P}_e^{-1} = blockdiag\left\{\mathbf{P}_{e_1}^{-1}, \ldots, \mathbf{P}_{e_m}^{-1}\right\}$ and
- $\mathbf{Y} = \begin{bmatrix} \mathbf{J}_q \hat{\mathbf{q}}_k + \hat{\mathbf{e}}_1 \\ \vdots \\ \mathbf{J}_q \hat{\mathbf{q}}_k + \hat{\mathbf{e}}_m \end{bmatrix} = \begin{bmatrix} \mathbf{J}_q \hat{\mathbf{q}}_k + [\hat{\mathbf{a}}_1 - \hat{\mathbf{q}}_k \oplus \hat{\mathbf{n}}_1] \\ \vdots \\ \mathbf{J}_q \hat{\mathbf{q}}_k + [\hat{\mathbf{a}}_m - \hat{\mathbf{q}}_k \oplus \hat{\mathbf{n}}_m] \end{bmatrix}$

Then, the covariance matrix of the NLLS estimator is known to be [11]:

$$\mathbf{P}_{q_{min}} = \left[\mathbf{J}_q^T \mathbf{P}_c^{-1} \mathbf{J}_q\right]^{-1} \tag{10}$$

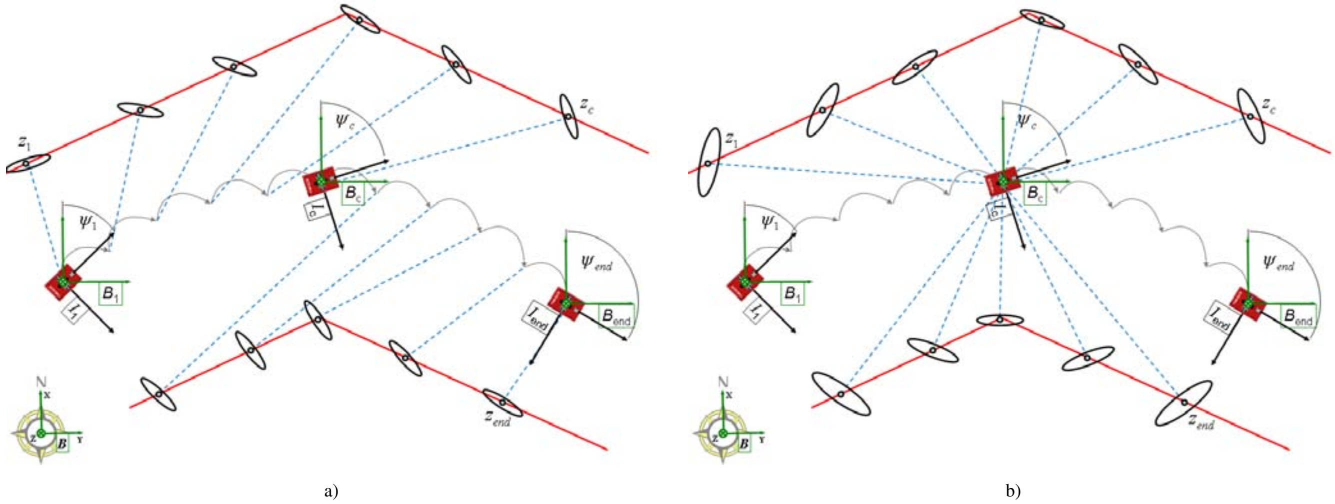


Fig. 4. The scan forming process. a) Each beam is grabbed in different robot's poses. The sequence of the scan lines is simplified for clarity. b) The beams of a full scan are referenced in the center $\{I_c\}$ frame of the robots' trajectory covering that scan.

which actually is the inverse of the Hessian of the cost function.

Nevertheless, [12] has shown cases that the Hessian method provides pessimistic estimations of the covariance matrix. In the same work the author proposed an approach formerly proposed by the computer vision community [13]. This method, is supposed to allow more accurate propagation of the uncertainty of the sonar points to the uncertainty of the scan matching solution, and is in our line of the future research. After the covariance estimation, we can easily fuse the scan matching estimation in the SLAM algorithm.

V. SLAM Algorithm

The proposed pose-based SLAM algorithm uses an ASEKF for the scan poses estimation. In this implementation of the stochastic map [14], the estimate of the positions of the vehicle at the center of each full scan $\{\mathbf{x}_1 \ldots \mathbf{x}_n^B\}$ are stored in the state vector $\hat{\mathbf{x}}$:

$$\hat{\mathbf{x}}_k^B = \left[\hat{\mathbf{x}}_{n_k}^B \;\ldots\; \hat{\mathbf{x}}_{i_k}^B \;\ldots\; \hat{\mathbf{x}}_{1_k}^B\right]^T \tag{11}$$

and the covariance matrix for this state is defined as:

$$\mathbf{P}_k^B = E([\mathbf{x}_k^B - \hat{\mathbf{x}}_k^B][\mathbf{x}_k^B - \hat{\mathbf{x}}_k^B]^T) \tag{12}$$

Note that, a full scan is defined as the final 360° polar range image obtained after compounding, along the path, the robot pose with the range and bearing data, which is the output from the MSISpIC algorithm.

A. Map Initialization

All the elements on the state vector are represented in the map reference frame B. Although this reference frame can be defined arbitrarily, we have chosen to place its origin on the initial position of the vehicle and orient it to the north, so compass measurements can be easily integrated.

The pose state $\mathbf{x}_i$ is represented as:

$$\mathbf{x}_i^B = [x \;\, y \;\, \psi]^T \tag{13}$$

where, x, y and ψ is the position and orientation vector of the vehicle in the global frame B. The state and the map are initialized from the first full scan obtained by the MSISpIC algorithm.

B. Prediction

Let

- $\mathbf{x}_{n_k}^B \equiv N(\hat{\mathbf{x}}_{n_k}^B,\ \mathbf{P}_{q_k}^B)$ be the last robot pose and
- $\hat{\mathbf{q}}_n^{B_n} \equiv N(\hat{\mathbf{q}}_n^{B_n},\ \mathbf{P}_{q_n}^B)$ be the robot displacement during the last scan, estimated through dead reckoning

then the prediction / state augmentation equation is given by:

$$\hat{\mathbf{x}}_k^{B+} = \hat{\mathbf{x}}_k^B \odot \hat{\mathbf{q}}_{n_k}^{B_n} = \tag{14}$$

$$\left[\;\hat{\mathbf{x}}_{n-1_k}^B \odot \hat{\mathbf{q}}_{n_k}^{B_n} \,|\, \hat{\mathbf{x}}_{n-1_k}^B \ldots \hat{\mathbf{x}}_{i_k}^B \ldots \hat{\mathbf{x}}_{1_k}^B\;\right]^T \tag{15}$$

where, given that B and B_n frames are both north aligned, the operator $\odot$ is defined as:

$$\mathbf{x} \odot \mathbf{q} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \odot \begin{bmatrix} d \\ e \\ f \end{bmatrix} = \begin{bmatrix} a+d \\ b+e \\ f \end{bmatrix} \tag{16}$$

being $\mathbf{J}_{1\odot}$ and $\mathbf{J}_{2\odot}$ the corresponding linear transformation matrices:

$$\mathbf{J}_{1\odot} = \begin{bmatrix} \mathbf{I}_{2x2} & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{J}_{2\odot} = \mathbf{I}_{3x3}$$

and the predicted pose uncertainty $\mathbf{P}_k^{B+}$ is computed as:

$$\mathbf{P}_k^{B+} = \mathbf{F}_k \mathbf{P}_k^B \mathbf{F}_k^T + \mathbf{G}_k \mathbf{P}_{q_i}^B \mathbf{G}_k^T \tag{17}$$

where,

$$\mathbf{F}_k = \begin{bmatrix} \mathbf{J}_{1\odot} & \mathbf{0}_{3x3} & \ldots & \mathbf{0}_{3x3} \\ \mathbf{0}_{3x3} & \mathbf{I}_{3x3} & \ldots & \mathbf{0}_{3x3} \\ \vdots & \vdots & \ldots & \vdots \\ \mathbf{0}_{3x3} & \mathbf{0}_{3x3} & \ldots & \mathbf{I}_{3x3} \end{bmatrix} \quad \mathbf{G}_k = \begin{bmatrix} \mathbf{J}_{2\odot} \\ \mathbf{0}_{3x3} \\ \vdots \\ \mathbf{0}_{3x3} \end{bmatrix}$$

C. Scan Matching

In order to execute the modified pIC algorithm, given two overlapping scans (S_i, S_n) with related poses $(\mathbf{x}_i^B, \mathbf{x}_n^B)$, an initial guess of their relative displacement is necessary. This initial guess $[\hat{\mathbf{q}}_i^{I_i}, \mathbf{P}_i^{I_i}]$ can be easily extracted from the state vector using the tail-to-tail transformation [14]:

$$\hat{\mathbf{q}}_i^{I_i} = \ominus\hat{\mathbf{x}}_i^B \oplus \hat{\mathbf{x}}_n^B \tag{18}$$

Since the tail-to-tail transformation is actually a nonlinear function of the state vector $\hat{\mathbf{x}}_k^B$, the uncertainty of the initial guess can be computed by means of the Jacobian of the nonlinear function:

$$\mathbf{P}_{q_i}^{I_i} = \mathbf{H}_k \mathbf{P}_k^B \mathbf{H}_k^T \tag{19}$$

where

$$\mathbf{H}_k = \left. \frac{\partial \ominus \hat{\mathbf{x}}_i^B \oplus \hat{\mathbf{x}}_n^B}{\partial\, \mathbf{x}_k^B} \right|_{(\mathbf{x}_k^B = \hat{\mathbf{x}}_k^B)} \tag{20}$$

Moreover, as shown in [14], the Jacobian for the tail-to-tail transformation $\mathbf{x}_{a_c} = \ominus\mathbf{x}_{b_a} \oplus \mathbf{x}_{b_c}$, is:

$$\frac{\partial \ominus \mathbf{x}_{b_a} \oplus \mathbf{x}_{b_c}}{\partial\,(\mathbf{x}_{b_a}\mathbf{x}_{b_c})} = [\mathbf{J}_{1\oplus}\mathbf{J}_{\ominus} \quad \mathbf{J}_{2\oplus}] \tag{21}$$

where the $\mathbf{J}_{1\oplus}, \mathbf{J}_{2\oplus}$ and $\mathbf{J}_{\ominus}$ are the Jacobian matrices of the compounding and inverse transformations respectively.

Being in our case $\hat{\mathbf{x}}_{n_k}^B$ and $\hat{\mathbf{x}}_{i_k}^B$ components of the full state vector, the Jacobian of the measurement equation becomes:

$$\mathbf{H}_k = \frac{\partial \ominus\hat{\mathbf{x}}_{i_k}^B \oplus \hat{\mathbf{x}}_{n_k}^B}{\partial\, \mathbf{x}_k} = \\ [\mathbf{J}_{2\oplus_{3x3}} \quad \mathbf{0}_{3x3(n-i-1)} \quad \mathbf{J}_{1\oplus}\mathbf{J}_{\ominus_{3x3}} \quad \mathbf{0}_{3x3(i-1)}]$$

Once the initial displacement guess is available, the pIC algorithm can be used to produce an updated measurement of this displacement.

D. Loop Closing Candidates

Each new pose of a scan is compared against the previous scan poses that are in the nearby area defined by a threshold. Whenever enough points are overlapping, a new scan match puts a constrain between the poses updating the ASEKF. These constrains close the loops correcting the whole trajectory and bounding the drift.

Let

- $\mathbf{x}_{n_k}^B$ be the last scan pose and S_{n_k} the corresponding scan,
- $Overlap_k = \left\{S_{i_k} \;/\; \left\|\hat{\mathbf{x}}_{n_k}^B - \hat{\mathbf{x}}_{i_k}^B\right\| < threshold\right\}$ the set of overlapping scans and $\mathbf{O}_k = [S_{1_k},\ S_{2_k}\ \ldots\ S_{m_k}]$ the sequence of overlapping scans belonging to the $Overlap_k$ set

then $\forall\ [S_{i_k}, \mathbf{x}_i^B] \in \mathbf{O}_k$, perform a new scan matching between the scan poses $(\mathbf{x}_{n_k}^B, \mathbf{x}_{i_k}^B)$ with the corresponding scans (S_{n_k}, S_{i_k}) obtaining $[\hat{\mathbf{q}}_i^{I_i}, \mathbf{P}_{q_i}^{I_i}]$ the result of the scan matching. $\mathbf{P}_{q_i}^{I_i}$ is the corresponding uncertainty computed as described in Section IV. Finally the scan matching result is used to update the filter.

E. State Update

When two overlapping scans (S_i, S_n) with the corresponding poses $(\mathbf{x}_i^B, \mathbf{x}_n^B)$ are registered, their relative displacement defines a constrain between both poses. This constrain can be expressed by means of the measurement equation:

$$\mathbf{z}_k = h(\mathbf{x}_k, \mathbf{v}_k) \tag{22}$$

which again, in our case becomes:

$$\mathbf{z}_k = \ominus\hat{\mathbf{x}}_{i_k}^B \oplus \hat{\mathbf{x}}_{n_k}^B \tag{23}$$

where $\hat{\mathbf{x}}_{i_k}^B$ is the scan pose which overlaps with the last scan pose $\hat{\mathbf{x}}_{n_k}^B$. Now, an update of the stochastic map can be performed with the standard extended Kalman filter equations.

VI. Experimental Results

The method described in this paper has been tested with a dataset obtained in an abandoned marina located in the Catalan coast [15]. This dataset corresponds to structured environment but our algorithm does not take into account any structural information neither features and with the current sensor suite, it can been used wherever there is enough vertical information. The survey mission was carried out using ICTINEU$^{\text{AUV}}$ [16] traveling along a 600 m path. The MSIS was configured to scan a full 360° sector and it was set to maximum range of 50 m with a 0.1 m resolution and a 1.8° angular step. With those settings, the MSIS needed around 14 secs to complete a full scan. Dead-reckoning was computed using the velocity reading coming from the DVL and the heading data obtained from the MRU sensor, both merged using the described EKF. The whole dataset was acquired in 53 mins and the off-line execution of the algorithm implemented in MATLAB took around 14 mins in a simple Pentium M @2,00 GHz laptop, which gives good possibilities for real time implementation of the proposed algorithm.

Fig. 5 shows the trajectory and the map estimated with the proposed SLAM algorithm. With the dotted line is the trajectory from our previous work presented in [1]. The improvements can be easily appreciated when compered with the ground truth (DGPS) trajectory (solid line). In Fig. 6, both results are projected on an ortophotomap of the real environment. The improvements can be appreciated in the top part of marina, in the corridor and also in the less sparsity of the map.

VII. Conclusions

This paper proposes an extension to the MSISpIC algorithm in the pose-based SLAM framework. To deal with the motion induced distortion of the acoustic image, an EKF is used to estimate the robot motion during the scan. The filter uses a constant velocity model with acceleration noise for motion prediction and velocity (DVL) and attitude measurements (MRU) for updating the state. Through the compounding of the relative robot position within the scan, with the range and bearing measurements of the beams gathered with the sonar, the acoustic image gets undistorted. Assuming Gaussian noise, the algorithm is able to predict the uncertainty of the

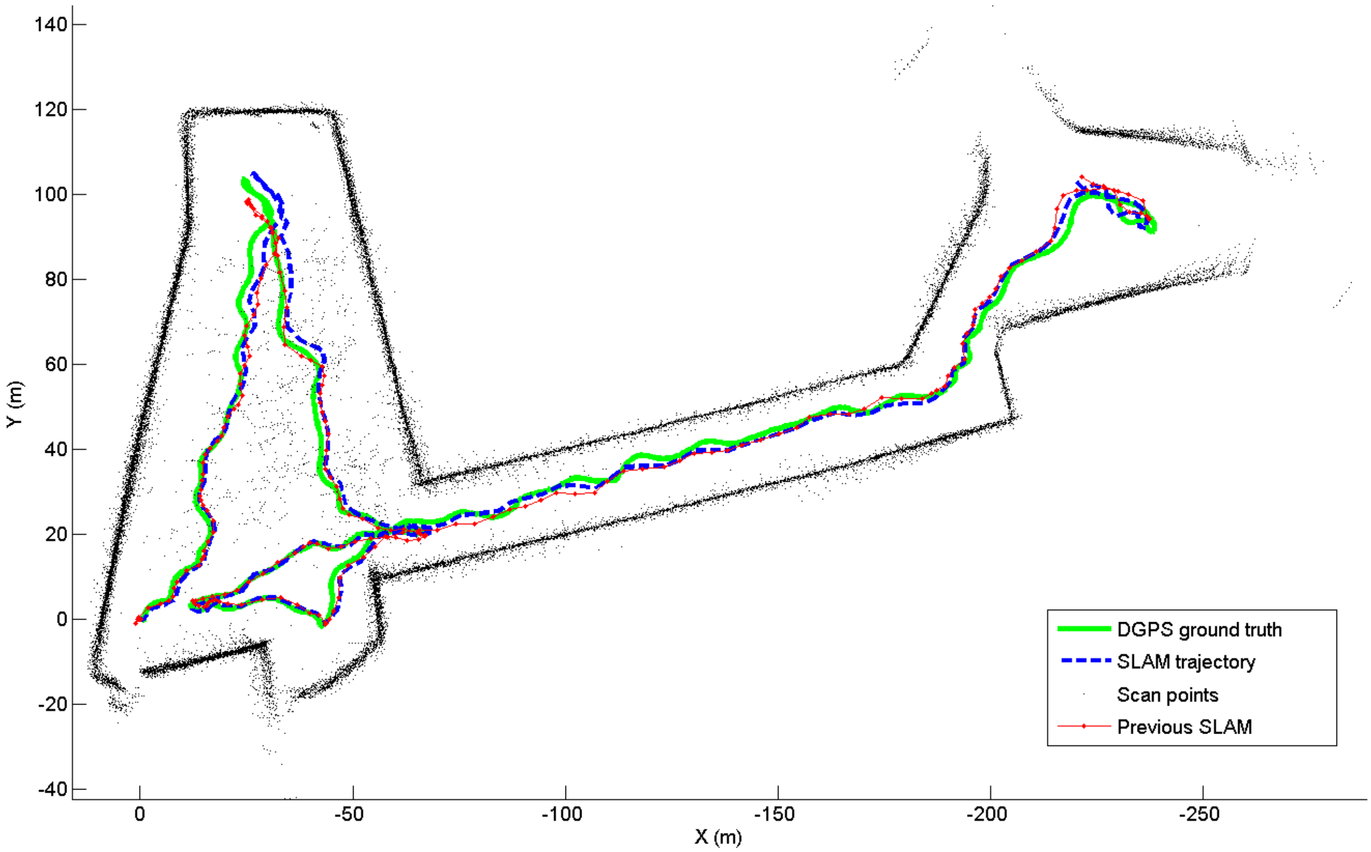


Fig. 5. SLAM Trajectory and map. In red is the SLAM trajectory from our previous work (dotted line), in solid green line the DGPS trajectory used as a ground truth and in blue (dash) the trajectory estimated with the SLAM algorithm.

a) b)

Fig. 6. Trajectory and map estimated with the SLAM algorithm projected on the orthophoto map. a) Results from [1]. b) Current results.

sonar measurements with respect to a frame located at the position occupied by the robot at the center of the scan. Each full scan pose is maintained in a second filter, an augmented EKF, and is cross registered with all the previous scan poses that are in a certain range applying a modified pIC algorithm. The uncertainty of the scan matching algorithm is computed and incorporated into the proposed SLAM algorithm based on the Hessian of the error function being minimized. The proposed SLAM method has been tested with a real world dataset including DGPS for ground truth acquired during a survey mission in an abandoned marina located in the Girona coast. The results are compared against our previous work in [1], and substantial improvements in trajectory correction and map reconstruction are notable.

Acknowledgment

This research was sponsored by the Marie Curie Research Training Network FREEsubNET (MRTN-CT-2006-036186) and by the Spanish government (DPI2005-09001-C03-01).

REFERENCES

[1] A. Mallios, P. Ridao, E. Hernandez, D. Ribas, F. Maurelli, and Y. Petillot, "Pose-based slam with probabilistic scan matching algorithm using a mechanical scanned imaging sonar," in *Oceans IEEE - EUROPE*, 11-14 May 2009.

[2] D. Hahnel, W. Burgard, D. Fox, and S. Thrun, "An efficient FastSLAM algorithm for generating maps of large-scale cyclic environments from raw laser range measurements," in *Intelligent Robots and Systems, 2003. (IROS 2003). Proceedings. IEEE/RSJ International Conference on*, vol. 1, 27-31 Oct. 2003, pp. 206–211.

[3] E. Hernandez, P. Ridao, D. Ribas, and A. Mallios, "Probabilistic sonar scan matching for an auv," in *Intelligent Robots and Systems, 2009. IROS 2009. IEEE/RSJ International Conference on*, Oct. 2009, pp. 255–260.

[4] L. Montesano, J. Minguez, and L. Montano, "Probabilistic scan matching for motion estimation in unstructured environments," in *Intelligent Robots and Systems, 2005. (IROS 2005). 2005 IEEE/RSJ International Conference on*, Aug. 2005, pp. 3499–3504.

[5] A. Burguera, Y. González, and G. Oliver, "A Probabilistic Framework for Sonar Scan Matching Localization," *Advanced Robotics*, vol. 22, no. 11, pp. 1223–1241, 2008.

[6] D. Ribas, P. Ridao, J. Tardós, and J. Neira, "Underwater SLAM in man made structured environments," *Journal of Field Robotics*, vol. 25, no. 11-12, pp. 898–921, July 2008.

[7] F. Lu and E. Milios, "Robot pose estimation in unknown environments by matching 2d range scans," *Journal of Intelligent and Robotic Systems*, vol. 18, no. 3, pp. 249–275, 1997.

[8] S. Pfister, K. Kriechbaum, S. Roumeliotis, and J. Burdick, "Weighted range sensor matching algorithms for mobile robot displacement estimation," in *Robotics and Automation, 2002. Proceedings. ICRA '02. IEEE International Conference on*, vol. 2, 2002, pp. 1667–1674.

[9] O. Bengtsson and A.-J. Baerveldt, "Robot localization based on scan-matching - estimating the covariance matrix for the idc algorithm," *Robotics and Autonomous Systems*, vol. 44, no. 1, pp. 29 – 40, Jun 2003, best Papers of the Eurobot '01 Workshop.

[10] J. Nieto, T. Bailey, and E. Nebot, "Recursive scan-matching SLAM," *Robotics and Autonomous Systems*, vol. 55, no. 1, pp. 39–49, Jan 2007.

[11] Y. Bar-Shalom, X. Li, and T. Kirubarajan, *Estimation with applications to tracking and navigation*. Wiley-Interscience, 2001.

[12] A. Censi, "An accurate closed-form estimate of ICP's covariance," in *Proceedings of the IEEE International Conference on Robotics and Automation (ICRA)*, Rome, Italy, apr 2007, pp. 3167–3172. [Online]. Available: http://purl.org/censi/2006/icpcov

[13] R. Haralick, "Propagating covariance in computer vision," *Performance Characterization in Computer Vision*, vol. 1, pp. 95 – 115, 1998.

[14] R. Smith, M. Self, and P. Cheeseman, *Estimating uncertain spatial relationships in robotics*. New York, NY, USA: Springer-Verlag New York, Inc., 1990, pp. 167–193.

[15] D. Ribas. (2009, March) David Ribas homepage. [Online]. Available: http://eia.udg.edu/∼dribas

[16] D. Ribas, N. Palomer, P. Ridao, M. Carreras, and E. Hernandez, "Ictineu AUV wins the first SAUC-E competition," in *IEEE International Conference on Robotics and Automation*, Roma, Italy, apr 2007.