RT^2: A Real-Time Ray-Tracing Method for Acoustic Distance Evaluations among Cooperating AUVs

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***Abstract*— The paper deals with the problem of distributed acoustic localization of teams of Autonomous Underwater Vehicles (AUVs) and proposes a novel algorithm, Real-Time Ray-Tracing (RT^2), for evaluating the distance between any pair of AUVs in the team. The technique, based on a modified formulation of the non-linear sound-ray propagation laws, allows efficiently handling the distorted and reflected acoustic ray paths, induced by the anisotropy of the underwater medium. Further it can be easily implemented on-board of low-cost AUVs. Indeed it just requires the presence, on each vehicle, of a simple acoustic modem and a pair of look-up tables, a-priori constructed via the assumed knowledge of the depth-dependent sound velocity profile. On such a basis, every AUV can easily on-line compute its distance w.r.t. to any other neighbour team member, through time-of-flight measurements and the exchanges of depth information *only*. Further, since the proposed RT^2 algorithm makes available accurate distance evaluations (despite the distorted acoustic rays), the effective filtering techniques normally used by terrestrial mobile robots for distributed localization are expected to be transferable to the underwater field.**

I. Introduction

The paper deals with acoustic distributed localization of teams of Autonomous Underwater Vehicles (AUVs) and, more specifically, it focuses on the problem of accurately evaluating the distances between vehicles, by explicitly keeping into account the propagation anisotropy of the underwater medium (i.e. the changes in the sound velocity profile as a function of depth). Problems posed by ray distortions are very well known in the underwater acoustic literature and induce significant implications on the problem of localization. Indeed, differing from what happens in air, where radio communication is available, an underwater device receiving an acoustic signal cannot precisely estimate its distance from the sound source, through the *sole* measurement of the Time-of-Flight (ToF) of the received wave. To achieve an underwater accurate acoustic localization, it is indeed necessary to combine a physical parameter extracted from the received acoustic signal (such as the ToF, the ray Angle of Arrival (AoA), or the level of attenuation of the signal) with the knowledge of the sound-ray propagation laws applied to the specific sound-speed profile of the considered area.

In fact, the more advanced acoustic localization devices, such as some of the most recent USBL/SBL and LBL systems (see for instance [1]), can provide corrections to range estimates, based on ray-tracing theory and measurements of ToF or AoA parameters. Despite their efficiency, there is a concrete limit in the applicability of the above systems in the considered framework of a team of cooperating AUVs. Such instruments are conceived for tracking the position of a mobile underwater device, while they do not allow the underwater device be aware of its position, which remain an information available only at a *centralized* level; instead, for achieving a good level of team-cooperation, it is mandatory that each team member knows its position with the maximum achievable precision. Moving from this consideration, the paper is intended to give a contribution within the lines of development of algorithms and methods based on ToF measurements and depth information sharing, as recently proposed in [2].

Indeed, the availability of acoustic modems, able to provide ToF measurements together with the communication functionality opens the way for implementing on-board the vehicles ray-tracing capabilities.

Previous treatment of the acoustic path distortion effects can be found in [3], [4], where however LBL systems are considered, and the ray correction is not performed on board the vehicle(s); in [5] the solution of considering straight paths is proposed, using, as sound speed, the weighted depth average of the sound speed. This latter solution may be useful when propagation occurs mostly on the vertical, but it may be not enough accurate at oblique angles. In this paper a novel algorithm is proposed, based on a modified formulation of the sound-ray propagation laws which allows to efficiently handle the distorted and reflected acoustic ray paths in a way suitable for producing accurate evaluations of distances between pairs of AUV within a team, as well as between an AUV and the buoy elements of an LBL system. Further the

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implementation of the proposed Real-Time Ray-Tracing (*RT*^{2}) technique is very simple. It just requires the presence, on-board each vehicle, of an acoustic modem (already necessary for coordination purposes) and of a pair of look-up tables, a-priori constructed on the basis of a preliminary knowledge of the depth-dependent sound velocity profile. As a consequence, the proposed RT^2 can be easily hosted even on-board of low-cost AUVs. Among the others, we can here mention the AUV "Folaga" [6], specifically developed by our research team, on which we expect to experiment the here proposed RT^2 method.

Other than presenting details of the proposed RT^2 method, the paper will also show how its adoption reduces the distributed localization problem for AUV teams to its aerial analogous one, for which efficient filtering techniques for distributed localization already exist [7], [8], [9]. Some of these techniques are therefore expected to be transferable to the underwater field, thus significantly reducing the high level of criticality generally posed by underwater medium anisotropies. It is also expected that the joint use of the here proposed RT^2 method *and* at least the most classical parametric filtering techniques (quite effective and easy to implement) shall be soon experimented as part of the activities of the two EU-supported research projects, CO^{3}AUV (Cognitive Cooperative Control of AUVs) and UAN (Underwater Acoustic Networks), in which our research group is currently actively involved.

The paper is organized as follows: in section two a preliminary brief review of sound-ray propagation theory is given; in section three the basic functional algorithms for the RT^2 algorithm are described; in the fourth section the RT^2 algorithm is described. Section five briefly discusses the distributed localization problem. Finally conclusions are given.

II. Brief Review of Sound-Ray Theory

Let us consider a cylindrical coordinate system, with the origin at the sea surface and the z axis pointing downward toward the sea bottom. It is assumed that the sound speed in water can vary only with depth and not with range from the source (*depth-dependent* sound velocity profile). With these assumptions, acoustic propagation can be represented in the plane (z, r) of depth and range, independent from the azimuth angle. The coordinate system for acoustic propagation is illustrated in Fig. 1, right. Acoustic propagation will be treated with a ray-theory approach, which is a valid approximation within the usually high (above kHz) transmission frequencies used in underwater localization and communication. Moreover, the assumption of a range independent environment allows for the use of the ray equations for a stratified media as reported in [10], pp.176. The acoustic source is assumed to be at 0 horizontal range. By letting $c(z) > 0$ denoting sound velocity, consider the associated Snells law:

$$cos(\theta) = kc(z) \tag{1}$$
$$k = \frac{cos(\theta_0)}{c(z_0)} \in \left[0; \frac{1}{c(z_0)}\right]; \, \theta_0 \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$$

being θ_0 the angle with the horizontal axis of a ray starting at depth z_0 and θ the same one at a generic depth z, among those successively traversed by the same ray, as in Fig. 1.

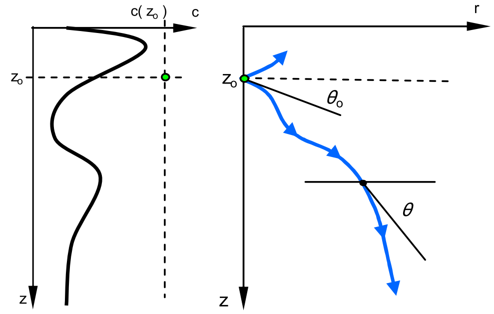

Fig. 1. Left: sound velocity profile as a function of depth in the water column; Right: cylindrical coordinate system, sample ray path and corresponding angles accordingly with Snell's law.

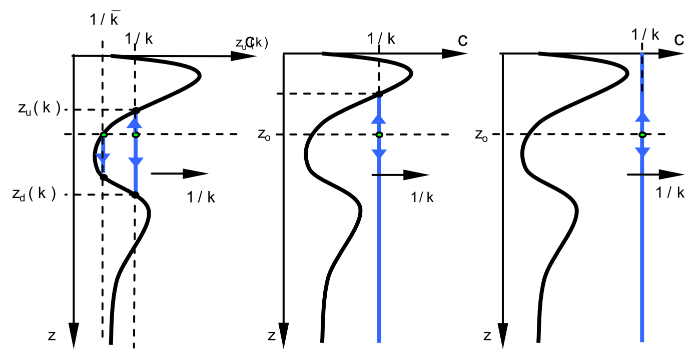

Fig. 2. Sound speed profile as a function of depth and different lines $1/k$ corresponding to different couples of ray solutions. The intersection of any $1/k$ line with the sound speed profile determines the upper and lower depths within which the rays will be confined.

In representation (1), since $(z_0; k) \iff (z_0; \pm\theta_0)$, when providing the left information we actually refer to a *couple* of trajectories. Note that, since it must necessarily be $1/k \geq c(z)$, it follows that, for any given starting conditions $(z_0; k)$, the couple of resulting trajectories will time-evolve within the depth range, including z_0:

$$z \in [z_u(k), z_d(k)] \quad ; \quad k \in [0, 1/c(z_0)] \tag{2}$$

with $z_u(k), z_d(k)$ the abscissas of the first-encountered upper and lower intersection points that the vertical line $1/k = c(z_0)$ forms with $c(z)$, when seen as starting from z_0, as indicated in Fig. 2 where, for a given z_0, different values of $1/k$ are shown.

Clearly for any given value of parameter k, $z_u(k)$ must set to zero (surface level) and $z_d(k)$ set to z_M (bottom level) whenever the corresponding vertical segment $1/k$ results with one or both its end-points on one or both such extreme levels. Thus, depending from the shape of $c(z)$, some rays may remain confined between two definite depths included between surface and bottom, giving rise to the so called *acoustic channel*. By now considering the kinematic equations for a generic ray, $\dot{r} = c(z)cos(\theta)$, $\dot{z} = c(z)sin(\theta)$, and constraint

(1), we have

$$\left\{\begin{array}{l}\dot{r}=kc^2(z)\\ \dot{z}=\pm c(z)\sqrt{1-k^2c^2(z)}\\ \theta=\pm arccos(kc(z))\end{array}\right. \tag{3}$$

where the "+" sign indicates the downward starting motion and the "-" sign refers to the upward starting one and where for given $(z_0;k)$, the couple of equations for z are defined within the interval (2).

Moreover upon integration, each one of such equations monotonically reaches its associated interval extreme, respectively at the finite times (see equation 3.101 in [10]):

$$\left\{\begin{array}{l}t_d(z_0,k)\triangleq\displaystyle\int_{z_0}^{z_d^-}f(k,z)dz\\ t_u(z_0,k)\triangleq\displaystyle\int_{z_u^+}^{z_0}f(k,z)dz\end{array}\right. \tag{4}$$

with

$$f(k,z)\triangleq\frac{1}{c(z)\sqrt{1-k^2c^2(z)}}$$

in correspondence of which, in case both the extremes do not coincide with the bottom and the surface, respectively (as in Fig. 2, left) it results $\dot{z}=0$. Obviously such occurrences correspond to the achievement of the finite-time equilibria points $z_u(k), z_d(k)$. In terms of the sole kinematic equations (3), these equilibria can be shown to be both unstable; it follows that any perturbation will almost surely force each z-motion having reached one of the extrema to reverse its direction, yielding to periodic evolutions. Note that the same result can be obtained more rigorously including all the terms from the wave equation, in particular the dynamics for k and for $\chi=sin(\theta)/c(z)$, see for instance [10], pp.152; this is the case usually referred in physics as total reflection. In case instead one or both of the extrema coincide with the surface and/or the bottom (Fig. 2, middle and right) we must account for surface/bottom reflections, also imposing a periodicity to the rays capable of reaching such extrema. This is qualitatively depicted in Fig. 3, which is relevant to the case of Fig. 2, middle.

From (4), the time period $T(k)$ (see Fig. 3) can be then easily calculated as:

$$T(k)\triangleq 2\int_{z_u^+}^{z_d^-}f(k,z)dz \tag{5}$$

Further the ratio of the second and the first of (3) leads to:

$$\frac{dz}{dr}=\pm\frac{1}{kc(z)}\sqrt{1-k^2c^2(z)} \tag{6}$$

which establishes a *one-to-one* correspondence between the pair of time-z motions $z(z_0,k,t)$ resulting from the second in (3) with the couple of distance-z motions $z(z_0,k,r)$ resulting from the integration of (6). The representation of the pair of distance-z motions on the (z,r) plane also exhibits periodic behaviours, which are qualitatively similar to those of their companion time-z motions; this explains why we have actually used the same Fig. 3 for representing both type of motions. Still within the strict analogy with the time-z motions, note that with regard to the resulting distance-period $R(k)$ we consequently have (by causally inverting (6) within a period, see also equation 3.97 in [10])

$$R(k)\triangleq\int_{z_u^+}^{z_d^-}g(k,z)dz \quad ; \quad g(k,z)\triangleq\frac{kc(z)}{\sqrt{1-k^2c^2(z)}} \tag{7}$$

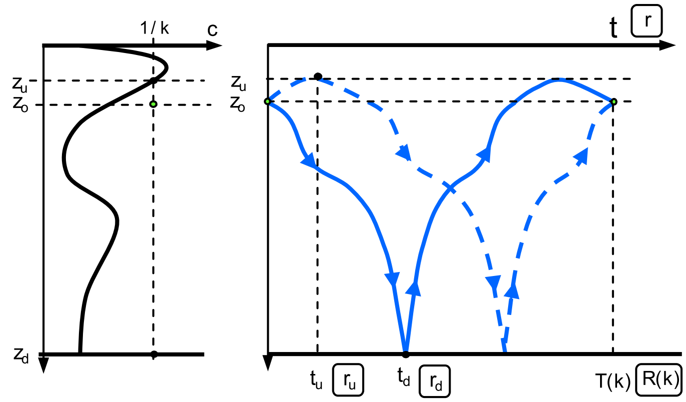

Fig. 3. Left: sound speed profile as a function of depth, with indication of upper extremum z_u for a given k; Right: the couple of ray-paths corresponding to the given k; note that ray evolution can be expressed either as a function of depth and time (t) or of depth and range (r).

III. Basic Algorithms

Given a pair of depths z_a, z_b, for any assigned admissible parameter k (satisfying $1/k \geq \max\{c(z_a), c(z_b)\}$ the data sets $T_{ab}(k)$ and $R_{ab}(k)$ can be defined, as respectively constituted by all the possible travel-times and their one-to-one associated horizontal-travel-distances connecting the depths z_a, z_b. By now noting that the horizontal-travel-distance (or equivalently the travel-time) of any ray connecting the depths z_a, z_b is constituted by an integer number of horizontal-distance-periods (time-periods) plus one out of four possibilities of connection (as shown in Fig. 4, where notation is introduced), the following expressions for the sets $T_{ab}(k)$ and $R_{ab}(k)$ can be stated:

$$\left\{\begin{array}{l}T_{ab}(k)=\bar{T}_{ab}(k)+nT(k)\\ R_{ab}(k)=\bar{R}_{ab}(k)+nR(k)\end{array}\right. ;$$
$$\left\{\begin{array}{l}\bar{T}_{ab}(k)\triangleq\left\{\tau_{ab}^j(k); j=1,\ldots,4\right\}\\ \bar{R}_{ab}(k)\triangleq\left\{r_{ab}^j(k); j=1,\ldots,4\right\}\end{array}\right. ; \tag{8}$$

where the finite sets $\bar{T}_{ab}(k)$ and $\bar{R}_{ab}(k)$, respectively, admit the following representations:

$$\bar{T}_{ab}(k)=\left\{\begin{array}{lcl}\tau_{a,b}^1&=&\Delta_{a,b}\\ \tau_{a,b}^2&=&\Delta_{a,b}+2\Delta_{b,d}\\ \tau_{a,b}^3&=&\Delta_{a,b}+2\Delta_{a,u}\\ \tau_{a,b}^4&=&\Delta_{a,b}+2\Delta_{a,u}+2\Delta_{b,d}\end{array}\right\}_{(k)} \tag{9}$$

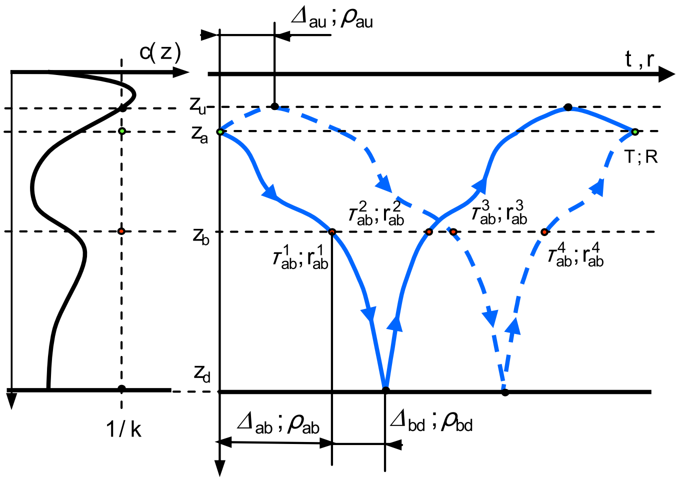

Fig. 4. Left: sound speed profile as a function of depth, with indication of the chosen depths z_a, z_b and of the chosen k. Right: the couple of ray paths corresponding to the given k showing the 4 intersections of the ray with the depth z_b within one period, either in time or in range. $\Delta_{i,j}$ represents the travel-time between points i, j, while $\rho_{i,j}$ represents the horizontal-travel-distance between the points i, j.

$$\bar{R}_{ab}(k) = \left\{ \begin{array}{lcl} r^1_{a,b} & = & \rho_{a,b} \\ r^2_{a,b} & = & \rho_{a,b} + 2\rho_{b,d} \\ r^3_{a,b} & = & \rho_{a,b} + 2\rho_{a,u} \\ r^4_{a,b} & = & \rho_{a,b} + 2\rho_{a,u} + 2\rho_{b,d} \end{array} \right\}_{(k)} \tag{10}$$

From (8), and (9), (10) it is easy to see that, in correspondence of any given triple z_a, z_b, k any element of the resulting $T_{ab}(k)$, $R_{ab}(k)$, sets can be generated by knowing a finite number of parameters: i.e. the period $T(k)$ or $R(k)$ plus the finite sets $\bar{T}_{ab}(k)$, $\bar{R}_{ab}(k)$. As a consequence a pair of look-up tables, termed as *partial-travel-time (PTT)* and *partial-travelled-distance (PTD)* containing data for on-line evaluating such parameters can be conveniently constructed as hereafter explained.

Assume the sound velocity profile function $c(z)$ is a-priori available. Then, for any $1/k$ (made gradually increasing via suitable step increments) detect its associated segments where $1/k \geq c(z)$. In correspondence of each one of them, evaluate the following integral functions (by arbitrarily considering either the "+" or the "-" forms)

$$\left\{ \begin{array}{lcl} \Delta_{\bar{z}}(k,z) & = & \pm \displaystyle\int_{\bar{z}}^{z} f(k,z)dz \\ \rho_{\bar{z}}(k,z) & = & \pm \displaystyle\int_{\bar{z}}^{z} g(k,z)dz \end{array} \right. ; z \in (z_u, z_d) \tag{11}$$

where $\bar{z}$ is the abscissa of any absolute minima of within the associated segment (z_u, z_d) (Fig. 5).

Once both PTT and PTD tables have been filled with terms $\Delta_{\bar{z}}(k,z)$ and $\rho_{\bar{z}}(k,z)$, for any given triple z_a, z_b, k, the terms appearing in (9), (10) can be directly evaluated via a finite number of simple look-up operations:

$$\left\{ \begin{array}{lcl} \Delta_{a,b}(k) & = & |\Delta_{\bar{z}}(k,z_a) - \Delta_{\bar{z}}(k,z_b)| \\ \Delta_{b,d}(k) & = & |\Delta_{\bar{z}}(k,z_b) - \Delta_{\bar{z}}(k,z_d)| \\ \Delta_{a,u}(k) & = & |\Delta_{\bar{z}}(k,z_a) - \Delta_{\bar{z}}(k,z_u)| \end{array} \right. \tag{12}$$

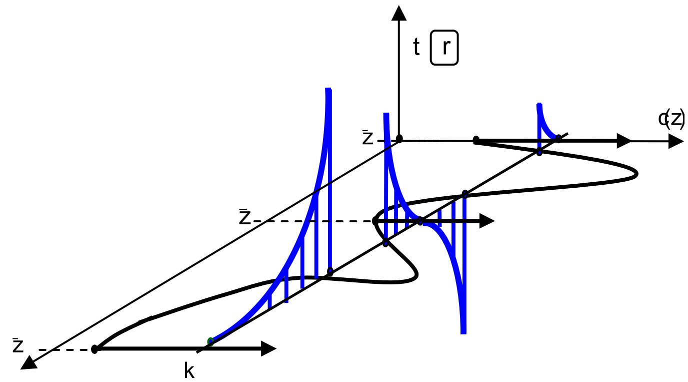

Fig. 5. Sound speed profile as a function of depth and corresponding integral as defined in (11) either in time or horizontal range.

$$T(k) = 2\,|\Delta_{\bar{z}}(k,z_u) - \Delta_{\bar{z}}(k,z_d)| \tag{13}$$

$$\left\{ \begin{array}{lcl} \rho_{a,b}(k) & = & |\rho_{\bar{z}}(k,z_a) - \rho_{\bar{z}}(k,z_b)| \\ \rho_{b,d}(k) & = & |\rho_{\bar{z}}(k,z_b) - \rho_{\bar{z}}(k,z_d)| \\ \rho_{a,u}(k) & = & |\rho_{\bar{z}}(k,z_a) - \rho_{\bar{z}}(k,z_u)| \end{array} \right. \tag{14}$$

$$R(k) = 2\,|\rho_{\bar{z}}(k,z_u) - \rho_{\bar{z}}(k,z_d)| \tag{15}$$

from which $\bar{T}_{ab}(k)$, $\bar{R}_{ab}(k)$, as well as also $T_{ab}(k)$, $R_{ab}(k)$, follow immediately.

The PTT and PTD look-up tables, whose computation has just been described, can now be considered as a unique basic functional block that, when receiving in input the triple z_a, z_b, k, it immediately provides as output the data vector $V_{ab}(k) \triangleq \left[T, \bar{T}_{ab} | R, \bar{R}_{ab}\right](k)$.

Such block will represent one of the fundamental components of the complete processing system for horizontal distance evaluations that will be hereafter described. Other necessary processing ingredients are now the so called *travel-time-checker (TTC)* and *travelled-distance-checker (TDC)* functional blocks, respectively accomplishing the following tasks: given a couple of depths z_a, z_b, a travel-time τ (a horizontal travel-distance r) and a conditioning parameter k; check if the two depths can be connected at time τ (with distance r) by a ray with parameter k; and in case provide the corresponding associated travelled distance $r_{ab}(k,\tau)$ (travel time $\tau_{ab}(k,r)$).

With reference to the above tasks, the following couple of very similar algorithms can be used:

1) TTC-Algorithm. Inputs: $V_{ab}(k), \tau$
   - $\bar{n} \triangleq \lfloor \tau / T(k) \rfloor$
   - $\bar{\tau} \triangleq \tau - \bar{n}T(k)$
   - if $\bar{\tau} \in \bar{T}_{ab}(k)$
     - set k as valid
     - $\bar{i} =$ index of $\bar{\tau}$
     - $r_{ab}(k,\tau) = r^{\bar{i}}_{ab}(k) + \bar{n}T(k)$
   - else
     - set k as invalid
2) TDC-Algorithm. Inputs: $V_{ab}(k), r$
   - $\bar{n} \triangleq \lfloor r / R(k) \rfloor$
   - $\bar{r} \triangleq r - \bar{n}R(k)$

- if $\bar{r} \in \bar{R}_{ab}(k)$
  - set k as valid
  - $\bar{i} =$ index of $\bar{r}$
  - $\tau_{ab}(k, \tau) = \tau_{ab}^{\bar{i}}(k) + \bar{n}R(k)$
- else
  - set k as invalid

IV. RT^2: Horizontal Distance Evaluation

By composing the processing ingredients introduced in the previous section, we are now ready to describe the complete implementation of the Real-Time Ray-Tracing (RT^2) algorithm for horizontal distance evaluation.

It is assumed that an underwater vehicle B is located at z_b depth, known since provided by its on-board depth-meter. Vehicle B receives a signal from an other vehicle A transmitting from depth level z_a. Such signal is further assumed encoding the time instant t_a of its transmission, plus its departing depth z_a. The vehicle clocks are assumed to be synchronized. It is finally assumed that vehicle B is able to discriminate the first arrival of the signal from vehicle A, and to separate it from the subsequent, lower intensity, replicas due to multi-path. Note that the assumptions just made are similar to those routinely used in quite a number of existing commercial or prototypal equipments. In addition to the previous assumptions, it is further required to the vehicles to have the pair of look-up-tables *PTT* and *PTD* (a-priori off-line evaluated from the assumed knowledge of the sound profile $c(z)$).

Under the above assumptions, vehicle B can evaluate (via trivial difference) the travel time τ spent by the sound ray that, as the *first one* received among many possible others, has reached B starting from A. From the computed travel time, the following processing mechanism (see Fig. 6) provides a minimal set of possible candidate horizontal distances from vehicle A.

First of all, the functional block *PPT-PTD*, containing the look-up tables, receives as input the couple of considered depths z_a, z_b. Then, at each stage i; $i = 1, \ldots, N$ of the algorithm, such a block receives the value $1/k_i$, which is used for accessing the look-up tables and computing the vector $V_{ab}(k_i)$, as described in the previous section. The resulting $V_{ab}(k_i)$ is then given as input to the unique TTC block, to a group of, say M, TDC blocks and to a shift buffer.

As indicated in the above related algorithm, the job of the TTC block is to check, at every stage, if the measured travel time τ can be obtained by the data contained in the input vector $V_{ab}(k_i)$. If such is the case, the resulting distance $r_{ab}(i) \triangleq r_{ab}(k_i, \tau)$ represents one of the candidate horizontal distances compatible with the measured travel time τ. Such a distance is hence given as input to the first available TDC block.

At the beginning of the algorithm every TDC block is inactive, with its output set to zero. When, at a certain stage, say the p-th, an inactive TDC, say the j-th, receives a candidate distance $r_{ab}(p)$, it becomes active ($b_j = 1$) and stores the value $r_{ab}(p)$. Then, at any successive i-th stage ($i = p+1, \ldots, N$),

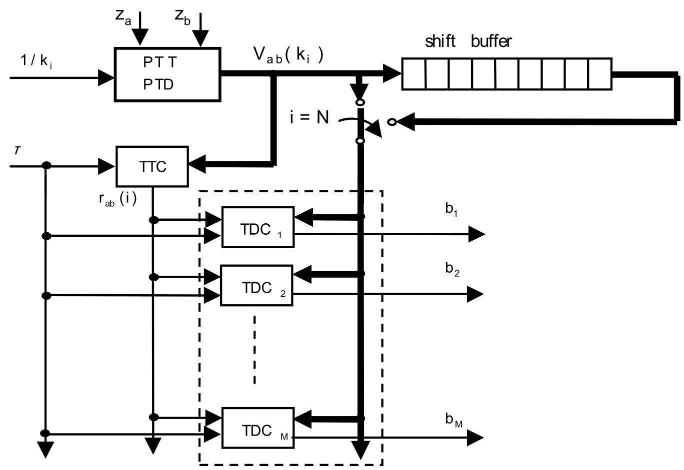

Fig. 6. Block diagram of the algorithm to evaluate the minimal set of horizontal distance candidates between the vehicles A, B.

it receives a new vector $V_{ab}(k_i)$ and checks if the associated horizontal distance $r_{ab}(p)$ could have been covered by a different ray, now parametrized by the new value $k_i \neq k_p$.

If such an occurrence is found at a later stage, say q, the considered TDC block then computes the corresponding "virtual" travel-time $\tau_{ab}(k_q, r_{ab}(p)$ and compares it with the "real" time τ, actually measured. Two cases can happen:

1) $\tau \leq \tau_{ab}(k_q, r_{ab}(p))$ which implies that the considered "virtual" ray would have arrived not before than the "real" one;
2) $\tau > \tau_{ab}(k_q, r_{ab}(p))$ which implies that the considered "virtual" ray would have been received before the "real" one, if the vehicle were at the considered horizontal-distance $r_{ab}(p)$.

In the first case, the block maintains its output as active and keeps on working in the same way for any successive i-th stage ($i = q+1, \ldots, N$). In the second case, instead, the TDC block becomes inactive ($b_j = 0$) and the associated candidate horizontal distance $r_{ab}(p)$ is discarded. Indeed, if the vehicle were at the horizontal distance $r_{ab}(p)$, it would have been reached by the ray parametrized by $k = k_q$ at an earlier time than τ; i.e. against the previous assumption that the first pulse is received at time τ.

The overall process is then continued till the N-th stage. At that point, the set of TDC blocks with a non-zero output bit contains the set of, say m, still-valid candidate distances among which looking for the *real* one.

In case $m = 1$ the only still-valid candidate distance is obviously coincident with the real one and the procedure is completed. Otherwise a simple further attempt to reduce the set of candidates may be performed. To this aim note that, in the procedure so far described, for any computed candidate distance $r_{ab}(p)$, the associated TDC block performed consistency checks only by considering the set of vectors $V_{ab}(k_i)$; $i = p+1, \ldots, N$. Since it is certainly convenient performing consistency checks w.r.t. *all* the possible vectors $V_{ab}(k_i)$, also the set of vectors obtained for $i = 1, \ldots, p-1$ should be considered for sake of completeness. To this aim a "shift buffer" is introduced in Fig. 6, for temporarily storing the sequence of vectors $V_{ab}(k_i)$, in order to (eventually) allow the execution of all the still pending checks, once stage N has been reached.

At the end of the overall process, the remaining output bits set to one indicate the resulting *minimal* set of horizontal distance candidates.

An example where the *minimal* set contains more than one solution, even if unlikely from the point of view of oceanographic conditions and source-receiver geometry, is the one where receivers at different ranges, but at the same depth, may receive as first arrival a signal from the same source at the same time.

In [3] a classification has been proposed for *isotemporal curves*, i.e., the set of all points in the depth-range plane that are reached by the acoustic rays coming from the same source at the same time and before any other acoustic ray has reached them. Examples reported in [3] shows that indeed there are cases in which the isotemporal curve has contribution from both direct arrivals and bottom/surface or totally reflected arrivals, corresponding to different ranges at the same depth.

In the following we give a possible self-contained algorithm to uniquely determine the *real* range based on the acoustic intensity of the arrival, and derived in the spirit of the computing blocks and look-up table developed so far. However, it has to be remarked that the non-uniqueness problem is not crucial in multiple vehicles localization: in fact, in order to completely localize one vehicle, range estimates from multiple vehicles are needed, in order to set up a triangulation or trilateration scheme. The scheme itself will rule out the possible range solutions not corresponding to the physical situations (it may be argued that there may still be cases in which the ambiguity is not resolved with three or even more vehicles; however such cases require a combination of particular geometric symmetries in the relative position of the vehicles together with specific environmental conditions that their practical occurrence at sea can be ruled out remorselessly). Moreover, even with two vehicles, a change in relative position (for instance in depth) will lead to the loss of the multiple solutions not corresponding to the true one; hence the presence of spurious solution is unstable and potentially resolvable within the trilateration or triangulation computation.

Notwithstanding the consideration of the previous paragraph, for the sake of completeness an additional computational block is now described to solve the range ambiguity whenever it may occur. The entire previous analysis has been based on the acoustic travel-time alone, without consideration of acoustic intensity loss. This is obviously not true, since for increasing travelled length, the acoustic intensity associated to any ray suffer from increasing loss (assuming spherical propagation, the loss is proportional to the square of the travelled distance). Surface/bottom reflections generate additional loss mechanisms. On this basis, similarly to what has been done for the sets of travel-times and horizontal-travel-distances $T_{ab}(k)$, $R_{ab}(k)$, we may introduce the data set $L_{ab}(k)$, comprising all the *travel-lengths* (ray-path lengths) connecting two given depths z_a, z_b for a given admissible k. This new data set also results one-to-one related with each set $T_{ab}(k)$, $R_{ab}(k)$; in particular,

$$L_{ab}(k) = \bar{L}_{ab}(k) + nL(k); \qquad (16)$$
$$\bar{L}_{ab}(k) \triangleq \left\{ l^j_{ab}(k); j = 1, \dots, 4 \right\}$$

with $L(k)$ now analogously representing the *length-period*, while $\bar{L}_{ab}(k)$ is the associated finite set of connections (similarly to (8)), only composed by four elements, for which we have (in analogy with (9) and (10)):

$$\bar{L}_{ab}(k) = \left\{ \begin{array}{lcl} l^1_{a,b} & = & \Lambda_{a,b} \\ l^2_{a,b} & = & \Lambda_{a,b} + 2\Lambda_{b,d} \\ l^3_{a,b} & = & \Lambda_{a,b} + 2\Lambda_{a,u} \\ l^4_{a,b} & = & \Lambda_{a,b} + 2\Lambda_{a,u} + 2\Lambda_{b,d} \end{array} \right\}_{(k)} \qquad (17)$$

The terms in (17) can be directly computed, jointly with the length-period $L(k)$, as:

$$\left\{ \begin{array}{lcl} \Lambda_{a,b}(k) & = & |\Lambda_{\bar{z}}(k, z_a) - \Lambda_{\bar{z}}(k, z_b)| \\ \Lambda_{b,d}(k) & = & |\Lambda_{\bar{z}}(k, z_b) - \Lambda_{\bar{z}}(k, z_d)| \\ \Lambda_{a,u}(k) & = & |\Lambda_{\bar{z}}(k, z_a) - \Lambda_{\bar{z}}(k, z_u)| \end{array} \right. \qquad (18)$$

$$L(k) = 2\,|\Lambda_{\bar{z}}(k, z_u) - \Lambda_{\bar{z}}(k, z_d)| \qquad (19)$$

The various terms in (18) and (19) can be stored in an additional look-up table, strictly similar to PTT-PTD and termed *Partial-Travel-Lengths (PTL)*, which is built on the basis of the length abscissa integral function

$$\Lambda_{\bar{z}}(k, z) = \pm \int_{\bar{z}}^{z} \sqrt{1 + g^2(k, z)}dz \qquad (20)$$

At this point, by adding the PTL look-up-table to the algorithm in Fig. 6, thus making the new PTL/PTT/PTD block outputting the augmented vector $\left[V_{ab}|L, \bar{L}_{ab}\right](k_i)$, we end-up with the m horizontal distance candidates joined with their associated total *travel-lengths*. Knowing the source intensity, and assuming to measure also the received intensity, simple loss models can now provide the unique solution to the range localization problem.

V. Application to Distributed Localization

The framework underlying the problem of AUVs group localization is represented by a set of more than one AUV, each one equipped with its on-board acoustic modem. All this as appears depicted in Fig. 7, where the presence at surface level of one or more ASVs (or elements of an LBL system), each one endowed with GPS absolute localization capabilities, has been also assumed. Let us assume that any generic j-th vehicle of the group (via modem and while moving) intermittently broadcast a pulse immediately followed by a data-stream containing (for a total of *eight* numbers) the information

1) Its identification code j;
2) The pulse-emission instant $\bar{t}^j_i; i = 1, 2, \dots$
3) Its depth $\bar{z}^j_i$ at the pulse-emission instant $\bar{t}^j_i$

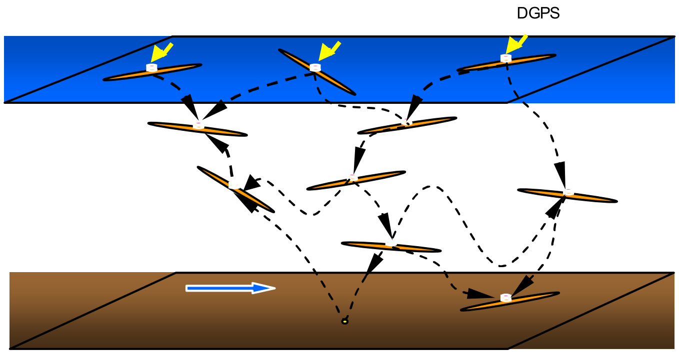

Fig. 7. Conceptual configuration of the team localization scheme, in which localization is obtained with respect to neighbouring vehicles and ultimately to the GPS-linked surface vehicles.

4) The *prediction* $\hat{\bar{\chi}}^j_{i/i-1}$ location $\bar{\chi}^j_i$ at instant $\bar{t}^j_i$
5) The variance $\bar{p}^j_{i/i-1}$ of $\hat{\bar{\chi}}^j_{i/i-1}$

It is meanwhile required that any other generic k-th vehicle differing from the surface ones must also be able to translate its previous *prediction* $\hat{\chi}^k_{i/i-1}$ of its horizontal location χ^k_i (relevant to the pulse-reception instant $t^j_i > \bar{t}^j_i$ and generally obtained via time-integration of its own odometric data) into the upgraded *estimate* $\hat{\chi}^k_{i/i}$ of the same location at same time.

Note that we are actually requiring the production, by part of a vehicle, of prediction and estimates relevant to its *horizontal positions* only; this because the shared knowledge of depths straightforwardly projects the problem into the horizontal plane. Moreover, the possibility offered by the RT^2 technique of the previous section of directly measuring the horizontal distances between AUVs even in presence of non-negligible ray distortions and reflections, just reduces the problem to be of the same type of those generally considered within (planar) in-air and range-based distributed localization applications. As a consequence, most part of the already existing in-air techniques for such problem is expected to be adoptable also for the underwater environment, without any sensible further modification. More precisely, it is deemed here that among the most appreciable advantages that could emerge from the adoption of the here proposed RT^2 method, the following should actually reveal a particular significance.

In terrestrial applications, the Gaussianity of the unavoidable noise superimposed to any distance evaluation is recognized to be a very reasonable assumption; and this is actually a basic support to the use of parametric filtering methods, (like Kalman Filters, Extended Kalman Filters, or even Iterated Kalman Filters; [7], [8], [9]) that generally revealed as effective, and meanwhile much more easy to implement than non-parametric method (like for instance Particle Filtering, [7], [8], [9]) Then, accordingly with the RT^2 possibility of direct measuring of underwater distances, most probably the above comment could be extended also to the underwater environment. This in the sense that assuming the horizontal distance measurements superimposed by Gaussian noise may now reveal less critical than when roughly used without keeping into account sound-wave distorting and reflecting effects. As a matter of fact, the need of avoiding the criticality posed by Gaussian assumptions, when in presence of ray propagation anisotropies and reflections, might be one of the basic motivation underlying the recent proposal of use of much more sophisticated (but also more difficult to implement) non-parametric filtering techniques [11] for attempting to manage underwater anisotropies. However note how the here proposed RT^2 method does not at all prevent the use of non-parametric filters, in case deemed, for some reasons, necessary (e.g., outlier removal).

Within underwater distributed group localization, one approximate practical possibility for avoiding to fully face the problem of sound ray distortions and reflections (even those induced by possible acoustic channels) could however be that of trying to act in such a way to keep (horizontally and vertically) each vehicle sufficiently close to its neighbours. Instead, within the use of the proposed techniques, such type of structuring of the AUVs group should result not anymore necessary, since the formulation of the proposed technique does not actually make any difference between approximately straight, distorted, or even reflected rays along any path length (however provided it allows an adequate intensity at the receiver). The net consequence of this should be therefore to allow even small groups of AUVs effectively working with more sparse (and then more volumetric-extended) configurations.

VI. Conclusions

By assuming the depth dependent sound velocity profile a-priori available and the AUVs equipped with acoustic modems, the paper has shown how the sound-ray propagation laws can be managed in such a way to allow the preliminary construction of a pair of look-up tables. On the basis of the look-up tables, that can be updated as the environmental conditions change, a simple algorithm (the Real Time Ray Tracing - RT^2) can in turn be defined, for accurately evaluating the horizontal-distance separating two vehicles, even in the presence of non-negligible sound ray distortions and reflections.

Via this RT^2 method, the AUVs distributed localization problem becomes the same of its terrestrial analogous one, for which consolidated, easy to implement and generally effective, parametric filtering methods already exist that consequently could be transferred, with equal effectiveness, to the more complex underwater field. Nevertheless, the developed RT^2 method obviously does not prevent the use of nonparametric, nonlinear estimation methods.

In order to validate the proposed RT^2 technique, an accurate simulative campaign has been recently started with the design of a virtual environment where any given sound-depth profile can be easily introduced. In this way, in few months, it will be possible obtaining useful indications on the behaviour of the RT^2 technique under a number of different operative conditions.

After the simulative phase, the implementation of the algorithm on-board a team of Folaga vehicles is foreseen for the second half of 2010, together with the necessary successive set of sea trials.

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