Comparing of LPC-EKF, LPC-UKF in UUV Bearings-Only Tracking Systems

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***Abstract*- Unmanned underwater vehicles have various missions within civilian, military and academic sectors. They have the ability to explore areas unavailable to manned assets and to perform duties that are risky to humans. In particular, UUVs have the ability to perform bearings-only tracking in shallow areas near shorelines. This article compares a Logarithmic Polar Coordinates EKF (LPC-EKF) and Logarithmic Polar UKF (LPC-UKF) for the estimation problem. Its objective is to track the kinematics (position, velocity and course) of a moving target using noise-corrupted bearing measurements.**

I. Introduction

The Unscented Transform has been used in the KF framework and the resulting filter is referred to as the Unscented Kalman Filter (UKF) [1]. It uses the intuition that it is easier to approximate a probability distribution function than to approximate an arbitrary nonlinear function or transformation. Thus the state distributions are approximated by the Gaussian density, which is represented by a set of deterministically chosen sample points [2]. The idea of nonlinear filtering using the Gaussian representation of the posterior density via a set of deterministically chosen sample points has been proposed by several authors. The whole class of nonlinear filters (including UKF) is referred to as the linear regression Kalman filter, because they are all based on statistical linearization rather than analytical linearization (as the EKF). The statistical linearization is performed via the linear regression through the regression (sample) points [3]. These sample points (Sigma points) completely capture the true mean and covariance of the Gaussian density. When propagated through nonlinear systems, they capture the true mean and covariance accurately to the second order (Taylor series expansion) of any nonlinearity [4].

Bearings-only tracking is inherently a nonlinear problem. The nonlinearity can either be in the state equations or in the measurement equations. If a traditional Cartesian Kalman filter is used, the state dynamics are linear while the measurement relationships are nonlinear. The traditional Cartesian state vector consists of the relative position and relative velocity of the target in both the x and the y direction.

Transforming the problem into Logarithmic polar coordinates causes the state equations to be nonlinear but the[1] measurement equations to be linear. The state vector then contains the bearing and the Logarithmic reciprocal of range of the target from the observer's perspective. The measurement vector has been simplified since it is solely concerned with the bearing. Another benefit of using Logarithmic polar coordinates is the state vector is partially decoupled. The only component that is not observable prior to an observer maneuver is the Logarithmic reciprocal of range. This allows the state estimates to behave as one would theoretically expect, even in the face of errors, unlike the coordinates.

In this article, the LPC-EKF and LPC-UKF will be compared in the UUV Bearings-Only Tracking systems. The observation platform needs to manoeuvre in order to estimate the target range. This need for own UUV manoeuvre and its impact on target state observability have been explored extensively. Three zigzag manoeuvre tracking scenarios will be used in this Simulation.

II. Model In Cartesian And Lpc System

The problem of BOT is the following: the own-ship (O) moves in the plane and periodically measures the bearing of a target (T) that we assume traveling with constant velocity [**5**] (see Fig. 1). Target state, consisting of position and velocity in the (x, y) plane, is estimated by processing the bearing and the knowledge of the own-ship location and velocity [**6**]. It is well known that the own-ship has to maneuver properly in order to render the problem observable [**7**].

A. State vector in Cartesian Coordinates:

Historically, BOT is presented in the Cartesian system. Let us define target state at time k:

Let $\mathrm{X_T(k)}$ (respectively$\mathrm{X_O(k)}$) be the target (respectively own-ship) state vector in absolute Cartesian Coordinates at time k:

$$\mathrm{X_T(k) = [x_T(k) \quad y_T(k) \quad \dot{x}_T(k) \quad \dot{y}_T(k)]^T} \tag{1}$$

$$\mathrm{X_O(k) = [x_O(k) \quad y_O(k) \quad \dot{x}_O(k) \quad \dot{y}_O(k)]^T} \tag{2}$$

It is of common use to describe the state equation in the relative own-ship referential, though it is not compulsory with Cartesian Coordinates, and we define the state vector to be:

$$\mathrm{X(k) = X_T(k) - X_O(k)} \tag{3}$$

$$\mathrm{X(k) = [x(k) \quad y(k) \quad \dot{x}(k) \quad \dot{y}(k)]^T} \tag{4}$$

The state equation corresponding to the hypothesis of uniform motion for the target is then:

$$\mathrm{X(k+1) = AX(k) + U(k+1,k) + \Gamma w(k+1)} \tag{5}$$

Where A describes the classical constant velocity model; the own-ship maneuvers between time k and time k+1 are taken into account by the deterministic command term $\mathrm{U(k+1,k)}$:

$$\mathrm{U(k+1,k)} = \begin{bmatrix} \mathrm{x_o(k+1) - x_o(k) - T\dot{x}_o(k)} \\ \mathrm{\dot{x}_o(k+1) - \dot{x}_o(k)} \\ \mathrm{y_o(k+1) - y_o(k) - T\dot{y}_o(k)} \\ \mathrm{\dot{y}_o(k+1) - \dot{y}_o(k)} \end{bmatrix} \tag{6}$$

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$$A = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \Gamma = \begin{bmatrix} \frac{T^2}{2} & 0 \\ 0 & \frac{T^2}{2} \\ T & 0 \\ 0 & T \end{bmatrix}$$

The stochastic system measurement equation is:

$$Z(k) = \tan^{-1}\left(\frac{x(k)}{y(k)}\right) + v(k) \qquad (7)$$

Where $v(k) \sim \mathcal{N}(0, \sigma_\beta^2)$ and $w(k) \sim \mathcal{N}(0, Q)$

B. State vector in LPC:

Nardone and Aidalahave demonstrated that no information on range exists as long as the observer is not maneuvering. So the idea consists of using a coordinates system for which unobservable component (range) is not coupled with the observable part. This is also the motivation of Aidala and Hammel for defining the MPC system:

$$Y_t = \begin{bmatrix} \beta_t & r_t & \dot{\beta}_t & \frac{\dot{r}_t}{r_t} \end{bmatrix}^T \qquad (8)$$

Thus, the target states at time t is defined by eq. (8), where $r_t$and β_t are the relative bearing and target range. Thomas Br´ehard and Jean-Pierre Le Cadre propose in the following section a slight modification of the MPC system, named Logarithmic Polar Coordinates (LPC) system. The only difference is that the second component is not $r_t$but $\ln r_t$. Even if this tiny difference appears very minor, it will be shown that it is instrumental for deriving a closed-form of the PCRB. The BOT equation system state Y_t is expressed in the logarithmic Polar Coordinates (LPC) system:

$$Y_t = \begin{bmatrix} \beta_t & \ln r_t & \dot{\beta}_t & \frac{\dot{r}_t}{r_t} \end{bmatrix}^T \qquad (9)$$

We just have $f_c^{lp}(X_t)$and $f_{lp}^c(X_t)$ respectively LPC-to-Cartesian and Cartesian -to- LPC state mapping functions [**8**] such that:

$$Y_t = f_c^{lp}(X_t) = \begin{bmatrix} \arctan\left(\frac{r_x(t)}{r_y(t)}\right) \\ \ln\left(\sqrt{r_x^2(t) + r_y^2(t)}\right) \\ \frac{v_x(t)r_y(t) - v_y(t)r_x(t)}{r_x^2(t) + r_y^2(t)} \\ \frac{v_x(t)r_x(t) + v_y(t)r_y(t)}{r_x^2(t) + r_y^2(t)} \end{bmatrix} \qquad (10)$$

And

$$X_t = \begin{cases} f_{lp}^c(Y_t) \text{ if } r_y(t) > 0 \\ -f_{lp}^c(Y_t) \text{ if } r_y(t) < 0 \end{cases}$$

$$\text{With } f_{lp}^c(Y_t) = \begin{bmatrix} \sin\beta_t \\ \cos\beta_t \\ \dot{\beta}_t \cos\beta_t + \frac{\dot{r}_t}{r_t}\sin\beta_t \\ \dot{\beta}_t \sin\beta_t + \frac{\dot{r}_t}{r_t}\cos\beta_t \end{bmatrix} \qquad (11)$$

Thus, the stochastic system is:

$$f(Y_t, k+1, k) = \begin{cases} f_c^{lp}\left(Af_{lp}^c(Y_t) + U(k+1,k) + w(k+1)\right) \text{ if } r_y(t) > 0 \\ -f_c^{lp}\left(Af_{lp}^c(Y_t) + U(k+1,k) + w(k+1)\right) \text{ if } r_y(t) > 0 \end{cases}$$

$$\begin{cases} Y_{t+1} = f(Y_t, k+1, k) \\ Z_t = \beta_t + V_t \end{cases} \qquad (12)$$

Where $v(k) \sim \mathcal{N}(0, \sigma_\beta^2)$ and $w(k) \sim \mathcal{N}(0, Q)$

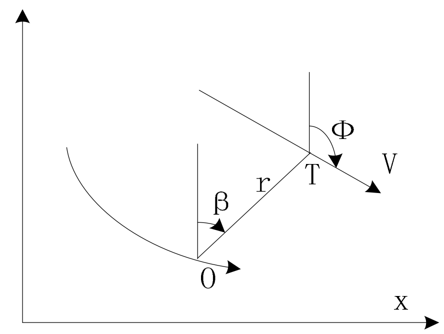

Figure 1. Typical two-dimensional UUV bearings-only tracking geometry

III. FILTERS DESCRIPTION

One of the first algorithms employed to solve the bearings-only tracking problem was the EKF in Cartesian coordinates. Simulation studies have shown that EKF in Cartesian coordinates exhibits unstable behavior. To overcome these difficulties, a new EKF was proposed, which is formulated in modified polar coordinates. This coordinates system was shown to be well suited for angle-only target motion analysis because it automatically decouples the observable and un-observable components of the estimated state vector. Such decoupling prevents covariance matrix ill-conditioning which is the primary cause of filter instability.

Observe that in the LPC, the state dynamics equation is nonlinear while the measurement equation is linear, whereas is true in Cartesian coordinates [**9**]. Straightforward application of the EKF and UKF to eq. (12) will now yield the LPC-EKF and LPC-UKF described below:

A. EKF in logarithmic Polar Coordinates

$\hat{Y}_{0|0}$= initial estimate of LPC state vector

$P_{0|0}$= initial estimate of LPC state vector error covariance matrix

$$\hat{Y}_{k+1|k} = f(\hat{Y}_{k|k}, k+1, k)$$

$$F_{k+1,k} = \left.\frac{\partial f(Y_k, k+1, k)}{\partial Y}\right|_{Y=\hat{Y}_{k|k}}$$

$$P_{k+1|k} = F_{k+1,k} P_{k|k} F'_{k+1,k}$$

$$H = [1 \quad 0 \quad 0 \quad 0]$$

$$K_{k+1} = P_{k+1} H' \left[H P_{k+1|k} H' + \sigma_\beta^2\right]^{-1}$$

$$Y_{k+1|k+1} = \hat{Y}_{k+1|k} + K_{k+1}\left[\beta_{k+1} - H\hat{Y}_{k+1|k}\right]$$

$$P_{k+1|k+1} = [I - K_{k+1}H] P_{k+1|k}$$

B. *UKF in logarithmic Polar Coordinates*

$\hat{Y}_{0|0}$= initial estimate of LPC state vector

$P_{0|0}$= initial estimate of LPC state vector error covariance matrix

Calculate sigma points:

$$\chi_{k-1} = \left[\hat{Y}_{k-1} \ \ \hat{Y}_{k-1} + \gamma\sqrt{P_{k-1}} \ \ \hat{Y}_{k-1} - \gamma\sqrt{P_{k-1}}\right]$$

Time update:

$$\chi^*_{k|k-1} = f(\chi_{k-1}, k, k-1)$$

$$\hat{Y}_{k|k-1} = \sum_0^{2N} \mathcal{W}_i^m \chi^*_{i,k|k-1}$$

$$P_{k|k-1} = \sum_0^{2N} \mathcal{W}_i^c \left[\chi^*_{i,k|k-1} - \hat{Y}_{k|k-1}\right]\left[\chi^*_{i,k|k-1} - \hat{Y}_{k|k-1}\right]'$$

$$\chi_{k|k-1} = \left[\hat{Y}_{k-1} \ \ \hat{Y}_{k|k-1} + \gamma\sqrt{P_{k|k-1}} \ \ \hat{Y}_{k|k-1} - \gamma\sqrt{P_{k|k-1}}\right]$$

$$H = [1 \quad 0 \quad 0 \quad 0]$$

$$Y_{k|k-1} = H\chi_{k|k-1}$$

$$Z_{k|k-1} = \sum_0^{2N} \mathcal{W}_i^m \chi_{k|k-1}$$

Measurement updates equations:

$$P_{Z_k Z_k} = \sum_0^{2N} \mathcal{W}_i^c \left[Y_{i,k|k-1} - Z_{k|k-1}\right]\left[Y_{i,k|k-1} - Z_{k|k-1}\right]'$$

$$P_{Y_k Z_k} = \sum_0^{2N} \mathcal{W}_i^c \left[\chi_{i,k|k-1} - Y_{k|k-1}\right]\left[Y_{i,k|k-1} - Z_{k|k-1}\right]'$$

$$K_k = P_{Y_k Z_k} P_{Z_k Z_k}$$

$$Y_{k|k} = \hat{Y}_{k|k-1} + K_k\left[Z_k - Z_{k|k-1}\right]$$

$$P_k = P_{k|k-1} - K_{kP_{Z_k Z_k}} K_k'$$

IV. Simulation Scenarios

A. Simulation Scenario 1and Scenario2:

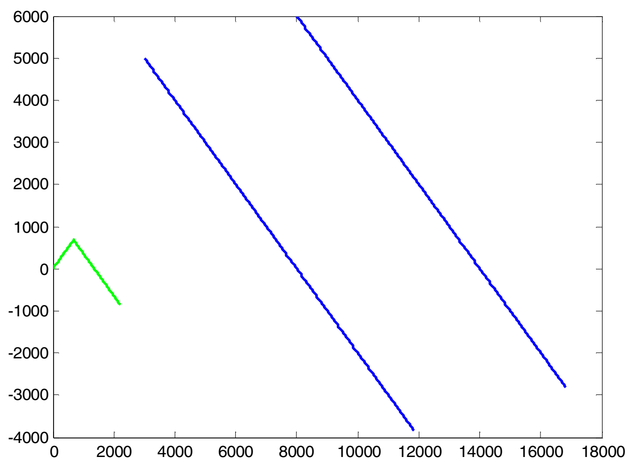

Figure 2 – simulation Scenario 1and Scenario 2

The first two scenarios composed of two legs for the UUV and one straight line for the target. The UUV starts at the origin and travels with a constant course 4m/s with initial heading 45°. After 4 min, it turns to a 315° heading. The maneuver lasts 9 min, and the scenario 13 min. The target moves at a constant speed of 16m/s with heading 315°. The target starts at the coordinates (8000, 6000) in Scenarios 1and (4800, 6400) Scenarios 2.

B. Simulation Scenario 2:

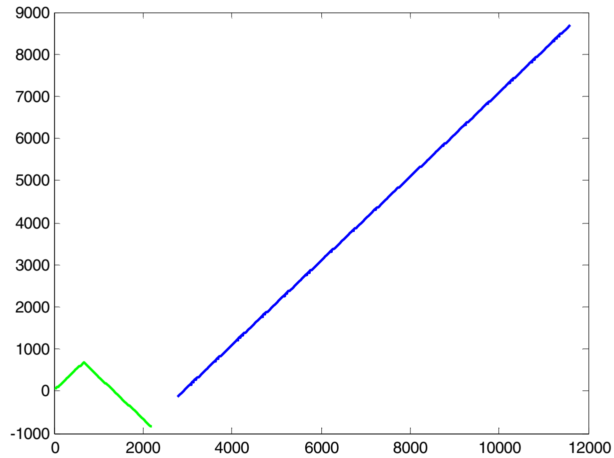

Figure 3 – simulation Scenario 3

The second class of scenarios is identical for the UUV but with a different heading and starting point for the target: the initial heading is set to 225° with an initial coordinates (11600, 8700). The scenarios last 13min. In the three Scenarios, the velocity of UUV is 4m/s and the true velocity of target is 16m/s.

V. Simulation Results And Discussion

A. Filter Initialization In LPC :

The sampling rate is T = 1 s, and the measurements are simulated with the exact geometry corrupted by an additive white zero mean Gaussian noise with$\sigma_\beta = 1^\circ$.

We use an a priori distance $r(0|0)$ in all our simulations with two strategies: the first $r(0|0)$ is exact value with $\sigma_r = 1\text{km}$ and the second have deviation from the exact value with $\sigma_r = 1\text{km}$ Then we use the first measurement$\beta(0)$ with its standard deviation σ_β to initialize theβ.

In UKF, for the unscented transformation parameters, we use $\alpha = 0.1$ and $\beta = 2$ k=0. The weights are then determined as in reference.

In order to obtain a reliable comparison of both algorithms, we use two ways to initial the state vector and its covariance matrix in LPC. The first way, we use the true value for the position, velocity and course initialization, the second initialization we use true value with a certain degree of deviation.

B. Simulation Results In LPC :

The following figures and tables show the results obtained on Monte- Carlo simulation of 1000 runs. On Fig. 4, 5 and 6, are plotted the RMS errors vs. time for course, position, velocity with the second initialization. In the table □, □ and □, we list the four locations of RMS errors with the first initialization.

In the following figure, the red denotes the UKF algorithm; the blue denotes the EKF algorithm and the cyan denotes the target true position.

From Fig. 4, 5 and 6, we can see that the LPC-UKF performs better than the LPC-EKF, LPC-UKF are more stable than LPC-EKF and LPC-EKF are more vulnerable to the impact of target unobservability. As the target of a priori inaccurate, the trajectory estimation of the LPC-EKF Deviate from the true target trajectory.

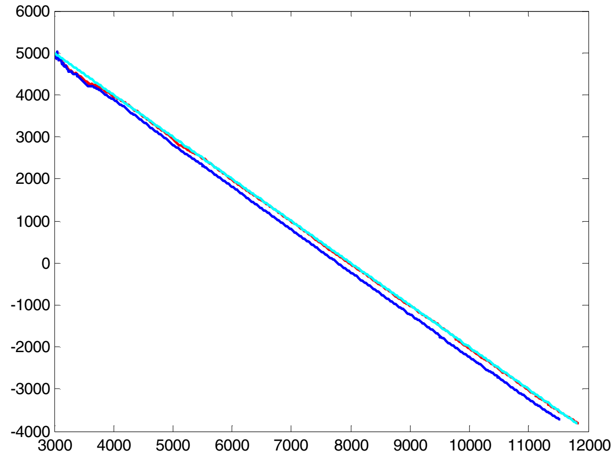

Figure 4–a simulation Scenarios 2 trajectory

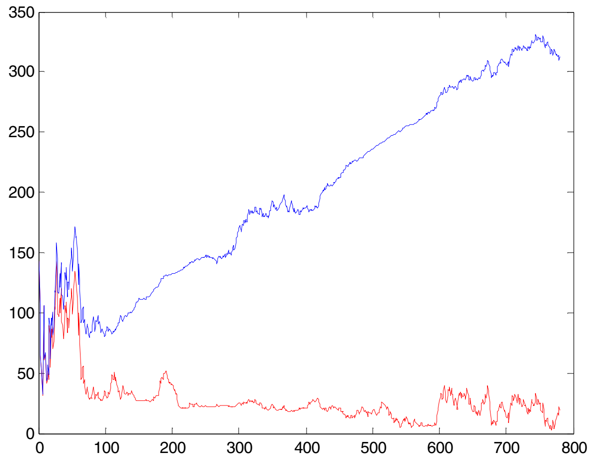

Figure 4 –b simulation Scenarios 2 RMS Position errors

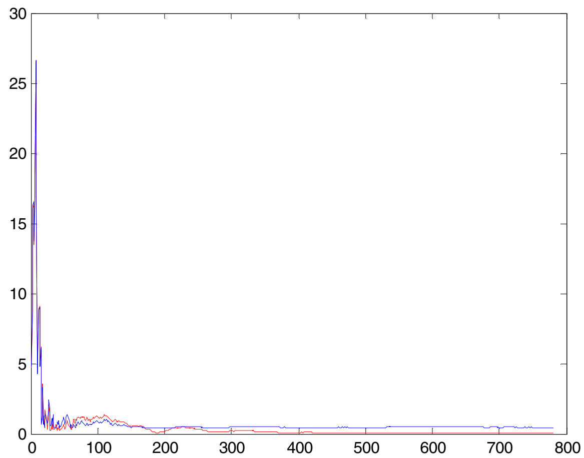

Figure 4–c simulation Scenarios 2 RMS velocity errors

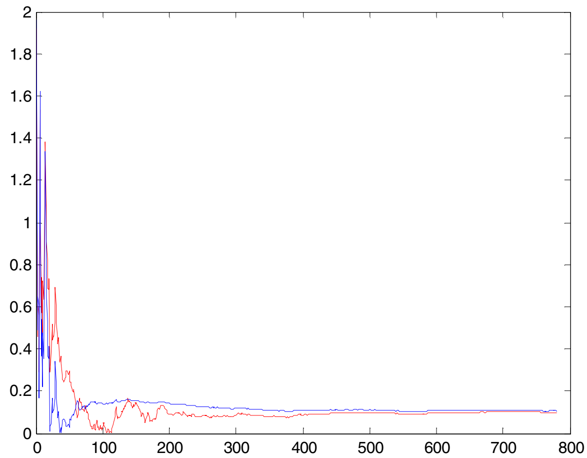

Figure 4–d simulation Scenarios 2 RMS course errors

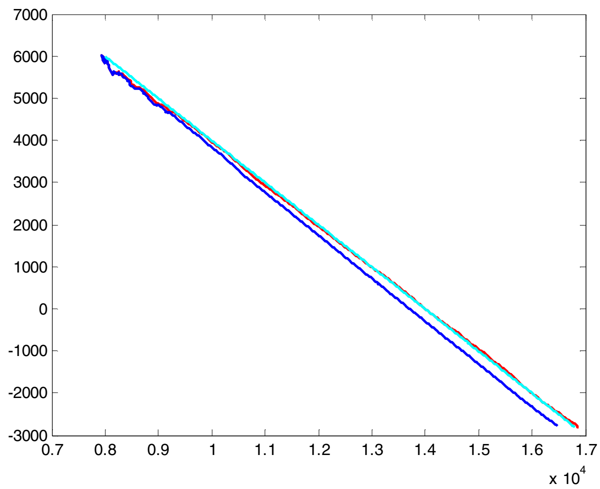

Figure 5–a simulation Scenarios 1 trajectory

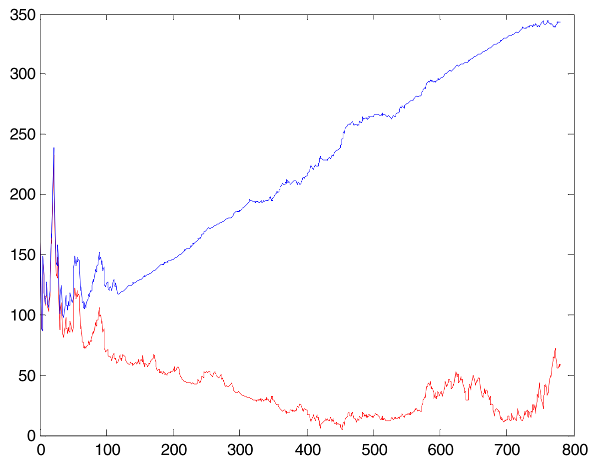

Figure 5 –b simulation Scenarios 1 RMS Position errors

From Fig. 4-b, 5-b and 6-b, we can see that in the position estimation, LPC-EKF generates a great deal of error, while the estimation results of LPC-UKF are relatively accurate. The position estimation of LPC-EKF has Started Divergence before the UUV maneuver.

In the velocity and course estimation, before the UUV maneuver, the LPC-EKF performs equivalently, when the UUV start maneuver, the LPC-UKF is a slightly better performance than LPC-EKF.

From the Fig. 4 and Fig. 5, we can see that the LPC-UKF and LPC-EKF are all performs better in the short distance. From the Fig. 6 with the target approach to the UUV, the performance of the two filters estimation is getting better.

In the three following tables, the two filters are initialized with true value. From the three tables, we can see that the performances of the two filters are better than initialization with Non-true value. The LPC-UKF is slightly better performance than The LPC-EKF.

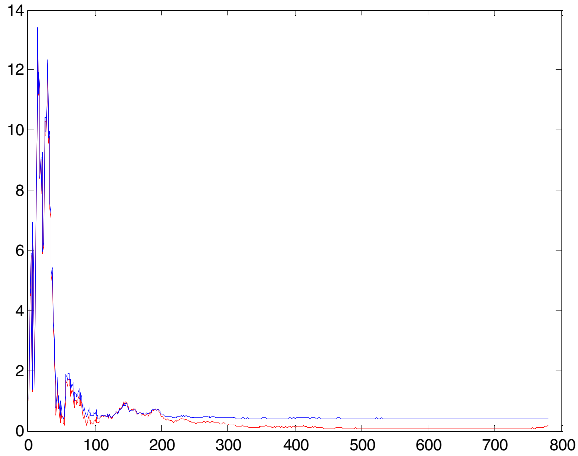

Figure 5–c simulation Scenarios 1 RMS velocity errors

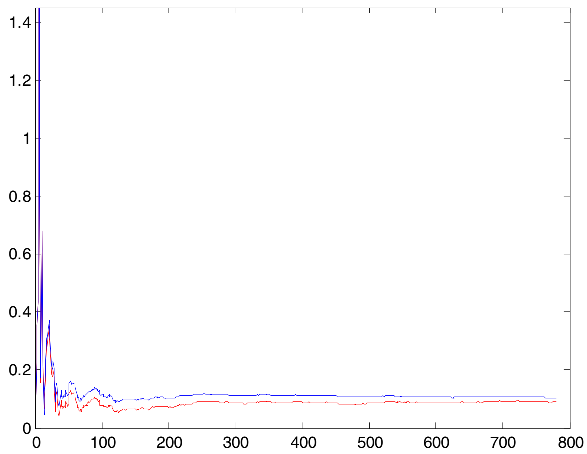

Figure 5–d simulation Scenarios 1 RMS course errors

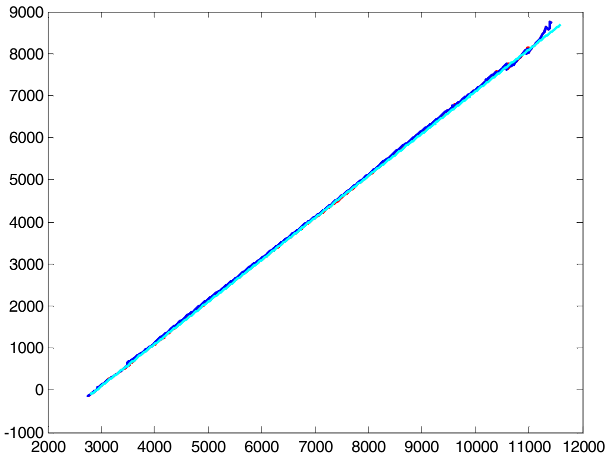

Figure 6–a simulation Scenarios 3 trajectory

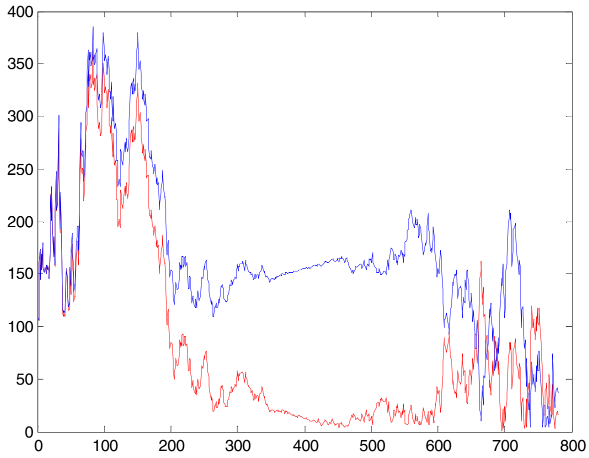

Figure 6 –b simulation Scenarios 3 RMS Position errors

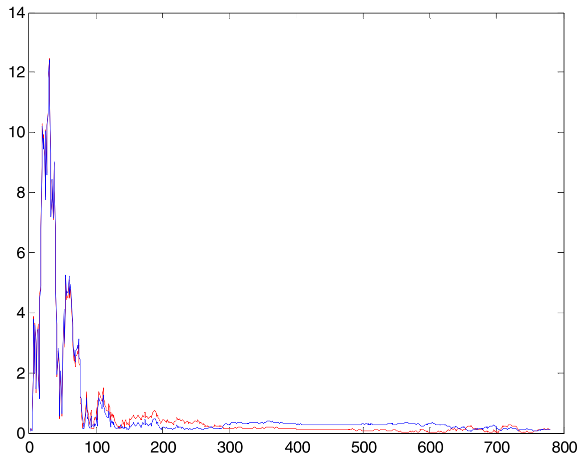

Figure 6–c simulation Scenarios 3 RMS velocity errors

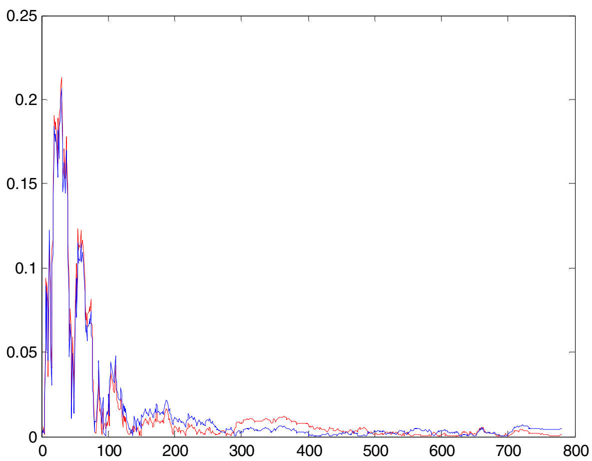

Figure 6–d simulation Scenarios 3 RMS course errors

TABLE I

SCENARIOS 1 RMS ERRORS WITH TRUE INITIAL VALUE

Coordinates	Filters	Position(m)	Velocity(m)	Course(m)
(9131,4869)	LPC-UKF	25.16	0.72	0.00645
	LPC-EKF	27	0.708	0.01199
(11394,2606)	LPC-UKF	23.83	0.131	0.00253
	LPC-EKF	45	0.081	0.00282
(13657,343)	LPC-UKF	97.98	0.142	0.00626
	LPC-EKF	167	0.198	0.00153
(16825, - 2825)	LPC-UKF	59.39	0.065	0.00366
	LPC-EKF	185	0.160	0.00008

TABLE II

SCENARIOS 2 RMS ERRORS WITH TRUE INITIAL VALUE

Coordinates	Filters	Position(m)	Velocity(m)	Course(m)
(4131,3869)	LPC-UKF	45.62	0.485	0.02503
	LPC-EKF	51.78	0.535	0.02848
(6394,1606)	LPC-UKF	146.36	0.522	0.02938
	LPC-EKF	153.09	0.557	0.03103
(8657,-657)	LPC-UKF	126.50	0.239	0.00778
	LPC-EKF	136.18	0.265	0.00876
(11824, -3824)	LPC-UKF	29.49	0.055	0.00075
	LPC-EKF	36.86	0.067	0.00037

TABLE III

SCENARIOS 3 RMS ERRORS WITH TRUE INITIAL VALUE

Coordinates	Filters	Position(m)	Velocity(m)	Course(m)
(10469, 7569)	LPC-UKF	184.3	1.981	0.03366
	LPC-EKF	184.29	2.023	0.03063
(8206, 5306)	LPC-UKF	12.82	0.057	0.00353
	LPC-EKF	12.82	0.136	0.00108
(5943, 3043)	LPC-UKF	50.99	0.094	0.00248
	LPC-EKF	50.99	0.053	0.00071
(2775, -125)	LPC-UKF	39.71	0.008	0.00047
	LPC-EKF	45.99	0.058	0.00175

VI. CONCLUSION

The paper compared the LPC-UKF and LPC-EKF in UUV BO-TMA. We hence can conclude that the LPC-UKF is better than LPC-EKF for UUV BOT-TMA solution. The LPC-EKF Are more susceptible to the initialization and the observability.

We have observed that the position estimation of the LPC-EKF sometimes diverge, probably due to a poor initialization as previously mentioned.

From the simulation results, we can see that the initialization is important for UUV BO-TMA problem. But in the really UUV BO-TMA problem, there is often no target true value to initialize the filter, so in the following studies, we will research in a relatively accurate method of initialization for UUV BO-TMA problem.

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