Path Planning of Autonomous Underwater Vehicles for Optimal Environmental Sampling

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***Abstract*- Dynamic variability in vast ocean environments and limitation in sampling resources have made optimal environmental sampling a hot topic of discussion recently. In this paper we consider the problem of path planning for Autonomous Underwater Vehicles (AUVs) carrying conductivity, temperature and depth (CTD) sensors, aiming to minimizing sound velocity profile prediction uncertainty after assimilating in-situ measurements. The problem is modeled as a non-linear deterministic optimization problem based on maximum a posterior probability (MAP) criterion. An approximated way to calculate the uncertainty reduction for each path group is utilized to save the computation time. Numeric simulation results have confirmed the effectiveness of the approach.**

I. INTRODUCTION

Currents, internal waves, eddies and wind driven flows are dominant oceanographic processes existed in shallow water. Dynamic variabilities are commonly observed in coastal regions due to those processes and their coupling. Those variabilities inevitably complicate the ocean environment; as a result, the temperature, salinity and density distribution in water may vary in a complex dynamic way in both time and space. To make it worse, those variabilities spanning a wide range of spatial and temporal scales can hardly be sufficiently measured especially at small and fast-varying scales. Consequently estimation of the ocean-acoustic environment parameters such as the Sound Velocity Profile (SVP) of the water column and the seabed properties via ocean acoustic tomography [1] is usually accompanied by great uncertainties. Those uncertainties often lead to acoustic prediction uncertainties and serious performance degradation of model-based sonar processing, for example Matched Field Processing (MFP). Quality of acoustic communication is also frequently affected, which certainly limits the application of underwater wireless sensor networks.

To reduce those estimation uncertainties, in-situ measurement data can be collected and assimilated with ocean process modeling. Unmanned Underwater Vehicles (UUVs) such as Autonomous Underwater Vehicles (AUVs) and Remotely Operated Vehicles (ROVs) carrying conductivity, temperature and depth (CTD) sensors are ideal platforms for such purpose of oceanographic sampling. They provide a flexible and wide-coverage solution for adaptively gathering oceanographic data. However, they are limited in operation time and units available due to energy constrains and budget considerations. Indeed the sampling resources are always limited in practice to some degree given the vastness of the environment. The problem comes as given the ocean-acoustic environment uncertainties and the total volume of the environment, how the limited resource should be deployed for optimal environmental sampling. This is indeed becoming a hot topic in the related research community lately.

One of the most recent relevant research work is conducted by Wang [2]. In this work, the optimal path planning for a single AUV is modeled as a shortest path routing problem where the AUV moves in a vertical plane. Multi-AUV operation has been considered in Yilmaz's work [3], where path planning for one or more AUVs is realized in a horizontal plane such as the sea surface. Two optimization methods based on the Mixed Integer Programming (MIP) are developed. However, no SVP correlation effect across path is considered, which certainly does not reflect the reality. Besides, there have been some other efforts of finding sub-optimal paths with certain pre-defined shapes [4]. It is also worth to mention that many sensor selection strategies introduced in the context of wireless sensor networks can be linked to the problem of UUV path planning [5][6].

In this paper, we focus on path planning for a group of AUVs moving in the vertical plane. The objective function used is the summation of the posterior acoustic estimation uncertainty of the entire two-dimensional (2D) SVP field, in the presence of SVP correlation effect. During the implementation, the posterior prediction uncertainty is updated from the prior estimation uncertainty through one step Kalman filtering, and the very path with the most uncertainty reduction is chosen as the optimal one. We also develop an approximate way to evaluate the uncertainty reduction, which together with the exhaustive searching approach is tested through numeric simulations.

II. PROBLEM AND METHODOLOGIES

In this section we formulate AUV path planning as an optimization problem. To simply the presentation, we mainly discuss path planning for a single AUV; extension of the results to multiple AUVs is then straightforward.

A. *Objective function*

Consider 2D sound velocity variation in water column with a prior SVP vector estimation $\mathbf{c}(-)$; each position p_i in this

2D plane is represented by its depth z_i and range r_i. As mentioned above, the SVP estimation is inevitably accompanied by uncertainty caused by environmental variabilities, which can be described using an uncertainty map representing a prior covariance matrix $\mathbf{R}(-)$. We model this covariance matrix as

$$r_{ij}(-) = \sigma(p_i)\sigma(p_j)\rho_{p_i p_j}, \tag{1}$$

where $\sigma(p_i)$ is the standard deviation associated with position p_i, and $\rho_{p_i p_j}$ is the correlation coefficient associated with positions p_i and p_j. In this paper we use a simple model below for $\rho_{p_i p_j}$:

$$\rho_{p_i p_j} = \exp\left(-\left(\left(\frac{r_i - r_j}{Lr}\right)^2 + \left(\frac{z_i - z_j}{Lz}\right)^2\right)/2\right), \tag{2}$$

where Lr and Lz denote correlation lengths of sound velocity variation along horizontal and vertical directions respectively.

The measurement model can be written as a linear equation

$$\mathbf{d} = \mathbf{Hc}(-) + \mathbf{v} \qquad \mathbf{v} \sim N(0, \sigma^2\mathbf{I}), \tag{3}$$

where $\mathbf{H}$ is a sparse matrix with only one element equal to 1 in each row, that is:

$$\mathbf{H} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}. \tag{4}$$

The places of those non-zero elements are associated with the positions where in-situ measurements are going to be carried out by AUVs. $\mathbf{v}$ is the measurement noise, which is often assumed to be white Gaussian.

Given the models above, the MAP estimation of sound velocity, with a selected AUV path characterized by matrix $\mathbf{H}$, is

$$\mathbf{c}_{MAP} = (\sigma^{-2}\mathbf{H}^T\mathbf{H} + \mathbf{R}(-)^{-1})^{-1}\mathbf{H}^T\mathbf{d}. \tag{5}$$

The estimation error has zero mean and covariance

$$\mathbf{R}(+) = (\sigma^{-2}\mathbf{H}^T\mathbf{H} + \mathbf{R}(-)^{-1})^{-1}. \tag{6}$$

The AUV path planning problem can now be formulated as

$$\underset{\mathbf{H}\in\Xi}{\text{minimize}}\ f(\mathbf{R}(+)), \tag{7}$$

where Ξ denotes all feasible sampling matrices. The objective function f could have different forms based on different measures of the quality of estimation. There are studies on maximizing the amount of information along the state space direction associated with the maximal estimation error [5], and on minimizing the volume of the η-confidence ellipsoid associated with $\mathbf{R}(+)$ [6]. In this paper we focus on the mean squared error in estimating the entire sound velocity field, thus the objective function is $tr(\mathbf{R}(+))$, where $tr(\cdot)$ denotes the trace operation. With some simple mathematical derivations, we can get

$$\mathbf{R(+)} = \mathbf{R(-)} - \mathbf{R(-)H^T[HR(-)H^T} + \sigma^2\mathbf{I}]^{-1}\mathbf{HR(-)}, \tag{8}$$

which is a familiar form similar to the error covariance matrices updating equation in Kalman filters. As a result, we now rewrite (7) as

$$\underset{\mathbf{H}\in\Xi}{\text{maximize}}\ Ud(\mathbf{H}), \tag{9}$$

where Ud represents uncertainty reduction defined by

$$Ud = tr\left\{\mathbf{R(-)H^T[HR(-)H^T} + \sigma^2\mathbf{I}]^{-1}\mathbf{HR(-)}\right\}. \tag{10}$$

Eq. (10) is obviously a complicated nonlinear optimization problem which is quite computationally intensive. We will introduce an approximated way to calculate uncertainty reduction later this Section.

B. *Path representation*

To define AUV paths and calculate uncertainty reductions, we have to discretize the 2D plane in both dimensions and construct a network graph from all possible AUV paths as shown in Fig. 1. Suppose that during a voyage an AUV is deployed from the left and moves on a vertical plane; at each range with $r = n\Delta r$, it chooses one from all possible waypoints of different depths; finally it floats up at the right and stops. As a result every path can be represented by a sequence of equal-horizontal distance waypoints $\{(0, z_1), (\Delta r, z_2) \cdots (n\Delta r, z_n)\}$, or a sequence of depth $\{z_1, z_2 \cdots z_n\}$.

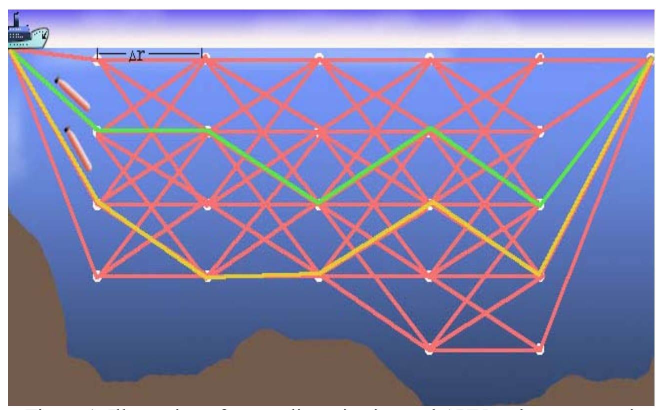


Figure 1. Illustration of ocean discretization and AUV path representation.

Now the optimization problem can be rewritten as

$$\underset{\{z_1, z_2 \cdots z_n\}\in\Gamma}{\text{maximize}}\ Ud(\{z_1, z_2 \cdots z_n\}). \tag{11}$$

C. *Uncertainty reduction approximation*

Even though simplified form (9), solving (11) is still a non-linear problem and may take too much time in performing exhaustive searching. In fact Lr is often less than 1 km because of various submeso- to small-scale oceanographic processes such as internal waves. Therefore the individual significance of an arbitrary in-situ measurement is only affected by other measurements within few kilometers. As here Δr is usually chosen to be 600-1000m, uncertainty reduction of every path segment denoted by a straight red line in Fig. 1 can be approximated as

$$Ud(\{z_i, z_{i+1}\}) \approx Ud(\{z_{i-1}, z_i, z_{i+1}\}) - Ud(\{z_{i-1}, z_i\}). \tag{12}$$

The total reduction of a selected path is just a summation across all its segments.

Let N be the number of stages, K be the number of waypoints to choose from at each stage, and T be the total number of discretized grids in the 2D plane. The computation burden for a selected path is almost the same as for a path segment because T is often larger than ten thousand, and we represent it as $\alpha(T)$. As a result the total computational complexity for exhaustive search is on the order of $K^N\alpha(T)$, while from (12) we can see that the total computational complexity for the approximated method is on the order of $((N-1)K^2+(N-2))K^3\alpha(T)$. If N is larger than 3, the later approach would save computation time dramatically.

D. Multiple-AUV case

In the case with multiple, say M , AUVs, we have to choose M waypoints at each stage. A simple way to route each AUV is that after path planning, waypoints with the smallest depth at each stage is assigned to the first AUV; waypoints with the greatest depth at each stage is assigned to the last AUV, and so on. By doing this the probability of collision is reduced naturally.

III. NUMERICAL SIMULATIONS

In order to investigate the feasibility as well performance of the approach above, we present here a series of numeric simulation results under a shallow water environment shown in Fig. 2. The SVP uncertainty map displays the variance of prior estimation error of the sound speed field. We are interested in water column within 4 km in range and between 50 m and 100 m deep. There are 6 waypoints to choose from at each stage, with an equal vertical discretization interval of 10 m. The horizontal and vertical correlation lengths of the sound speed field are 900 m and 5 m respectively. We also assume a random measure noise with a variance of $0.5\,(m/s)^2$.

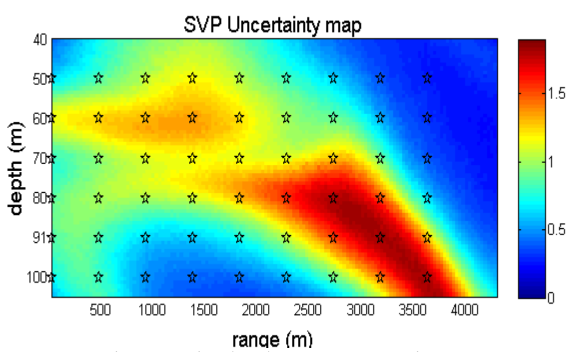


Figure 2. Simulated 2D SVP uncertainty map.

Fig. 3 shows the path of a single AUV and the residual uncertainty using (10). Attentions are put on areas with higher prior uncertainties, and the effect of SVP correlation is well taken care of. Fig. 4 is the result using (12). The total uncertainty reductions are 2993.6 $(m/s)^2$ and 2938.0 $(m/s)^2$ respectively. While it takes about 48 hours to conduct an exhaustive search to get a result in Fig. 3, it takes only 30 seconds to get a sub-optimal AUV path using the approximated method. Computational efficiency has been improved by 5760 times, as could be predicted theoretically according to Section II.C.

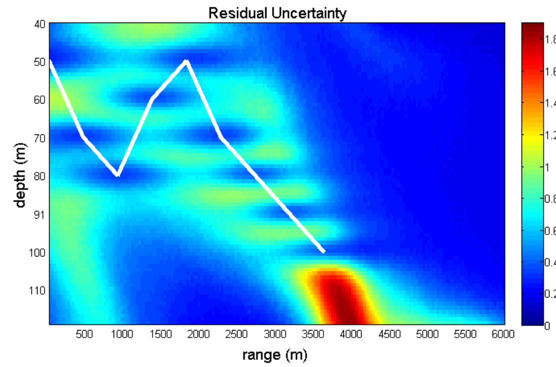


Figure 3. Optimal path and residual uncertainty map for a single AUV.

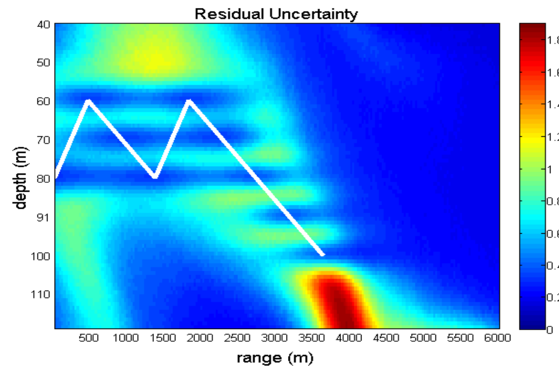


Figure 4. Sub-optimal path and residual uncertainty map for a single AUV.

We then present in Fig. 5 the paths for two AUVs and the corresponding posterior prediction uncertainty. The totlal uncertainty reduction is $4117.4\,(m/s)^2$. Comparing with results by a single AUV, we can see that the contribution of adding a second AUV to uncertainty reduction is only about $1000\,(m/s)^2$, thus implying the path planning strategy here is quite efficient.

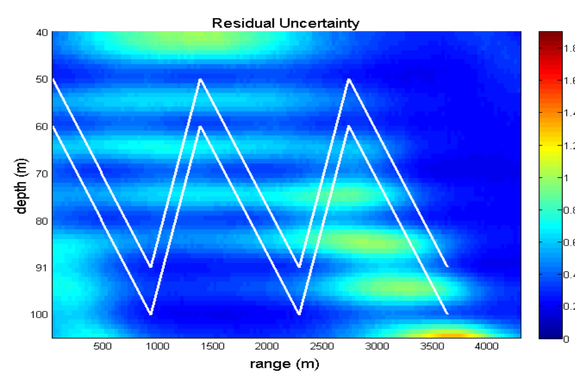
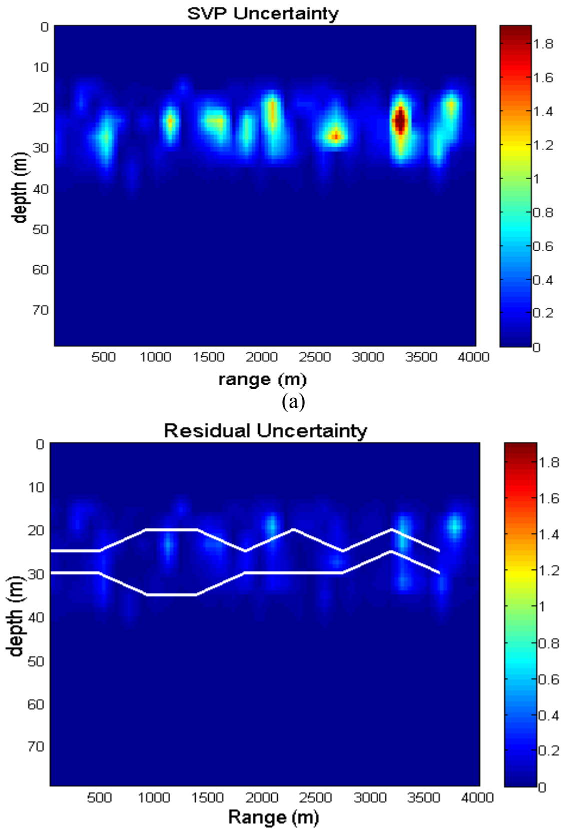


Figure 5. Sub-optimal paths and residual uncertainty map for two AUVs.

An internal-wave-introduced sound speed uncertainty map based on the modified Garret-Munk model [7] is given in Fig. 6(a). Here the buoyancy frequency used for the internal wave model is the same as the Yellow Sea 1996 experiment [8]. From Fig. 6(b) we can see that in-situ measurements through path planning using the developed approach are dedicated to water column with a depth between 15 and 35 meters, where great prior uncertainty is observed.



(b)

Figure 6. (a) Internal-wave-introduced sound speed uncertainty map; (b) Paths for two AUVs and the corresponding residual uncertainty map.

IV. CONCLUSION

We have developed a novel path planning strategy for AUVs to eliminate sound velocity profile prediction uncertainty in the water. The problem is formulated in the MAP estimation framework, and the objective function is the summation of the posterior acoustic prediction uncertainty of the entire 2D SVP field, which is updated from the prior estimation uncertainty through one step Kalman filtering. To save computation time, an approximated method to calculate the uncertainty reduction is utilized here. The proposed strategy is shown to be quite efficient in a number of numeric simulations.

For future effort, the developed approach can be implemented in an iterative manner in the framework of acoustic data assimilation [1]. Besides, a realistic AUV motion model including moving pattern and the associated energy consumption shall be taken into account.

ACKNOWLEDGMENT

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