Uncertainty in Bearing Estimation with The Volume Search Sonar

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Abstract- **The Volume Search Sonar uses a cylindrical receiver array consisting of 40 staves of 9 elements each mounted parallel with the axis of the cylinder, itself aligned with the fore-aft axis of the towbody on which it is installed. 27 fore-aft beam pairs are formed with 27 sub-arrays of 16 contiguous staves. Previous work [Brogan et al., J. Acoust. Soc. Am., 115(5-2), 2547, 2004] has shown that a combination of monopulse and conjugate product processing applied to beam pairs yielded the time and angles of arrival necessary to obtain batymetry and seafloor acoustic backscatter imagery from seafloor echoes received at the VSS array. Here we analyze the estimation uncertainties for these angles of arrival. In monopulse processing applied to fore-aft beams with common phase centers, a signal-to-noise ratio (SNR) in excess of 15 dB is necessary for the standard deviation of the angle estimate to remain below 0.5º in long-range mode (±4º about broadside), whereas an SNR in excess of 20 dB is required to obtain the same angular uncertainty in short-range mode (±8º about broadside). For the conjugate product processing applied to pairs of adjacent across-track beams with distinct phase centers, an SNR in excess of 24 dB is required to maintain the standard deviation of angle estimates below 1º, highlighting the need to improve such estimates by filtering the real and imaginary parts of the conjugate product output.**

I. INTRODUCTION

Given the two-way travel time, t (s), and angles of arrival (azimuth φ, and polar θ) for an echo received from the bottom by an echo-sounder, the depth and horizontal distance coordinates of the sounding are obtained by:

$$R = C\,t/2 \text{ (m), with sound speed } C \text{ (m/s)},$$
$$z(R, \theta) = R\cos\theta,$$
$$x(R, \theta, \varphi) = R\sin\theta\cos\varphi,$$
$$y(R, \theta, \varphi) = R\sin\theta\sin\varphi$$

The expressions above assume a straight path between the echo-sounder and the bottom at the end points of the actual refracted ray path, and the sound speed is the harmonic mean sound speed along that straight path.

The estimates of range and angles are uncorrelated and independent, but the estimates of polar and azimuth angles (θ, and φ) might be correlated, therefore, the uncertainties in depth and horizontal distance are related to the uncertainties in range and angles by:

$$\sigma_z^2 = \sigma_R^2 (\partial z/\partial R)^2 + \sigma_\theta^2 (\partial z/\partial \theta)^2 = \sigma_R^2 \cos^2\theta + \sigma_\theta^2 R^2 \sin^2\theta$$

$$\begin{aligned}\sigma_x^2 &= \sigma_R^2 (\partial x/\partial R)^2 + \sigma_\theta^2 (\partial x/\partial\theta)^2 + \sigma_\varphi^2 (\partial x/\partial\varphi)^2 \\ &\quad + 2\sigma_{\theta\varphi} (\partial x/\partial\theta)(\partial x/\partial\varphi) \\ &= \sigma_R^2 \sin^2\theta \cos^2\varphi + \sigma_\theta^2 R^2 \cos^2\theta \cos^2\varphi \\ &\quad + \sigma_\varphi^2 R^2 \sin^2\theta \sin^2\varphi - 2\sigma_{\theta\varphi} R^2 \cos\theta \cos\varphi \sin\theta \sin\varphi\end{aligned}$$

$$\begin{aligned}\sigma_y^2 &= \sigma_R^2 (\partial y/\partial R)^2 + \sigma_\theta^2 (\partial y/\partial\theta)^2 + \sigma_\varphi^2 (\partial y/\partial\varphi)^2 \\ &\quad + 2\sigma_{\theta\varphi} (\partial y/\partial\theta)(\partial y/\partial\varphi) \\ &= \sigma_R^2 \sin^2\theta \sin^2\varphi + \sigma_\theta^2 R^2 \cos^2\theta \sin^2\varphi \\ &\quad + \sigma_\varphi^2 R^2 \sin^2\theta \cos^2\varphi + 2\sigma_{\theta\varphi} R^2 \cos\theta \sin\varphi \sin\theta \cos\varphi\end{aligned}$$

where σ_z^2, σ_x^2, and σ_y^2 are the variances of the (x, y, z) coordinates of the sounding, σ_R^2, σ_θ^2, and σ_φ^2 are the variances of the range and angle estimates, and $\sigma_{\theta\varphi}$ is the covariance of the polar and azimuth angle estimates.

In the following, we analyze the uncertainties in the estimation of arrival angles (variances σ_θ^2, and σ_φ^2 above) for bottom echoes received by the Volume Search Sonar (VSS).

We begin with a review of the VSS receive array geometry and beamforming methodologies, and we consider two methods of bottom echo detection: (1) a monopulse technique for cophasal beam pairs steered from the same array, and (2) a conjugate product technique for pairs of beams, or groups of beams, formed from different subarrays.

II. VOLUME SEARCH SONAR (VSS) RECEIVE ARRAY GEOMETRY AND BEAMFORMING

The Volume Search Sonar uses a cylindrical receiver array consisting of 40 staves of 9 elements each mounted parallel with the axis of the cylinder. The axis of the cylinder is aligned with the fore-aft axis of the towbody on which it is installed. The staves are spaced at 7.16º increments in the roll plane, covering ±143.2º about nadir, and every other stave is offset fore-aft by half a wavelength to reduce grating lobes.

27 fore-aft beam pairs are formed with 27 sub-arrays of 16 contiguous staves. In the roll plane, the phase centers of these sub-arrays are 7.16º apart, and the axis of each beam pair is broadside to its 16-stave sub-array [Fig. 1(a)]. In the pitch plane, the beam pair is obtained by steering at either ±8º fore-aft using the 5 centermost elements in each stave in short-range mode, or ±4º fore-aft using all 9 stave elements in long-range mode [Fig. 1(b)]. Therefore, each beam pair is obtained with a 3-D array of 16 × 9 elements in long-range mode, or 16 × 5 elements in short-range mode.

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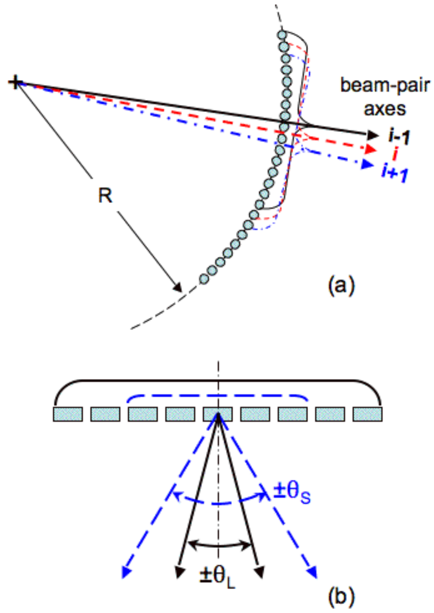

Figure 1. Beamforming geometry of the VSS cylindrical receive array. (a) In the roll plane, the array contains 40 staves spaced at 7.16º intervals on a cylinder of radius R = 196.85 mm. (b) In the pitch plane, each stave contains 9 elements aligned fore-aft. 16 contiguous staves are used to form a fore-aft beam pair, and 27 such beam pairs are formed at 7.16º increments around the cylinder (same angular spacing between beam-pair axes i and i+1 as between staves). In short-range mode, the central 5 elements in each stave are used to steer the beam pair fore-aft at $\theta_S = \pm 8º$. In the long-range mode, all 9 elements in each stave are used to steer the beam pair fore-aft at $\theta_L = \pm 4º$.

Because the array is cylindrical, all the beams have the same nominal beamwidth for a given mode of operation: 9.5º across-track by 8.25º fore-aft in long-range mode, and 9.5º across-track by 16.5º fore-aft in short-range mode. In addition, from about 20º to 70º incidence the angular dependence of seafloor acoustic backscatter is relatively level, and adjacent beams across-track should have roughly the same signal to noise ratio for any signal arriving from the bottom at an angle halfway between the maximum response axes of the two beams. The same reasoning applies to signals arriving from the bottom halfway between combinations of adjacent beams, however in such cases the signal to noise ratio (SNR) should decrease in proportion to the reduction in overlap between the beam groups. Conversely, for most sedimentary bottoms (sands and muds in any combination) we should expect variations of 5-10 dB in the backscattered level received by adjacent beams between 0º and 20º incidence, and beyond about 70º incidence.

Previous work [1-3] has shown that seafloor echoes received by the VSS could be used to obtain swath bathymetry and seafloor acoustic backscatter imagery. In [2-3], this was achieved with two general methods to determine the time and angles of arrival of seafloor echoes received at the VSS array: (1) a monopulse technique applied to each fore-aft beam pair because beams in such a pair have the same phase center, and (2) a conjugate product technique applied in the roll plane to adjacent beams, or groups of adjacent beams with distinct phase centers.

In both methods, it was initially assumed that the angle of arrival lies exactly halfway between the two beams or groups of beams considered. Then, the corresponding time of arrival of bottom echoes was estimated with a weighted mean-time technique for fore-aft beam pairs, or with the zero-crossing of the phase difference between adjacent across-track beams (or groups of adjacent beams). However, the angle of arrival is rarely known exactly and, as shown in the introduction, uncertainty in the bathymetry depends on the uncertainties in the estimations of both the time and the angle of arrival for each sounding. We examine the angle uncertainties in sections III and IV below.

III. VSS Beam Pointing Uncertainty In The Fore-Aft Beam Pairs

For each 3D subarray of 16 adjacent staves, the VSS forms fore-aft beam pairs at ±8º and ±4º about broadside in short-range mode, and long-range mode respectively.

Nielsen [4-5] provides expressions for the Cramer-Rao lower bounds (CRLB) on the azimuth and elevation angle estimates made with an arbitrary 3D array of omni-directional sensors. Here, we simplify the analysis by considering a line array of 9 equally-spaced "elements" for the long-range mode, and the central 5 "elements" of this array for the short-range mode. This simplified geometry ignores the actual staggered stave arrangement with every other stave offset by half a wavelength to lessen grating lobes, but it should not affect the beam pointing estimates.

Nielsen [6] compares measurements of the arrival angle of a signal at a uniform line array (ULA) by either (1) conventional discrete Fourier transform (DFT) beamforming that uses the output from all the elements to form the angle estimate, or (2) forming a set of beams based on the entire array of sensors, and using the outputs of the two beams that encompass the arrival angle. He shows that the CRLB for the angle estimate obtained with approach #2 is bounded from below by the CRLB of approach #1. His results are adapted here to the simplified VSS array geometry described above.

At the outputs of the matched filter for each of the M array elements (M = 9 or 5) spaced d apart, the basebanded quadrature outputs (I, Q) of the ith array element (i = 0, 1, …, M-1) are:

$$x_I(i) = A\cos\left(\left(i - \frac{M-1}{2}\right)\frac{2\pi d \sin\theta}{\lambda} + v_0\right) + w_I(i),$$

$$x_Q(i) = A\sin\left(\left(i - \frac{M-1}{2}\right)\frac{2\pi d \sin\theta}{\lambda} + v_0\right) + w_Q(i),$$

where $\lambda = f_0/c$ is the wavelength at the sonar's center frequency f_0, and v_0 is the phase angle at the origin of the array. The array is centered at the origin of the coordinate system, and the angle of arrival θ is relative to the broadside of the array.

The quadrature components of the noise, $w_I(i)$ and $w_Q(i)$, are assumed to be zero-mean stationary white Gaussian processes with the same variance σ_w^2. Noise sequences received by different sensors are assumed to be uncorrelated.

We form a set of $N \geq M$ beams equally spaced in $u = \sin\theta$ using a length N DFT (padding with zeros for $N > M$). To obtain beams at ±8º and ±4º about broadside requires respectively N = 8 with M = 5 in short-range mode, and N = 16 with M= 9 in long-range mode.

In the following expressions, the SNR in dB is defined by $SNR = 10 \log_{10}(\eta) = 10 \log_{10}(MA^2 / 2\sigma_w^2)$. The CRLB for a ULA of M sensors (approach #1) is [6]:

$$CRLB_M(\theta) = \frac{M}{2\eta \left(\frac{2\pi d}{\lambda}\right)^2 \cos^2(\theta) \frac{M(M^2-1)}{12}}$$

For approach #2, let beams k and (k+1) encompass the desired angle θ. When the signal arrival angle is at (k + 1/2), halfway between beams k and (k+1), $\sin\theta = (k + 1/2)\lambda/Nd$, and with $\beta = \pi/2N$ the monopulse CRLB is:

$$CRLB_{k+1/2}(\theta) = \frac{M\sin^2(\beta)\left(M - \frac{\sin(2M\beta)}{\sin(2\beta)}\right)}{\eta \left(\frac{2\pi d}{\lambda}\right)^2 \cos^2(\theta) \left(M\cos(M\beta) - \frac{\sin(M\beta)\cos(\beta)}{\sin(\beta)}\right)^2}.$$

These CRLB expressions are plotted in Figures 2 and 3 for the long- and short-range modes, respectively and various SNR values. In long-range mode (±4º about broadside), an SNR of 15 dB or better is necessary for the standard deviation of the angle estimate to remain below 0.5º. In short-range mode (±8º about broadside) an SNR of 20 dB or better is required to obtain the same angular uncertainty. Such SNR levels should be easily obtainable for fore-aft beam pairs in the near nadir region (0º to ±10º across-track), but they might more difficult to obtain for fore-aft beam pairs at across-track angle of 60º or more over a flat bottom.

These results are plausible, but they are not directly applicable to the VSS geometry because the fore-aft monopulse processing is done on the ±8º (±4º in long-range) beam pair about broadside, which corresponds to beams k and k+2 (rather than k and k+1 in the plots). However, the standard deviation of the broadside beam (0º) gives an indication of the uncertainty to be expected halfway between the two VSS fore-aft beams. A more extensive analysis is therefore required, and it should include the effects of element amplitude weighting (e.g. [7]). Non-uniform element weight will broaden the beams and increase the region of overlap between them.

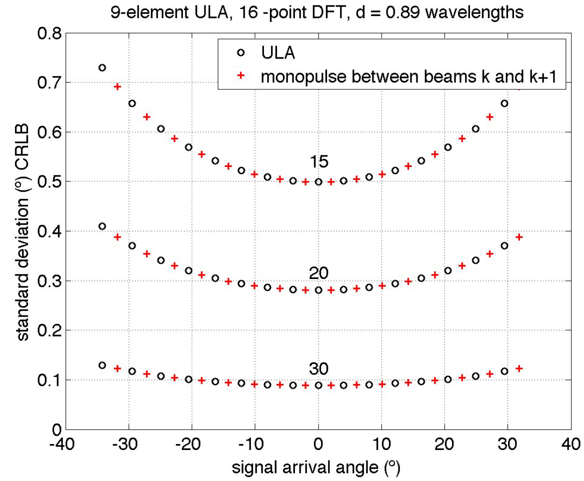

Figure 2. VSS Long Range Mode. Standard deviation of the angle of arrival estimate for a ULA of 9 elements with 0.89-wavelength spacing. Conventional 16-point DFT beamformer solution compared to monopulse processing between adjacent beams. Signal to noise ratio (dB) value shown at nadir decreases according to Lambert's law [$\cos^2(\theta_i)$, for angles of incidence θ_i at the seafloor].

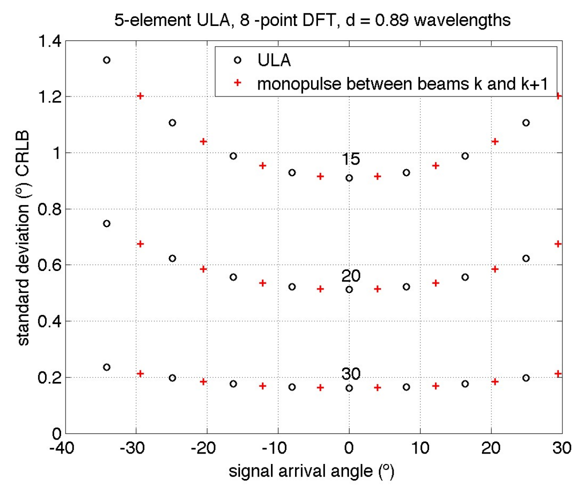

Figure 3. VSS Short Range Mode. Standard deviation of the angle of arrival estimate for a ULA of 5 elements with 0.89-wavelength spacing. Conventional 8-point DFT beamformer solution compared to monopulse processing between adjacent beams. Signal to noise ratio (dB) value shown at nadir decreases according to Lambert's law [$\cos^2(\theta_i)$, for angles of incidence θ_i at the seafloor].

IV. Angular Uncertainty In The Conjugate Product Processing

The conjugate product processing method consists in multiplying the complex output of one beam (maximum response axis at θ_p) by the conjugate of the output of an adjacent beam across-track (maximum response axis at θ_{p-1} or θ_{p+1}).

Beams are spaced at 7.16º (0.125 rad) across-track, which is also the spacing between the phase centers of their respective subarrays of 16 elements. Therefore, the distance between the phase centers (d_{cp}) of adjacent across-track beams is approximated by the corresponding arc length on a cylinder of radius 196.85 mm:

$$d_{cp} = 196.85 \times 10^{-3} \times 0.125 = 2.46 \times 10^{-2} \text{ m},$$

which is 0.615 wavelengths at a nominal frequency of 37.5 kHz with 1500 m/s sound speed.

Assuming that the bottom signal is received at roughly the same SNR in each beam, the conjugate product yields the phase difference, $\Delta\varphi = \varphi_{p+1} - \varphi_p$, between the two beams so that when the phase difference is zero, the arrival angle is halfway between the maximum response axis of each beam (θ_p, θ_{p+1}). For narrowband echoes from the bottom, or for single frequency bins in broadband signals, the arrival angle θ relative to the "broadside" axis between the beams ($\theta_p + \theta_{p+1})/2$ is related to the electrical phase difference $\Delta\varphi$ between the beams by:

$$\Delta\varphi = 2\pi\, d_{cp} \sin\theta / \lambda \quad \text{(rad)}.$$

When the angle is within ±4º of broadside, the small angle approximation can be applied to $\sin\theta$ such that $\sin\theta \approx \theta$ (rad), then $\theta \approx \Delta\varphi\, \lambda / (2\pi\, d_{cp})$ (rad).

This approximation is valid here because beams are 7.16º apart and the angle of interest is within ±3.58º of the "broadside" axis between the beams. The variance of the estimate $\underline{\theta}$ of θ is therefore proportional to the variance of the estimate $\underline{\Delta\varphi}$ of $\Delta\varphi$:

$$\mathrm{Var}[\, \underline{\theta} \,] = (\lambda / (2\pi\, d_{cp}))^2\, \mathrm{Var}[\, \underline{\Delta\varphi} \,]$$

This problem is equivalent to the target bearing estimation with a split-aperture correlator described by Burdic [8] who derives the bearing-estimate variance for sinusoidal signals (signal power $A^2/2$) in narrowband white Gaussian noise received at each half of the aperture (noise power σ_w^2). Neglecting beam pattern effects, this derivation is adapted here to the general case of high or low SNR, with output SNR $\eta_o = (A^2/2\sigma_w^2)$ in each beam (or half aperture):

$$\mathrm{Var}[\, \underline{\theta} \,] = (\lambda / (2\pi\, d_{cp}))^2\, \eta_o^{-1}\, [1 + 2/\eta_o\,]. \tag{1}$$

This expression is consistent with Burdic's result for high SNR [8]:

$$\mathrm{Var}[\, \underline{\theta} \,] \approx (\lambda / (2\pi\, d_{cp}))^2\, \eta_o^{-1} \tag{2}$$

For comparison, the uncertainty in the estimation of the angle of arrival for a 2-element array with element separation d_{cp} is obtained by setting M=2 in $\mathrm{CRLB}_M(\theta)$ in section III. At broadside (θ = 0º), the angular uncertainty is inversely proportional to the SNR at the array [$\eta = 2(A^2 / 2\sigma_w^2)$]:

$$\mathrm{CRLB}_{M=2}(0) = 2\, \lambda^2 / [\eta\, (2\pi\, d_{cp})^2],$$

which is identical to (2) since $\eta = 2\, \eta_o$.

For a bottom echo received between adjacent across-track beams ($\theta \le \pm 3.58$º), the conjugate processing should yield an rms angular uncertainty of the arrival angle measured relative to the broadside of the two beams given by:

$$\theta_{rms} \approx (\eta_o)^{-1/2}\, [1 + 2/\eta_o\,]^{1/2}\, \lambda / (2\pi\, d_{cp}). \tag{3}$$

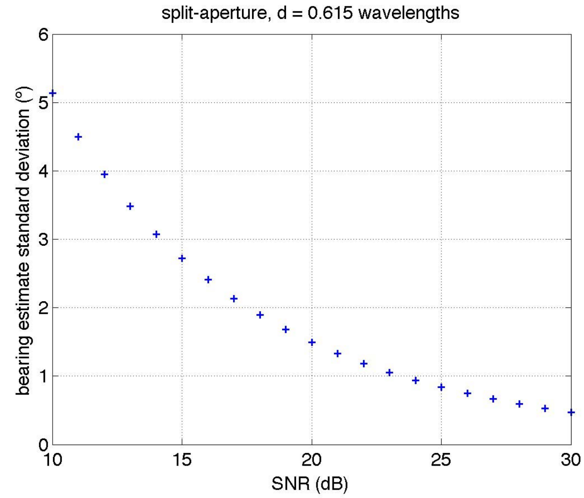

Figure 4. Standard deviation the bearing estimate near broadside of a split-aperture as a function of the output SNR = 10 log10(η_o): $\theta_{rms} \approx (\eta_o)^{-1/2}\, [1 + 2/\eta_o\,]^{1/2}\, \lambda / (2\pi\, d_{cp})$ with d_{cp}/λ = 0.615.

Fig. 4 is a plot of (3) where we see that an SNR in excess of 24 dB is needed to maintain the rms uncertainty in bearing angle below about 1º. In practice, noise reduction is achieved while estimating the phase difference by filtering the real and imaginary parts of the conjugate product output (e.g. [9]):

$$\underline{\Delta\varphi} = \mathrm{atan2}\, \frac{\sum_{n=1}^{N_s} h^2[n]\, \mathrm{Im}\{x_{p+1}[n]\, x_p^*[n]\}}{\sum_{n=1}^{N_s} h^2[n]\, \mathrm{Re}\{x_{p+1}[n]\, x_p^*[n]\}}$$

which is an unbiased estimate of the phase difference obtained from the conjugate product of the outputs of the two adjacent beams: x_p and x_{p+1}, respectively. In this expression h[n] is the impulse response of a finite impulse response filter (e.g. hanning window) acting on N_s data samples, and atan2 is the four-quadrant arctangent operator.

Computation of the variance of this filtered phase difference estimate requires a more extensive analysis (e.g. [10]) left for future work, but a first approximation is given by:

$$\theta_{rms} \approx (N_s\, \eta_o)^{-1/2}\, [1 + 2/\eta_o\,]^{1/2}\, \lambda / (2\pi\, d_{cp}). \tag{4}$$

The number of samples N_s is determined by assuming that the bottom is flat at the scale of a beam footprint, and by differentiating $R = z/\cos\theta_p$ with respect to the across-track beam angle θ_p relative to vertical:

$$dR/d\theta_p = z \tan\theta_p / \cos\theta_p.$$

In the small angular sector $d\theta_p$ (e.g. $d\theta \le 3$º) centered on the beam angle θ_p, the number of across-track range samples contained in the elemental across-track range dR is:

$$N_s = dR / \Delta R_s = z\, d\theta_p \tan\theta_p / (\Delta R_s \cos\theta_p),$$

with range sampling rate [$\Delta R_s = C / 2f_s$ (m)] at sampling frequency f_s (Hz). In practice, $d\theta_p$ decreases as the beam angles increase across-track to avoid averaging over too many range samples.

V. CONCLUSION

The uncertainties associated with estimates of the angles of arrival of bottom echoes received by pairs of VSS beams are primarily driven by the SNR obtained at the output of the chosen processing method. In monopulse processing applied to fore-aft beams with common phase centers, an SNR in excess of 15 dB is necessary for the standard deviation of the angle estimate to remain below 0.5º in long-range mode (±4º about broadside), whereas an SNR in excess of 20 dB is required to obtain the same angular uncertainty In short-range mode (±8º about broadside). These SNR values are realistic for near-specular backscattered echoes.

For the conjugate product processing applied to pairs of adjacent across-track beams with phase centers separated by d_{cp} = 0.615 wavelengths, a relatively high SNR in excess of 24 dB is required to maintain the standard deviation of angle estimates below 1º. Such high angular uncertainty will yield high uncertainties in the corresponding bathymetric values and the resulting bottom maps will appear noisy. In practice, the angular estimates are improved by filtering the real and imaginary parts of the conjugate product output, hence reducing the standard deviation of angle estimates by up to $(N_s)^{-1/2}$.

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