Near-Field Beamforming for A Multi-Beam Echo Sounder: Approximation and Error Analysis

Ying Jiang, Wen Xu*, Linsheng Chen, and Xiang Pan
Department of Information Science and Electronic Engineering
Zhejiang University
Hangzhou, China, 310027
*Email: wxu@zju.edu.cn

***Abstract*-For a Multi-Beam Echo Sounder (MBES) operating in very shallow water environments or close to seafloor, near-field beamforming shall be used to estimate the direction and range of the backscattering sources along the bottom. The most common beamforming method is the so-called weight-delay-sum (WDS) beamformer. In this paper, we present an approximation method to calculate time delays between individual array elements that can be applied to an arbitrarily-shaped array operated in near-field conditions. The result is applied to a practical MBES configuration with a curved receiving array. Phase-shift error analysis and comparison between the accurate beam patterns and the approximate beam patterns are implemented, and the results demonstrate that the developed method is quite efficient in computation while being enough accurate.**

I. INTRODUCTION

As a widely-used swath bathymetry sonar system, Multi-Beam Echo Sounder (MBES) can provide both wide cross-track coverage and high spatial resolution. Compared to a bathymetric side-scan sonar, MBES is more robust in the presence of complex bottom structures, due to its angular resolution of multiple beamforming. In the beamforming process, plane wave propagation from the far field of the acoustic array is often assumed. However, for short operation ranges typical in very shallow water environments, the far field condition may not be satisfied. Under those circumstances, the plane wave assumption shall be replaced by the spherical wave assumption to better describe the source signal propagation, and near-field beamforming shall be used to estimate the direction and range of the backscattering sources along the bottom.

The most common beamforming method is the so-called weight-delay-sum (WDS) beamformer [1]. For the far field narrow band signal model, the time delays between individual array elements can be converted to phase-shifts accordingly, and thus be readily evaluated. Under the near field condition, evaluation of time-delays relies on accurate range information which depends on both the element positions and the backscattering point locations. A typical MBES system simultaneously generates more than one hundred beams, thus imposing significant challengers to accurate real-time evaluation of the received signal time delays.

Numerous array pattern synthesis approaches have been developed for far-field beamforming. A well-known solution is given in Ref. [2] using the theory of Chebyshev polynomials, which, however, can only be used for uniform linear arrays. Other algorithms have been developed for beam pattern design of non-uniform arrays [3, 4]. Besides, Ref. [5] presents two optimal pattern synthesis approaches based on convex optimization of second-order cone programming.

For near-field beamforming, the straightforward method of pattern synthesis is to use the results of far-field pattern synthesis and apply time delays to compensate for the differing propagation delays assuming spherical propagation [6]. However, few methods can achieve the desired near-field compensation responses with enough accuracy uniformly across all angles [7].

Ref. [7] presents a beamforming method using a radial beampattern transformation that achieves the desired near-field pattern exactly. To reduce computational complexity, constrained optimization methods used in far-field pattern synthesis have been applied to near-field conditions. For example, a constrained least-square method is presented for near-field beamforming that can be used for arbitrary-geometry arrays [8]. However most of those methods has to design different weighting coefficients for different source distances, which involves complicated iterative algorithms. Considering that, methods based on far-field weighting coefficients are still quite good choices particularly if the computational efficiency can be further improved to allow more accurate approximation for near-field applications.

In this paper, we present a fast approximation method to calculate distance differences of signal propagation between individual array elements that can be applied to an arbitrarily-shaped array operated in near-field conditions. The rest of the paper is organized as follows. Section Ⅱ describes a near-field compensation method used in conventional near-field beamforming. Based on that, an approximation method is then developed in Section Ⅲ. Section Ⅳ applies this method to a practical MBES configuration with a curved receiving array. Phase-shift error analysis and comparison between the accurate beam patterns and the approximate beam patterns are implemented. Finally, Section V concludes the paper.

II. NEAR FIELD BEAMFORMING

In this section, a near-field beamforming method developed in Ref. [7] is briefly described, which is based on applying time delays to compensate for different propagation paths of spherical wave signals.

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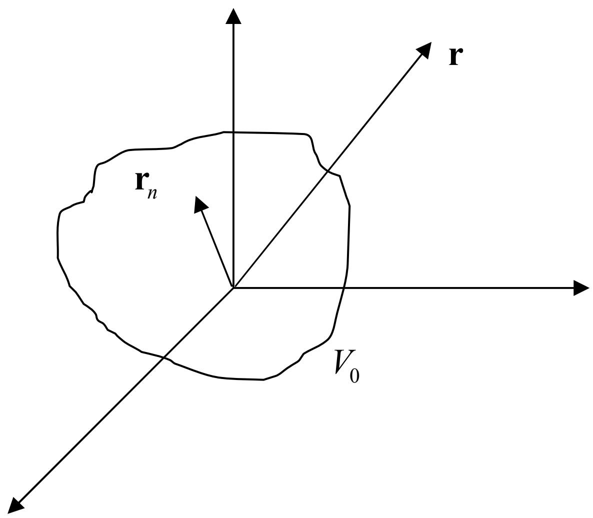

Figure 1. Geometry of array configuration.

To illustrate the near-field beamforming, consider an array of N sensors in a volume V_0 (see Fig. 1). A narrowband signal model is assumed and each sensor is weighted with a complex number. $\mathbf{r}_n$ is the position of the nth sensor of the array. The response of the array to a signal at position $\mathbf{r}$ is [7]:

$$b_{nf}\left(\mathbf{r}\right)=\sum_{n=0}^{N-1} w_n^{'} \frac{r}{d_n\left(\mathbf{r}\right)} e^{j2\pi fc^{-1}\left(d_n(\mathbf{r})-r\right)}, \tag{1}$$

where $w_n^{'}$ is the complex weight on the nth sensor, f is the frequency of operation, c is the speed of wave propagation, $d_n\left(\mathbf{r}\right)$ is the distance from the source to the nth sensor, and $r=\left|\mathbf{r}\right|$ is the source distance from the original point.

While in far-field beamforming, the response of the array to a signal at direction $\hat{\mathbf{r}}$ is

$$b\left(\hat{\mathbf{r}}\right)=\sum_{n=0}^{N-1} w_n e^{j2\pi fc^{-1}\mathbf{r}_n\cdot\hat{\mathbf{r}}}, \tag{2}$$

where w_n is the weight on the nth sensor, and $\hat{\mathbf{r}}$ is the unit vector in the direction of $\mathbf{r}$, that is

$$\mathbf{r}=r\hat{\mathbf{r}}\ . \tag{3}$$

The goal of near-field compensation is to transform the near-field response to that of the far field. By comparing (1) and (2), the compensated near-field response can be written as [7]:

$$b_{nf}\left(\mathbf{r}\right)=\sum_{n=0}^{N-1} w_n\psi_n \frac{r}{d_n\left(\mathbf{r}\right)} e^{j2\pi fc^{-1}\left(d_n(\mathbf{r})-r\right)}, \tag{4}$$

where

$$\psi_n=r^{-1}d_n\left(\mathbf{r}\right)e^{j2\pi fc^{-1}\left(r-d_n(\mathbf{r})+\mathbf{r}_n\cdot\hat{\mathbf{r}}\right)}. \tag{5}$$

Thus the complex weight on the nth sensor in near-field beamforming is

$$w_n^{'}=w_n\psi_n\ . \tag{6}$$

As seen from (5), the difficulty lies in the evaluation of $w_n^{'}$, which varies with both signal's direction $\hat{\mathbf{r}}$ and distance r.

III. APPROXIMATION METHOD FOR NEAR-FIELD BEAMFORMING

To reduce the evaluation complexity of near-field beamforming, we inspect (5) and (6) and rewrite the complex weight $w_n^{'}$ as follows:

$$w_n^{'}=\left(w_n e^{j2\pi fc^{-1}\mathbf{r}_n\cdot\hat{\mathbf{r}}}\right)\left(\frac{e^{j2\pi fc^{-1}r}}{r}\right)\left(d_n\left(\mathbf{r}\right)e^{-j2\pi fc^{-1}d_n(\mathbf{r})}\right). \tag{7}$$

From (7), $w_n^{'}$ is the product of three expressions. The first expression is a function of signal's direction $\hat{\mathbf{r}}$, the second expression is a function of signal's distance r, and the third expression varies with both signal's direction $\hat{\mathbf{r}}$ and distance r. For a given signal's direction, the result of the first expression can be applied to all signal distances. Similarly, the result of the second expression can be applied to all signal directions. Thus the complexity of calculating $w_n^{'}$ arises mostly from the third expression, which must be updated for each new signal direction and each new signal distance.

The key of calculating the third expression is in evaluation of $d_n\left(\mathbf{r}\right)$ or $\Delta d_n\left(\mathbf{r}\right)=d_n\left(\mathbf{r}\right)-r$. Hence reducing computations required for $d_n\left(\mathbf{r}\right)$ or $\Delta d_n\left(\mathbf{r}\right)$ is crucial for simplifying near-field beamforming implementation. We rewrite $\Delta d_n\left(\mathbf{r}\right)$ as follows:

$$\begin{aligned}\Delta d_n\left(\mathbf{r}\right)&=d_n\left(\mathbf{r}\right)-r\\&=\sqrt{\left(\mathbf{r}-\mathbf{r}_n\right)\cdot\left(\mathbf{r}-\mathbf{r}_n\right)}-r\\&=\sqrt{r^2+r_n^2-2r\mathbf{r}_n\cdot\hat{\mathbf{r}}}-r.\end{aligned} \tag{8}$$

where r_n is the distance of the nth element form the reference point. Define $\Delta d_{0n}=\mathbf{r}_n\cdot\hat{\mathbf{r}}$, which is the distance discrepancy between the nth element and the reference point when the source is in far field. Rearranging (8), we have

$$\begin{aligned}\Delta d_n\left(\mathbf{r}\right)&=\sqrt{r^2+r_n^2+2r\Delta d_{0n}}-r\\&=\sqrt{\left(r+\Delta d_{0n}\right)^2+r_n^2-\Delta d_{0n}^2}-r\\&=\left(r+\Delta d_{0n}\right)\sqrt{1+\frac{r_n^2-\Delta d_{0n}^2}{\left(r+\Delta d_{0n}\right)^2}}-r.\end{aligned} \tag{9}$$

Applying the well-known Taylor series expansion

$$\sqrt{1+x}=1+\frac{1}{2}x-\frac{1}{8}x^2+\ldots \tag{10}$$

to (9), we obtain an approximation for calculating $\Delta d_n\left(\mathbf{r}\right)$:

$$\Delta d_n\left(\mathbf{r}\right)\approx\Delta d_{0n}+\left(r_n^2-\Delta d_{0n}^2\right)\big/2\left(r+\Delta d_{0n}\right). \tag{11}$$

Compared with (8), Eq. (11) is much simple to evaluate which eliminates the complicated square root operation.

IV. EXAMPLE ERROR ANALYSIS

In this section, we apply the above approximation method to a practical MBES configuration with a curved receiving array (solid line in Fig. 2). For a typical MBES system, the transmission array projects narrow beams in the along-track direction, thus signals scattered back by the seafloor are mainly within the cross-track plane. Hence in this example, only the beampattern in the cross-track plane (x-y plane in Fig. 2) is considered. As shown in Fig. 2, the coordinate of the nth element is $\left(x_n,y_n\right)$, the coordinate of the near field source is

$(r\sin\theta, r\cos\theta)$, and the original point is selected as the reference point for computing the received signal time-delays associated with each array element.

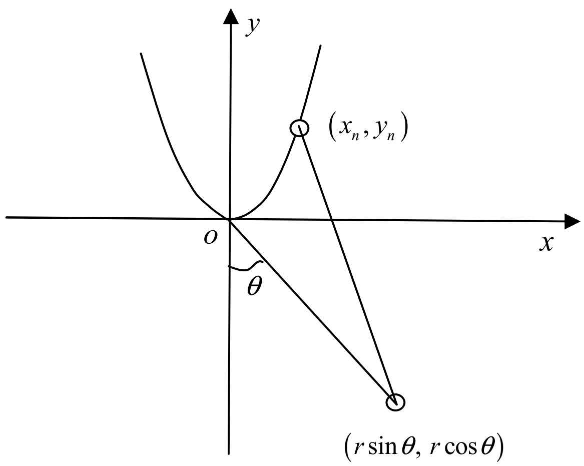

Figure 2. Geometry of a practical MBES configuration.

For signal propagation from direction θ, the distance discrepancy between the nth element and the reference point is

$$\Delta d_n(r,\theta) = \sqrt{(r\sin\theta - x_n)^2 + (r\cos\theta + y_n)^2} - r, \quad (12)$$

and the corresponding time delay is $\Delta t_n = \Delta d_n / c$, where c is the sound speed in water. Approximation in (11) can then be applied with

$$\Delta d_{0n} = -x_n \sin\theta + y_n \cos\theta \quad (13)$$

and

$$r_n = \sqrt{x_n^2 + y_n^2}\,. \quad (14)$$

For simulation analysis, we assume that the receiving array has 128 elements approximately half-wavelength spaced, the operating frequency is 165 kHz, and the sound propagation speed is 1500m/s. The distance at which the far-field approximation starts to be valid is commonly chosen as

$$r = \frac{2L^2}{\lambda}, \quad (15)$$

where L is the largest array dimension, and λ is the operating wavelength [7]. For the array configuration in this example, Eq. (16) specifies that near-field compensation shall be used for $r < 38$ m.

In the simulations, the convex optimization of second-order cone programming method [5, 9] is used to obtain two weighting coefficient sets when steering to angle 0 and 75 degree respectively. Achieved far-field beampatterns are shown in Figs. 3 and 6 respectively. Then the approximation in (11) and the near-field compensation in (4) and (5) are used to form near-field beampatterns. For comparison, the accurate near-field beampatterns are evaluated as well using (12), (4) and (5). Figs. 4 and 5 display two near-field beampatterns for source direction of 0 degree and source ranges of 2 m and 1 m respectively. The nonuniform far-field weighting of Fig. 3 is used in near-field beamforming.

Similarly, Figs. 7 and 8 display two near-field beampatterns for source direction of 75 degree and source ranges of 2 m and 1 m respectively. The nonuniform far-field weighting of Fig. 6 is used in near-field beamforming.

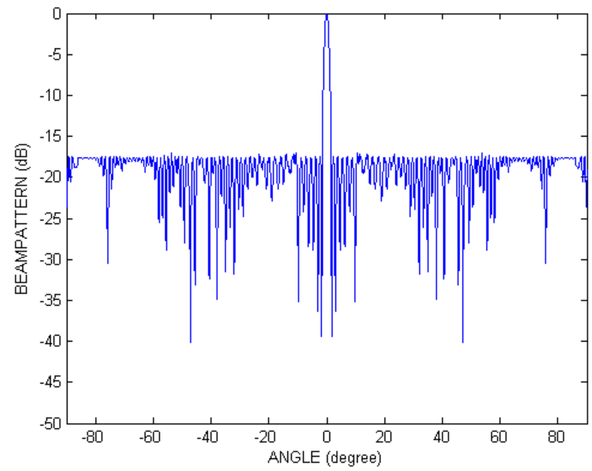

Figure 3. Example far field beampattern when steered to 0 degree.

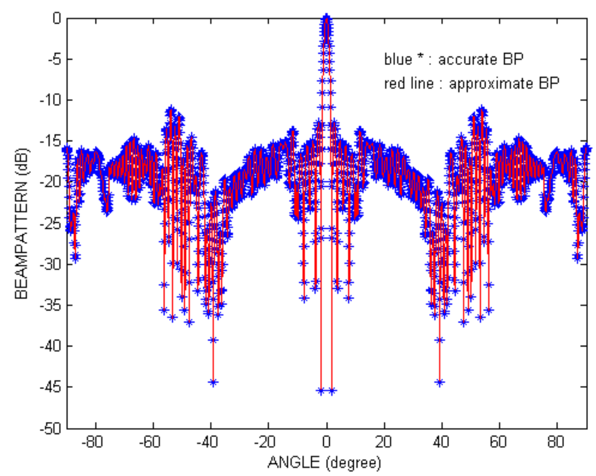

Figure 4. Example near field beampattern when steered to 0 degree: r=2 m.

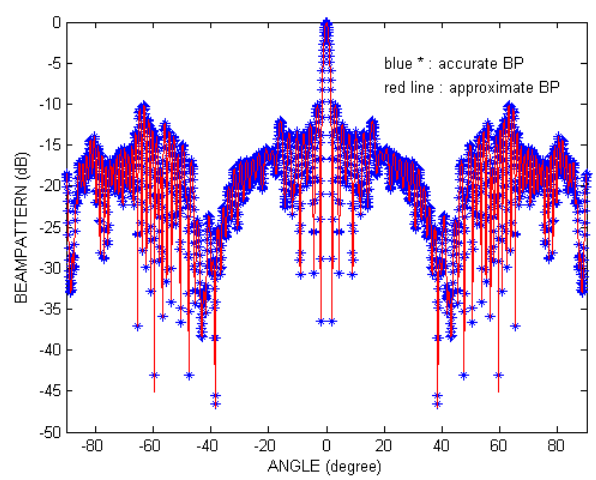

Figure 5. Example near field beampattern when steered to 0 degree: r=1 m.

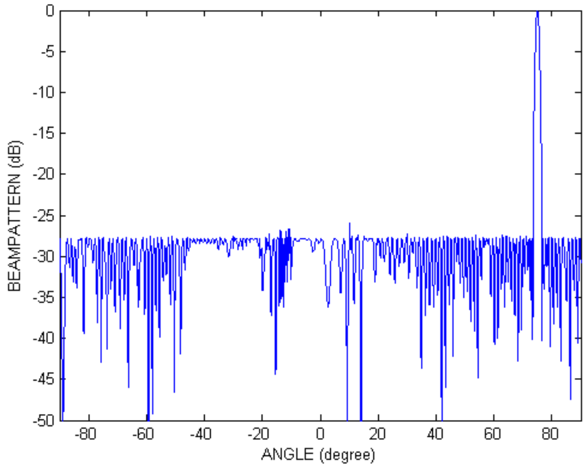

Figure 6. Example far field beampattern when steered to 75 degree.

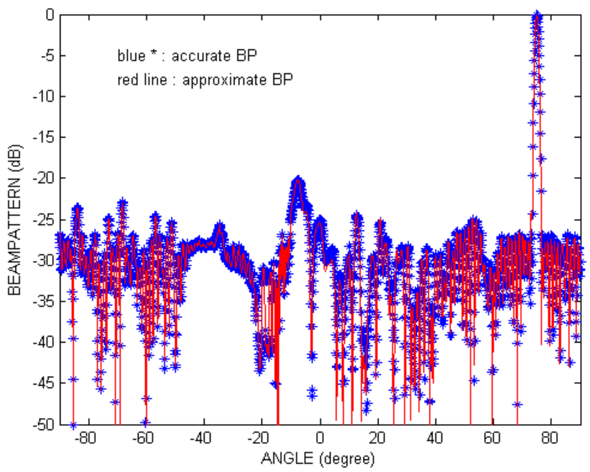

Figure 7. Example near field beampattern when steered to 75 degree: r=2 m.

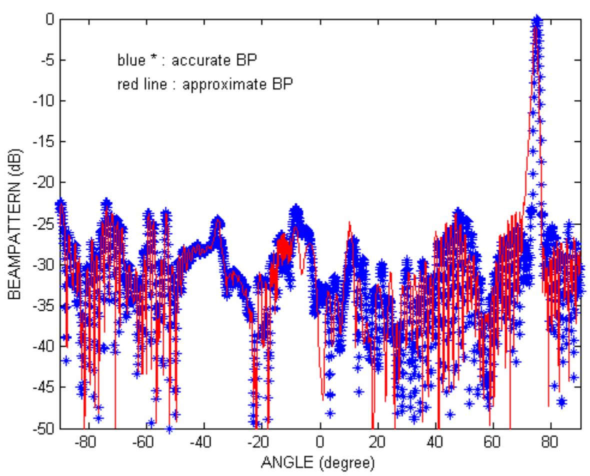

Figure 8. Example near field beampattern when steered to 75 degree: r=1 m.

From Figs. 4、5 and 7, we can see that the approximate beampatterns are almost exactly the same as the accurate beampatterns even when the source distance is as close as 1 m to the origin point. Fig. 8 displays slight differences mainly in the sidelobe region. Overall the approximation in (11) is shown to be fairly accurate.

To quantify the approximation precision of (11), we analyze the phase-shift error of the nth element, $\delta\varphi_n$, caused from approximation. From (10) and (11), the phase-shift error is about

$$\delta\varphi_n \approx 2\pi f c^{-1} \frac{\left(r_n^2 - \Delta d_{0n}^2\right)^2}{8\left(r + \Delta d_{0n}\right)^3} \tag{16}$$

From (16) we can see that the elements, which have large r_n and small Δd_{0n} would have large phase-shift error for a given signal distance. Fortunately, for a curved receiving array and signals scattered from a flat seafloor, large r_n usually means large Δd_{0n} , thus $\delta\varphi_n$ is always small enough so that it can be neglected. For example, consider a close signal source at $r = 1$ m and $\theta = 0^\circ$; from (13), the maximum phase-shift error $\delta\varphi_n \le 5.6^\circ$ which is far less than the "rule of thumb" criterion of maximum quadratic phase error of 22.5° at the edge of the array aperture [10].

V. CONCLUSION

In this paper, we present an approximation method to calculate time delays between individual array elements that can be applied to an arbitrarily-shaped array operated in near-field conditions. The result is applied to a practical MBES configuration with a curved receiving array. Phase-shift error analysis and comparison between the accurate beam patterns and the approximate beam patterns are implemented, and the results demonstrate that the developed method is quite efficient in computation while being enough accurate.

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