Multi-AUV Coordination in the Underwater Environment with Obstacles

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Abstract- **A concept of extended formation which is derived by combining the destination and obstacles in the underwater environment as elements into the traditional formation of AUVs is proposed. In extended formation, an improved potential-field-based approach is introduced to coordinate multiple autonomous underwater vehicles (AUVs). Artificial potential fields which define the control forces are constructed to correspond to a special destination and desired formation geometry. The extended formation with the improved potential fields can be regarded as a multi-body system with closed branches. In the system, the vehicles are interconnected in a configuration by forces derived from the potentials. Feasibility of the proposed approach is validated by using simulation results.**

I. INTRODUCTION

The coordination of multiple autonomous vehicles has been studied extensively in the last few years [1,2]. The good coordination of the individuals can bring “emergent” property such as efficient maneuver and significantly decision-making ability. Artificial potential field method is one of the commonly used approaches to coordination.

Artificial potential field method is initially used for exact motion planning and control of robot [3,4]. Progress has been made in using artificial potential fields in group tasks such as in mobile sensor network deployment, area coverage problem [5,6] and decentralized control of large swarm systems [7,8]. Leonard proposes virtual leader, artificial potential method [9] in which the interaction between neighboring vehicles is considered but no influence of obstacles.

In this paper we expand existing framework above proposing attraction potential field which is designed to guide the formation to the destination, interactive potential field which is used to build desired formation geometry, and repulsion potential field which is designed to avoid collisions with obstacles. These field functions only depend on the relative distance which can be measured by the sensors payload.

On the basis of the artificial potential fields above, we propose a concept of extended formation which is defined by combining the destination and the obstacles into the traditional formation of AUVs. In extended formation, artificial potential fields are constructed to control the individual and coordinate the group of AUVs. In this approach there exist three kinds of potential fields, respectively, the attractive potential field between AUV and the destination, the repulsive potential field between AUV and obstacles, the interactive potential field between AUV and its neighbors. A main objective of this approach is to implement distributed control and eliminate the ordering of the AUVs.

The remainder of this paper is organized as follows. In section II we expand existing framework proposing improved attraction potential function, repulsion potential function and interactive potential function which are constructed in extended formation. In section III we present the extended formation with artificial potential fields built and analyze the contribution of the forces derived from the potentials to the generalized force in the extended formation. In section IV we demonstrate our approach with simulations.

II. ARTIFICIAL POTENTIAL FIELDS CONSTRUCTION IN EXTENDED FORMATION

In this section, we focus on the construction of the artificial potential fields. The detail formulation and analysis of extended formation will be discussed in section III.

In our framework, artificial potential functions in the ocean environment are constructed based on the following principles: the destination should be at the global minimum point of the total potential while the obstacles and the AUVs should be at the local maximum point of the total potential. In light of the above principles, artificial potentials are defined as functions of relative distance, and corresponding force acting on the AUV is the minus the gradient of the potential function.

Virtual leader and artificial potentials method for multi-vehicle coordination is introduced in [9]. The approach of construction of artificial potential fields presented herein is the expansion of the concept proposed in that paper. Specially, avoidance of obstacles in the environment with obstacles is the main concern in this section.

An improved artificial-potential-based control approach is applied in the underwater environment. The virtual fields are constructed in such a way that each AUV is repelled by obstacles and the other AUVs, while attracted to the goal area.

Artificial potential fields constructed in underwater environments can be divided into three components: the attractive potential field due to the goal area, the repulsive potential field due to obstacles, and the interactive potential field due to adjacent AUVs. The attractive potential is constructed to drive AUVs to the goal position; the repulsive potential field is used to ensure that AUV is repulsed away from the obstacles, while interactive potential field is defined such that the AUVs keep desired distance from its neighbors.

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A. The Attractive Potential Field

To track the goal or drive the formation of AUVs to the destination, attractive potential field between AUV and goal area is constructed. For simplicity, we define M_g as the minimal distance between AUV and the goal area. The attractive potential function is given by

$$U_{att}=\begin{cases}0 & 0<M_g\le p\\ \frac{1}{2}k_a M_g^{\frac{3}{2}} & M_g>p\end{cases} \quad (1)$$

where k_a is a positive scalar control gain, p is the equivalent radius of the attractive area. Fig. 1 shows the contour of attractive potential which is like pan with a circle with radius p at the bottom. And the relevant attractive force can be written in the following form

$$\mathbf{F}_{att}=\begin{cases}0 & 0<M_g\le p\\ -\frac{3}{4}k_a\sqrt{M_g}\nabla M_g & M_g>p\end{cases} \quad (2)$$

which tends linearly to zero as the AUV approaches the goal area. As can be seen from piecewise function (2) above, there is no attractive force but still existence of interactive force when AUVs enter the equivalent attractive area. The function of this form ensures avoidance of collision between AUVs.

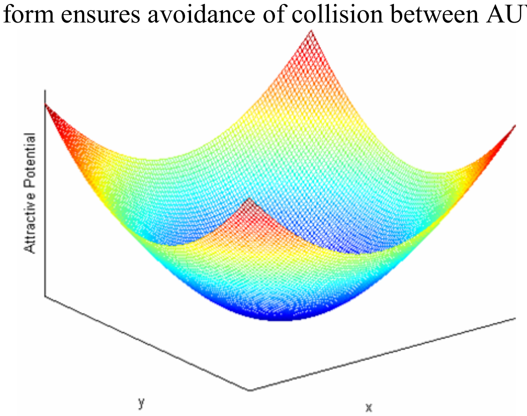

Figure 1. The contour of attractive potential.

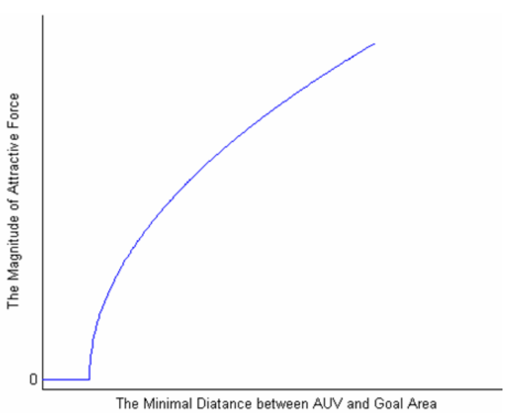

Figure 2. The component of the attractive force versus the minimal distance between AUV and the goal area.

Fig. 2 shows the magnitude of the attractive force. Note that the component of attractive force is depended on entirely the relative distance between AUV and the goal area, rather than their absolute positions vector. This allows us to compute the force directly from sensor data.

B. The Repulsive Potential Field

In this subsection, we presented the problem of the obstacle avoidance. The repulsive potential field is constructed between AUVs and obstacles. The most commonly used repulsive potential function has an inherent limitation which is usually called local minima problem.

To overcome this local minima problem, we introduce a harmonic function to build repulsive potential field in the underwater environment with obstacles. Harmonic function has some useful properties [10,11] which includes the following: 1) the principle of superposition; 2) Mean-value property; 3) the minimum principle. The above properties of harmonic function ensure that the repulsive potential function derived from harmonic function has only one global minimum. A typical example of harmonic function is

$$f(x)=\frac{1}{x} \quad (3)$$

Analogous to M_g, we define M_o as the minimal distance between AUV and the obstacle and it can be measured by sensor in real time. By additive and multiple transformations, one gets the repulsive potential in the following form

$$U_{rep}=\begin{cases}\frac{1}{2}k_r\left(\frac{1}{M_o}-\frac{1}{R}\right) & 0<M_o\le R\\ 0 & M_o>R\end{cases} \quad (4)$$

where k_r is a scalar control gain, R is a scalar constant denoting the distance of influence of the obstacle. This function can be viewed as the potential profile due to some charge distribution. According to the principle of electromagnetic, the repulsive force can be derived from (3)

$$\mathbf{F}_{rep}=\begin{cases}\frac{1}{2}k_r\frac{1}{M_o^{2}}\nabla M_o & 0<M_o\le R\\ 0 & M_o>R\end{cases} \quad (5)$$

Fig. 3 shows the repulsive potential field generated by a simple environment with only one obstacle. The repulsive potential derived from harmonic function ensures that there is only one minimum in the environment with obstacles clustered. And relevant repulsive force is shown in Fig. 4 in which the minus below the zero denotes the repulsive force.

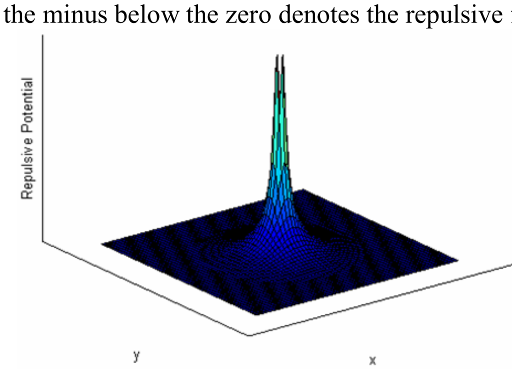

Figure 3. The contour of repulsive potential.

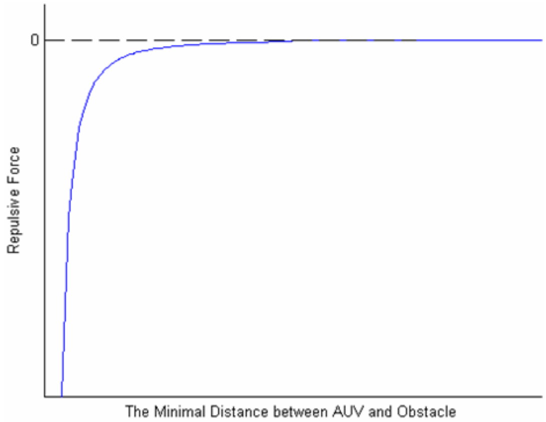

Figure 4. The component of repulsive force versus the minimal distance between AUV and obstacle

C. *The Interactive Potential Field*

To keep formation of AUVs in desired geometry and avoid collisions between them, we introduce interactive potential field which is constructed between adjacent AUVs. The corresponding interactive potential is the function of distance r_{ij} between neighboring AUVs. The function adopted is given by

$$U_I = \begin{cases} k_I(\ln(r_{ij}) + \dfrac{d_0}{r_{ij}}) + cr_{ij} & 0 < r_{ij} < d_1 \\ k_I(\ln(d_1) + \dfrac{d_0}{d_1}) + cd_1 & r_{ij} \geq d_1 \end{cases} \quad (6)$$

where r_{ij}=||$\mathbf{r}_{ij}$||=||$\mathbf{r}_i$-$\mathbf{r}_j$|| is the distance between AUV i and its neighbor AUV j, k_I is a scaling factor and denotes the strength of the potential, d_0 is constant which denotes the critical point between attraction and repulsion, d_1 is the distance of limit of interaction, namely when distance between AUVs exceed d_1, relevant interactive force becomes zero. Fig. 5 illustrates the contour of interactive potential.

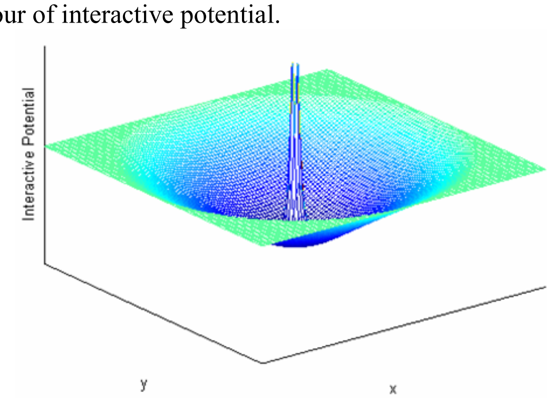

Figure 5. The interactive potential

Using (6), interactive force can be gained and the component of force takes the following form

$$F_I = \begin{cases} k_I(\dfrac{1}{r_{ij}} - \dfrac{d_0}{r_{ij}^2}) + c & 0 < r_{ij} < d_1 \\ 0 & r_{ij} \geq d_1 \end{cases} \quad (7)$$

which acts along a line connecting AUV i and AUV j. The relation between the interactive force and the distance between AUVs is illustrated in Figure 6 in which the plus quantity denotes the attraction and the minus quantity is repulsion.

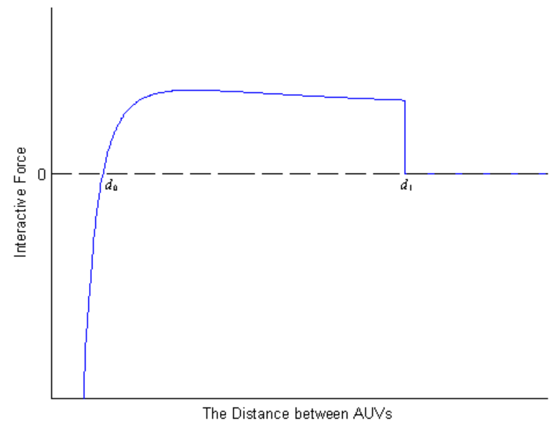

Figure 6. The interactive force versus the distance between AUVs.

III. The Extended Formation with Artificial Potential Field Constructed

In this section, we extend the concepts developed in the previous section and propose the definition of extended formation which combines the destination and obstacles in the ocean environment as elements into the traditional formation of AUVs and then constructs the artificial potentials fields. Three types of potentials are used in the extended formation: attractive potential field between AUVs and the destination, repulsive potential field between AUVs and obstacles, and the interactive potential field between AUVs. Specifically, an extended formation consisting of three AUVs is illustrated by Fig.7.

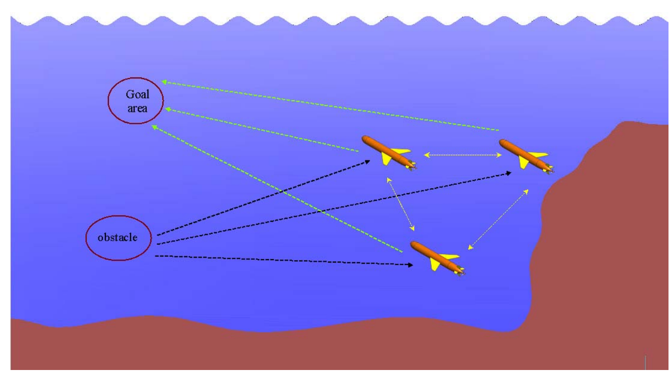

Figure 7. The extended formation with improved potential fields.

The formation of AUVs with artificial potential fields built can be viewed as a whole which can be analyzed by using multi-body theory. In the system, AUVs are interconnected in a configuration by forces derived from the artificial potentials. The configuration of AUV can be completely described by using six independent coordinates: three coordinates describing the location of the origin of the body axes and three coordinates describing the orientation of the body with respect to the inertial frame. In the extended formation, AUVs interact via interactive force $\mathbf{F}_I$ which is derived from interactive potential U_I. Considering the characteristics of interactive potential, the interactive force F_I between adjacent AUVs can be thought of as the force that arises because of the existence of a similar-spring-actuator element (Fig.8). In addition, attractive force $\mathbf{F}_{att}$ and repulsive force $\mathbf{F}_{rep}$ which is derived from the destination and the obstacles, respectively, can be regarded as external constraints.

Fig. 8 shows a system of two AUVs, AUV i and j, where the forces $\mathbf{F}^k_{att}$ and $\mathbf{F}^k_{rep}$ acting on the AUV k ($k = i, j$) are, respectively, defined by (2) and (5) whose components in terms of inertial frame are

$$\mathbf{F}^k_{att} = [F^k_{att1}, F^k_{att2}, F^k_{att3}]^T$$
$$\mathbf{F}^k_{rep} = [F^k_{rep1}, F^k_{rep2}, F^k_{rep3}]^T$$

In addition, the two AUVs are connected by a similar-spring-actuator element. The force that arises because of the existence of the similar-spring-actuator element is F_I which is defined by (7). And its direction must be directed along the straight line jointing AUVs i and j.

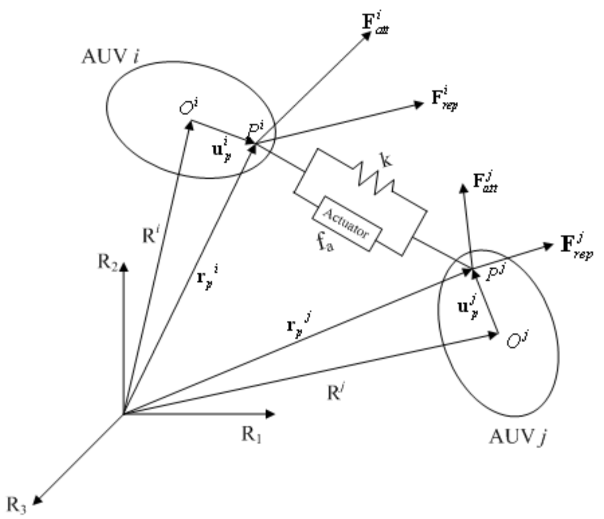

Figure 8. Similar-spring-actuator model.

In the spatial analysis we select the generalized coordinates of this system and they are given by [12]

$$\mathbf{q}^i_r = [R^i_1 \quad R^i_2 \quad R^i_3 \quad \phi^i \quad \theta^i \quad \psi^i]^T$$
$$\mathbf{q}^i_r = [R^j_1 \quad R^j_2 \quad R^j_3 \quad \phi^j \quad \theta^j \quad \psi^j]^T$$

which in vector notation is

$$\mathbf{q}^i_r = [\mathbf{R}^{i^T} \quad \boldsymbol{\theta}^{i^T}]^T$$
$$\mathbf{q}^j_r = [\mathbf{R}^{j^T} \quad \boldsymbol{\theta}^{j^T}]^T$$

where $\mathbf{R}^i$ and $\boldsymbol{\theta}^i$ are given by

$$\mathbf{R}^i = [R^i_1, R^i_2, R^i_3]^T$$
$$\boldsymbol{\theta}^i = [\phi^i, \theta^i, \psi^i]^T$$

The global position of a point P^i on AUV i can be written as

$$\mathbf{r}^i_p = \mathbf{R}^i + \mathbf{A}^i \mathbf{u}^i \tag{8}$$

where $\mathbf{R}^i$ is the position of the origin of the body axes with respect to the fixed reference frame, $\mathbf{u}^i$ is the local position of the point with respect to the body reference and this quantity can be gained by the sensor payload, $\mathbf{A}^i$ is the transformation matrix from the body coordinate system of AUV i to the global coordinate system and can be written in terms of Euler angles ϕ, θ, ψ as

$$\mathbf{A}^i = \begin{bmatrix} C_\psi C_\phi - C_\theta S_\phi S_\psi & -S_\psi C_\phi - C_\theta S_\phi C_\psi & S_\theta S_\phi \\ C_\psi S_\phi + C_\theta C_\phi S_\psi & -S_\psi S_\phi + C_\theta C_\phi C_\psi & -S_\theta C_\phi \\ S_\theta S_\psi & S_\theta C_\psi & C_\theta \end{bmatrix}^i \tag{9}$$

where S and C denote sine function and cosine function, respectively.

For AUV k (k = i, j), the sum of $\mathbf{F}^k_{rep}$ and $\mathbf{F}^k_{rep}$ can be expressed as

$$\mathbf{F}^k = \mathbf{F}^k_{att} + \mathbf{F}^k_{rep} \tag{10}$$

which in vector notation is

$$\begin{aligned} \mathbf{F}^k &= [F^k, F^k, F^k]^T \\ &= [F^k_{att1} + F^k_{rep1}, \quad F^k_{att2} + F^k_{rep2} \quad F^k_{att3} + F^k_{rep3}]^T \end{aligned}$$

The virtual change in $\mathbf{r}^i_p$ can be expressed as

$$\delta\mathbf{r}^i_p = \delta\mathbf{R}^i + \mathbf{B}^i \delta\boldsymbol{\theta}^i \tag{11}$$

where $\mathbf{B}^i$ is the partial derivative of $\mathbf{A}^i\mathbf{u}^i$ with respect to the rotational coordinates $\boldsymbol{\theta}^i$ of the body i.

$\delta\mathbf{r}^i_p$ can be written in a partitioned form as

$$\delta\mathbf{r}^i_p = \begin{bmatrix} \mathbf{I}_3 & \mathbf{B}^i \end{bmatrix} \begin{bmatrix} \delta\mathbf{R}^i \\ \delta\boldsymbol{\theta}^i \end{bmatrix} \tag{12}$$

The virtual work δW^i due to the force $\mathbf{F}^i$ acting on the AUV i is given by

$$\begin{aligned} \delta W^i &= \mathbf{F}^{i^T} \delta\mathbf{r}^i_p \\ &= \mathbf{F}^{i^T} \begin{bmatrix} \mathbf{I}_3 & \mathbf{B}^i \end{bmatrix} \begin{bmatrix} \delta\mathbf{R}^i \\ \delta\boldsymbol{\theta}^i \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{F}^{i^T} & \mathbf{F}^{i^T}\mathbf{B}^i \end{bmatrix} \begin{bmatrix} \delta\mathbf{R}^i \\ \delta\boldsymbol{\theta}^i \end{bmatrix} \end{aligned} \tag{13}$$

Similarly, the virtual work δW^j due to the force $\mathbf{F}^j$ acting on the AUV j is:

$$\begin{aligned} \delta W^j &= \mathbf{F}^{j^T} \delta\mathbf{r}^j_p \\ &= \mathbf{F}^{j^T} \begin{bmatrix} \mathbf{I}_3 & \mathbf{B}^j \end{bmatrix} \begin{bmatrix} \delta\mathbf{R}^j \\ \delta\boldsymbol{\theta}^j \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{F}^{j^T} & \mathbf{F}^{j^T}\mathbf{B}^j \end{bmatrix} \begin{bmatrix} \delta\mathbf{R}^j \\ \delta\boldsymbol{\theta}^j \end{bmatrix} \end{aligned} \tag{14}$$

It follows that force acting on a point P^i is equivalent to a force acting at the origin of the body reference frame and moment acting on this body.

In a similar way, we discuss the contribution of the interactive force to generalized forces. d_0 and r_{ij} in (7) corresponds to the undeformed length of the spring and the spring length, respectively. Let L denote the length of spring for convenience. Realizing that the spring force acts in a direction opposite to the direction of the increase in length, one may write the virtual work of the force F_I as

$$\delta W = -F_I \delta L \tag{15}$$

where δL is the virtual change in the spring length. Denoting the vector $\mathbf{r}_{ij}$ (the vector P^iP^j in Fig. 8) as $\mathbf{L}_s$ whose components are

$$\mathbf{L}_s = [l_1, l_2, l_3]^T$$

which can be evaluated from the relation

$$\mathbf{L}_s = \mathbf{r}_p^i - \mathbf{r}_p^j = \mathbf{R}^i + \mathbf{A}^i\mathbf{u}^i - \mathbf{R}^j - \mathbf{A}^j\mathbf{u}^j \tag{16}$$

The spring length L is

$$L = (\mathbf{L}_s^T\mathbf{L}_s)^{1/2} = [(l_1)^2 + (l_2)^2 + (l_3)^2]^{1/2} \tag{17}$$

Hence, the virtual change in the length δL can be written as

$$\begin{aligned}\delta L &= \frac{\partial L}{\partial l_1}\delta l_1 + \frac{\partial L}{\partial l_2}\delta l_2 + \frac{\partial L}{\partial l_3}\delta l_3 \\ &= \frac{1}{L}(l_1\delta l_1 + l_2\delta l_2 + l_3\delta l_3)\end{aligned} \tag{18}$$

which in vector notation can be written as

$$\delta L = \frac{1}{L}\mathbf{L}_s^T\delta\mathbf{L}_s \tag{19}$$

where $\delta\mathbf{L}_s$ is the virtual change of (16) and can be written as

$$\begin{aligned}\delta\mathbf{L}_s &= \delta\mathbf{R}^i + \mathbf{B}^i\delta\boldsymbol{\theta}^i - \delta\mathbf{R}^j - \mathbf{B}^j\delta\boldsymbol{\theta}^j \\ &= \begin{bmatrix}\mathbf{I}_3 & \mathbf{B}^i\end{bmatrix}\begin{bmatrix}\delta\mathbf{R}^i \\ \delta\boldsymbol{\theta}^i\end{bmatrix} - \begin{bmatrix}\mathbf{I}_3 & \mathbf{B}^j\end{bmatrix}\begin{bmatrix}\delta\mathbf{R}^j \\ \delta\boldsymbol{\theta}^j\end{bmatrix}\end{aligned} \tag{20}$$

Substitution of (19) into (15) yields

$$\begin{aligned}\delta W &= -F_I\delta L = -F_I\frac{1}{L}\mathbf{L}_s^T\delta\mathbf{L}_s \\ &= -F_I\frac{1}{L}\mathbf{L}_s^T\begin{bmatrix}\mathbf{I}_3 & \mathbf{B}^i\end{bmatrix}\begin{bmatrix}\delta\mathbf{R}^i \\ \delta\boldsymbol{\theta}^i\end{bmatrix} + F_I\frac{1}{L}\mathbf{L}_s^T\begin{bmatrix}\mathbf{I}_3 & \mathbf{B}^j\end{bmatrix}\begin{bmatrix}\delta\mathbf{R}^j \\ \delta\boldsymbol{\theta}^j\end{bmatrix} \\ &= \begin{bmatrix}-F_I\frac{1}{L}\mathbf{L}_s^T & -F_I\frac{1}{L}\mathbf{L}_s^T\mathbf{B}^i\end{bmatrix}\begin{bmatrix}\delta\mathbf{R}^i \\ \delta\boldsymbol{\theta}^i\end{bmatrix} \\ &\quad + \begin{bmatrix}F_I\frac{1}{L}\mathbf{L}_s^T & F_I\frac{1}{L}\mathbf{L}_s^T\mathbf{B}^j\end{bmatrix}\begin{bmatrix}\delta\mathbf{R}^j \\ \delta\boldsymbol{\theta}^j\end{bmatrix}\end{aligned} \tag{21}$$

It follows that the contribution of the interactive force to the generalized force depends on generalized derivative of the vector $\mathbf{L}_s$ which is the relative position between AUV i and AUV j.

In view of (13) and (21), one gets vectors of generalized force of AUV i associated with the generalized coordinates $\mathbf{R}^i$ and $\boldsymbol{\theta}^i$

$$\mathbf{G}_R^i = \mathbf{F}^i - F_I\frac{1}{L}\mathbf{L}_s \tag{22}$$

$$\mathbf{G}_\theta^i = \mathbf{B}^{i^T}\mathbf{F}^i - F_I\frac{1}{L}\mathbf{B}^{i^T}\mathbf{L}_s \tag{23}$$

In the similar way, using (14) and (21), the vectors of generalized force of AUV j associated with the generalized coordinates $\mathbf{R}^j$ and $\boldsymbol{\theta}^j$ is given by

$$\mathbf{G}_R^j = \mathbf{F}^j + F_I\frac{1}{L}\mathbf{L}_s \tag{24}$$

$$\mathbf{G}_\theta^j = \mathbf{B}^{j^T}\mathbf{F}^j + F_I\frac{1}{L}\mathbf{B}^{j^T}\mathbf{L}_s \tag{25}$$

IV. SIMULATION

According to characteristics of the specific formation in the ocean observation, a large number of simulations are conducted by using the ADAMS. In order to achieve asymptotic stability of the system, an additive term $\mathbf{f}_v$ is added to control law of the individual AUV. The additive term is designed to be zero when the AUV is moving at a desired velocity v_d and takes the following form

$$\mathbf{f}_v = \frac{k_d(v_i - v_d)}{v_i}\mathbf{v}_i \tag{26}$$

Fig. 9 shows the configuration of triangle formation consisting of three AUVs. The bigger green body denotes the obstacle in the environment. The temporal behavior of the formation is captured, which show plots of changes of trajectory of motion with time. The subplot (a) and (f) show the initial and final configuration of the AUV formation. Meanwhile, the trajectory of motion is always visible for the visual effects.

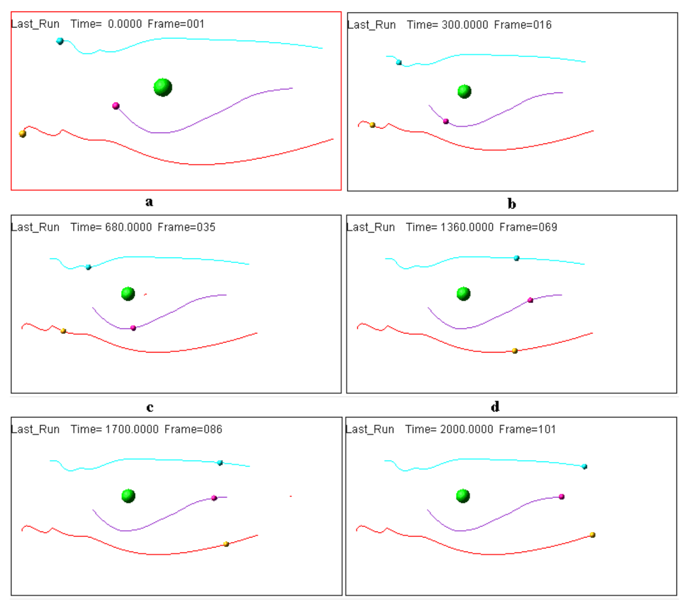

Figure 9. Formation of three AUVs rounded an obstacle.

V. CONCLUSIONS AND FUTURE WORK

In this paper we introduce an extended formation which is derived by combining the destination and obstacles in the underwater environment as elements into the traditional formation of AUVs. In extended formation, artificial potential fields are built to coordinate a group of AUVs. We have presented an improved construction of artificial potential functions which is prone to implementation. Three types of potentials are constructed in the extended formation: attractive potential, repulsive potential and the interactive potential. The performances of the extended formation as a whole are investigated with multi-body theory. The force derived from the interactive potential can be thought of as one that arises due to the existence of the similar-spring-actuator element. The forces derived from the attractive potential and repulsive potential can be interpreted as the external constraints. Meanwhile, we analyze the contribution of the forces derived from the potentials to the generalized force.

Simulation results have verified that the improved artificial potential fields can implement the coordination of multi-AUV. There are many directions which deserve attention, such as the influence of the control parameter on performance, and the coupling of this work with the orientation potential function. We plan to investigate and characterize these two problems in the future.

ACKNOWLEDGMENT

This work is supported by Key Program of National Natural Science Foundation of China (No.50835006), Science & Technology Support Planning Foundation of Tianjin (No.09ZCKFGX03000), and Natural Science Foundation of Tianjin (No.9JC2DJC23400). The authors appreciate all staff members in this research and development team.

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