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%
% Implements Monte Carlo ES (exploring starts) with first visit estimation to
% compute the action-value function for the black jack example.
%
% Written by:
% --
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%
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%
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% above email address.
%
%-----

close all;
clc;

N_HANDS_TO_PLAY=10;     % a numerical approximation of +Inf
N_HANDS_TO_PLAY=2*1e4;
N_HANDS_TO_PLAY=5e5;
N_HANDS_TO_PLAY=5e6;
%N_HANDS_TO_PLAY=1e7;

rand('seed',0); randn('seed',0);

%--
% implement hands of bj
%--

nStates       = prod([21-12+1,13,2]);
posHandSums   = 12:21;
nActions      = 2; % 0=>stick; 1=>hit
Q             = zeros(nStates,nActions);  % the initial action-value function
%pol_pi        = zeros(1,nStates);         % our initial policy is to always stick "0"
pol_pi        = ones(1,nStates);         % our initial policy is to always hit "1"
%pol_pi        = unidrnd(2,1,nStates)-1;   % our initial policy is random
firstSARewSum = zeros(nStates,nActions);
firstSARewCnt = zeros(nStates,nActions);

tic
for hi=1:N_HANDS_TO_PLAY,

  stateseen = [];
  deck = shufflecards();

  % the player gets the first two cards:
  p = deck(1:2); deck = deck(3:end); phv = handValue(p);
  % the dealer gets the next two cards (and shows his first card):
  d = deck(1:2); deck = deck(3:end); dhv = handValue(d); cardShowing = d(1);

  % disgard states who's initial sum is less than 12 (the decision is always to hit):
  while( phv < 12 )
    p = [ p, deck(1) ]; deck = deck(2:end); phv = handValue(p); % HIT
  end

  % accumulate/store the first state seen:
  stateseen(1,:) = stateFromHand( p, cardShowing );
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  % implement the policy specified by pol_pi (keep hitting till we should "stick"):
  si = 1;
  polInd         = sub2ind( [21-12+1,13,2], stateseen(si,1)-12+1, stateseen(si,2),
stateseen(si,3)+1 );
  pol_pi(polInd) = unidrnd(2)-1;      % FOR EXPLORING STARTS TAKE AN INITIAL RANDOM
POLICY!!!
  pol_to_take    = pol_pi(polInd);
  while( pol_to_take && (phv < 22) )
    p = [ p, deck(1) ]; deck = deck(2:end); phv = handValue(p); % HIT
    stateseen(end+1,:) = stateFromHand( p, cardShowing );

    if( phv <= 21 ) % only then do we need to querry the next policy action when we have
not gone bust
      si = si+1;
      %[ stateseen(si,1), stateseen(si,2), stateseen(si,3) ]
      polInd      = sub2ind( [21-12+1,13,2], stateseen(si,1)-12+1, stateseen(si,2),
stateseen(si,3)+1 );
      pol_to_take = pol_pi(polInd);
    end
  end
  % implement the fixed deterministic policy of the dealer (hit until we have a hand
value of 17):
  while( dhv < 17 )
    d = [ d, deck(1) ]; deck = deck(2:end); dhv = handValue(d); % HIT
  end
  % determine the reward for playing this game:
  rew = determineReward(phv,dhv);
  %fprintf( '[phv, dhv, rew] = \n' ); [ phv, dhv, rew ]

  % accumulate these values used in computing statistics on this action value function
Q^{\pi}:
  for si=1:size(stateseen,1),
    if( (stateseen(si,1)>=12) && (stateseen(si,1)<=21) ) % we don't count "initial" and
terminal states
      %[stateseen(si,1)]
      %[stateseen(si,1)-12+1, stateseen(si,2), stateseen(si,3)+1]
      staInd = sub2ind( [21-12+1,13,2], stateseen(si,1)-12+1, stateseen(si,2),
stateseen(si,3)+1 );
      actInd = pol_pi(staInd)+1;
      firstSARewCnt(staInd,actInd) = firstSARewCnt(staInd,actInd)+1;
      firstSARewSum(staInd,actInd) = firstSARewSum(staInd,actInd)+rew;
      Q(staInd,actInd)             =
firstSARewSum(staInd,actInd)/firstSARewCnt(staInd,actInd); % <-take the average
      [dum,greedyChoice]           = max( Q(staInd,:) );
      pol_pi(staInd)               = greedyChoice-1;
    end
  end
end % end number of hands loop
toc

% plot the optimal state-value function V^{*}:
%
mc_value_fn = max( Q, [], 2 );
mc_value_fn = reshape( mc_value_fn, [21-12+1,13,2]);
if( 1 )
  figure; mesh( 1:13, 12:21, mc_value_fn(:,:,1) );
  xlabel( 'dealer shows' ); ylabel( 'sum of cards in hand' ); axis xy; %view([67,5]);
  title( 'no usable ace' ); drawnow;
  fn=sprintf('state_value_fn_nua_%d_mesh.eps',N_HANDS_TO_PLAY); saveas( gcf, fn, 'eps2'
);
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  figure; mesh( 1:13, 12:21,  mc_value_fn(:,:,2) );
  xlabel( 'dealer shows' ); ylabel( 'sum of cards in hand' ); axis xy; %view([67,5]);
  title( 'a usable ace' ); drawnow;
  fn=sprintf('state_value_fn_ua_%d_mesh.eps',N_HANDS_TO_PLAY); saveas( gcf, fn, 'eps2' );

  figure;imagesc( 1:13, 12:21, mc_value_fn(:,:,1) ); caxis( [-1,+1] ); colorbar;
  xlabel( 'dealer shows' ); ylabel( 'sum of cards in hand' ); axis xy;
  title( 'no usable ace' ); drawnow;
  fn=sprintf('state_value_fn_nua_%d_img.eps',N_HANDS_TO_PLAY); saveas( gcf, fn, 'eps2' );
  figure;imagesc( 1:13, 12:21, mc_value_fn(:,:,2) ); caxis( [-1,+1] ); colorbar;
  xlabel( 'dealer shows' ); ylabel( 'sum of cards in hand' ); axis xy;
  title( 'a usable ace' ); drawnow;
  fn=sprintf('state_value_fn_ua_%d_img.eps',N_HANDS_TO_PLAY); saveas( gcf, fn, 'eps2' );
end

% plot the optimal policy:
%
pol_pi = reshape( pol_pi, [21-12+1,13,2] );
if( 1 )
  figure; imagesc( 1:13, 12:21, pol_pi(:,:,1) ); colorbar;
  xlabel( 'dealer shows' ); ylabel( 'sum of cards in hand' ); axis xy; %view([67,5]);
  title( 'no usable ace' ); drawnow;
  fn=sprintf('bj_opt_pol_nua_%d_image.eps',N_HANDS_TO_PLAY); saveas( gcf, fn, 'eps2' );
  figure; imagesc( 1:13, 12:21,  pol_pi(:,:,2) ); colorbar;
  xlabel( 'dealer shows' ); ylabel( 'sum of cards in hand' ); axis xy; %view([67,5]);
  title( 'a usable ace' ); drawnow;
  fn=sprintf('bj_opt_pol_ua_%d_mesh.eps',N_HANDS_TO_PLAY); saveas( gcf, fn, 'eps2' );
end

return;
```